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Transmission Loss Allocation via Loss Function Decomposition and Current Projection Concept

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***Abstract*—**One of the major problems in liberalized power markets is loss allocation. In this paper, a different method for allocating transmission losses to pool market participants is proposed. The proposed method is fundamentally based on decomposition of loss function and current projection concept. The method has been implemented and tested on several networks and one sample summarized in the paper. The results show that the method is comprehensive and fair to allocating the energy losses of a power market to its participants.

***Keywords*—**Transmission loss, loss allocation, current projection concept, loss function decomposition.

I. Introduction

THE electric power industry is experiencing important changes brought about by the deregulation. Electric power generators and users engage in power transactions which take place over the transmission system and create losses. Transmission losses represent up to 5-10% of the total generation, and cost millions of dollars per year. Consequently, the problem of “who should pay for losses” arises and the satisfactory distribution of the transmission system loss costs among all market participants has become a key issue. Unfortunately, losses are expressed as a nonlinear function of line flows, and it becomes almost impossible to calculate exactly the losses that are incurred by each generator, load or transaction in the system.

The difficulty presented when selecting a loss allocation method is the abesence of a standard means for comparing the different methods. Therefore, based on “fair and equitable” practice, any loss allocation algorithm shoud have most of the desirable properties stated below[1]:

- To be simple to understand and implement;
- To be consistent with power flow solution;
- To be able to promote efficient market operation, where the losses are reflected by network usage and the relative position of the bus in the network;
- To avoid volatility and provide appropriate economic signals.

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The principle difficulty in allocating losses to system agents is the fact that the transmission loss function is non-separable and nonlinear, which makes it impossible to divide the system losses into the sum of terms, each one uniquely attributable to a generation or a load [2]. Several approaches were proposed to solve this problem, but so far there is no one that really satisfies the generation and the load, i.e., there is no approach that does not involve any arbitrary action.

The most simple loss allocation procedure is the so-called pro-rata technique [3], in which the losses are allocated to the generators and loads by considering the level of active-power injection, pro-rata (P), or current injection, pro-rata (I). However, it “ignore” the network situation. In addition, it is unfair for the load located near the generating bus since it is allocated more losses. In the set of the marginal procedures, the losses are assigned to generators and loads through the so-called incremental transmission loss (ITL) coefficients [4] or by using the decomposed marginal cost [5]. The proportional sharing (PS) procedures use the results of the converged PF solution plus a linear proportional sharing principle [6-8]. Based on the assumption that the power injections are proportionally shared among the outflows of each bus, this method can determine the contribution of generators or loads to the line power flows. According to the same contribution ratio, each line loss will be allocated to each generator or load. Conejo et al. [2] proposed the Z-bus matrix in active transmission loss allocation. This method uses the current rather than power injections. Although this approach yields negative losses sometimes, only the absolute values are used, and consequently, the allocations must be normalized. In [9] the normalized “Loss Weight Factors” (LWF) is used to allocate the system losses. The LWFs are obtained from characteristics of the network, square current magnitude and the real part of the Z-bus matrix. After calculating the LWFs, these values will be normalized and total losses will be allocated to buses according to normalized LWFs.

In this paper, a different method for allocating transmission losses among loads and generators is presented. In this method, the first loss function is decomposed into two components, load losses and no load losses. Therefore by using current projection concept, share on branch current due to loads and generators are determined. Finally after obtaining the branch current contributions, allocated losses of each participant are calculated.

II. Methodology

It [10] has been represented that transmission losses can be considered as consisting of two parts. The first is due to line flows caused by load currents only when the generator voltages are all equal in both magnitude and angles, i.e. there is no circulating current flow. This part will be referred to as load losses. The second part is due to the circulating current resulting from differences in generator voltages. Being caused by mismatch in generator's voltages, this part will be referred to as no load losses.

A. Allocation of Losses to the Loads

Loss due to load currents are obtained by assuming that all generators act as ideal voltage sources with no circulating currents between them. In such a case, generation nodes are short circuited and the load nodes are considered as current sources. Therefore based on the equation proposed in [10], Y_{bus} can be written as:

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} Y_{GL} \\ Y_{LG} Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \quad (1)$$

The current injection at load bus is shown by I_L, the contribution of I_L to the generator bus voltage are zero and its contributions to the load bus voltage will be calculated according to bellow:

$$V_k^L = Y_{LL}^{-1} e I_L , \quad k = 1,...,n \quad (2)$$

Where l is number of loads and e is a $l \times 1$ dimension vector with value of 1 at position l and all the others equal 0.

The contribution of I_L to the voltage drop across branch is computed as

$$\Delta V_b^L = V_{bf}^L - V_{bt}^L \quad (3)$$

Where the subscripts bf , bt represent the "from" bus and the "to" bus of branch, respectively.

The contribution of I_L to the current through branch can be computed as

$$I_b^L = \Delta V_b^L / z_b \quad (4)$$

Where $z_b = r_b + jx_b$ is the complex serial impedance of branch b. Here the shunt elements of lines are not taken into account because we only focus on active loss allocation.

According to the superposition principle, the branch current contributions due to each current injection as follow:

$$I_b = \sum_{L=1}^{l} I_b^L \qquad , L = 1,...,l \quad (5)$$

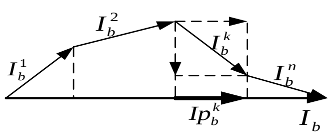

Fig. 1 projection of the branch current contributions due to current injection I_b

Since the real and imaginary components of the current are orthogonal, Kirchoff s current law applies to each of them separately. No physical device can transform a real current into an imaginary current or vice-versa. Therefore the branch current contributions due to each current injection are obtained accord with current projection concept which illustrated in Fig. 1.

As shown in Fig. 1, the branch current equals the algebraic sum of the branch current contributions due to each load current that are in its direction. In other words, branch current equals the algebraic sum of the projections of the branch current contributions due to each load current on its direction. Let Ip_b^L denote the projection vector of I_b^L in the direction of I_b, which is defined to be the current projection component of branch b produced by the current injection at load L and it is calculated as:

$$Ip_b^L = \left| I_b^L \right| \cos(\varphi_b^L - \varphi_b) e^{j\varphi_b} \quad (6)$$

Where φ_b^L , φ_b are phase angles of I_b^L, I_b ratio to angle of voltage bus reference.

After obtaining the branch current contributions due to load current injections, its allocated loss of branch b is accordingly given by the following formula:

$$\begin{aligned} L_b^L &= (V_{bf} - V_{bt}) Ip_b^{L*} \\ &= \left| I_b^L \right| \cos(\varphi_b^L - \varphi_b) e^{-j\varphi_b} I_b z_b \\ &= \left| I_b^L \right| \left| I_b \right| \cos(\varphi_b^L - \varphi_b) z_b \\ \Rightarrow P_{loss_b}^L &= \left| I_b^L \right| \left| I_b \right| \cos(\varphi_b^L - \varphi_b) r_b \end{aligned} \quad (7)$$

There, the total transmission loss allocation to injection attributed to load L is computed as follows:

$$PT_{loss}^L = \sum_{b=1}^{m} P_{loss_b}^L \quad , b = 1,...,m \quad (8)$$

B. Allocation of Losses to the Generators

Loss due to mismatch in generator's voltages are obtained by setting the load currents to zero. Setting I_L in (1) to zero, and keeping the generator voltage as obtained from the power flow solution.

The current injection at generator bus is shown by I_G, its contributions to the load bus voltage will be calculated according to bellow:

$$V_k^L = -(Y_{LL}^{-1}.Y_{LG}.Z_{GG}) e I_G \quad (9)$$

$$Z_{GG} = \left[[Y_{GG}] - [Y_{GL}][Y_{LL}]^{-1}.[Y_{LG}] \right]^{-1}$$

Also the contribution of I_G to the generator bus voltage can be computed as:

$$V_k^G = Z_{GG} e I_G \quad (10)$$

TABLE I
COMPARISON OF RESULTS OF SEVEN LOSS METHODS FOR THE BASIC 14-BUS NETWORK

Bus Num	Active power gen. Pg	Active load dem. Pd	Bus current inject. |I|	Distribution of active power losses Ploss=13.5 MW ; λ=50$/MWh						
				Proposed method	Z-bus	Pro-rata		PS	ITL	LWF
						P	I			
	MW	MW	A	$/h	$/h	$/h	$/h	$/h	$/h	$/h
1	232.7	0	1598	180	382	323	275	324	307	320
2	40	21.7	188	74	8	25	32	15	48	18
3	0	94.2	676	192	139	131	116	144	146	172
4	0	47.8	339	76	42	66	58	63	63	61
5	0	7.6	55	10	4	11	9	8	9	7
6	0	11.2	298	14	24	16	51	12	16	12
7	0	0	0	0	0	0	0	0	0	0
8	0	0.1	190	0	1	0	33	0	0	0
9	0	29.5	239	45	26	41	41	39	34	36
10	0	9	76	14	9	12	13	14	10	11
11	0	3.5	27	5	3	5	5	5	4	4
12	0	6.1	43	10	5	8	7	8	9	6
13	0	13.5	102	25	13	19	18	19	16	14
14	0	14.9	112	33	22	21	20	27	16	17
Sum	272.7	259.1	-	678	678	678	678	678	678	678

In this stage, like the previous one, Ip_b^G denote the projection vector of I_b^G in the direction of I_b, which is defined to be the current projection component of branch b produced by the current injection at generator G and it is calculated as:

$$Ip_b^G = \left| I_b^G \right| \cos(\varphi_b^G - \varphi_b) e^{j\varphi_b} \tag{11}$$

Where φ_b^G is phase angle of I_b^G, ratio to angle of voltage bus reference.

Consequently, allocated loss of branch b due to mismatch in generator's voltage derived as

$$P_{loss_b}^G = \left| I_b^G \right| \left| I_b \right| \cos(\varphi_b^G - \varphi_b) r_b \tag{12}$$

There, the total transmission loss allocation to injection attributed to generator G is computed as follows:

$$PT_{loss}^G = \sum_{b=1}^{m} P_{loss_b}^G \tag{13}$$

III. CASE STUDY

The proposed allocation method has been tested on a set of networks of varying sizes and types, and compared to some of the alternative algorithms described in the literature. In this paper one case study is summarized, namely the IEEE 14-bus network. This network also been used in [2].

The proposed method has been compared with Z-bus loss allocation [2], two Pro Rata methods [3] (one based on active power and the other on current magnitude injections), proportional sharing method [6-8], incremental transmission loss allocation method [4] and Loss Weight Factor (LWF) allocation method [9].

The dimension for evaluation of methods in this case study is chose to be "*dollar per hour*", which clearly describes the monetary impact of loss allocation and the significant differences among the various methods, and assume a system marginal price of 50$/MWh.

Table I compares the loss components allocated to each bus for different methods. Columns 5 through 11 represent the cost of the allocated bus losses for the seven different allocation methods. These are respectively: proposed method, Z-bus, *pro rata* based on active power injections (P), *and pro rata* based on current magnitude injections (I), proportional sharing (PS), incremental transmission loss method (ITL), and loss weight factor (LWF).

Also Fig. 2 shows the result of proposed method against other methods.

Because the power injection of bus 7 is zero, its share on the allocated losses equal to zero. It is clear that among generators, generator 1, gets the highest allocated, according to all methods. Similarly, among loads, all seven methods allocate the highest cost to the load at bus 3. The proposed method allocates 180$/h or 27% of the total losses to generator 1 and 192$/h or 29% of the total losses to load 3.

It is evident that other methods are available in Table I, the most present of loss allocated to generator 1 that seems unfair, but the proposed method is decreased this value to 27% that lead to "fair" results.

The proposed method may produce negative loss allocations. They could be physically understandable with the concept of counter flows and dominant flows [11]. Whether the negative loss allocations are acceptable or not, they depend on the market and its participants. If they are acceptable, they (NLA=Negative Loss Allocation) could actually send out locational signals to help reducing system

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losses. If not, several modifications could be made to remove the negatives. For examples, an unsubsidized strategy is presented in [11], where the negative allocations are shifted up by taking the smallest one as the basis.

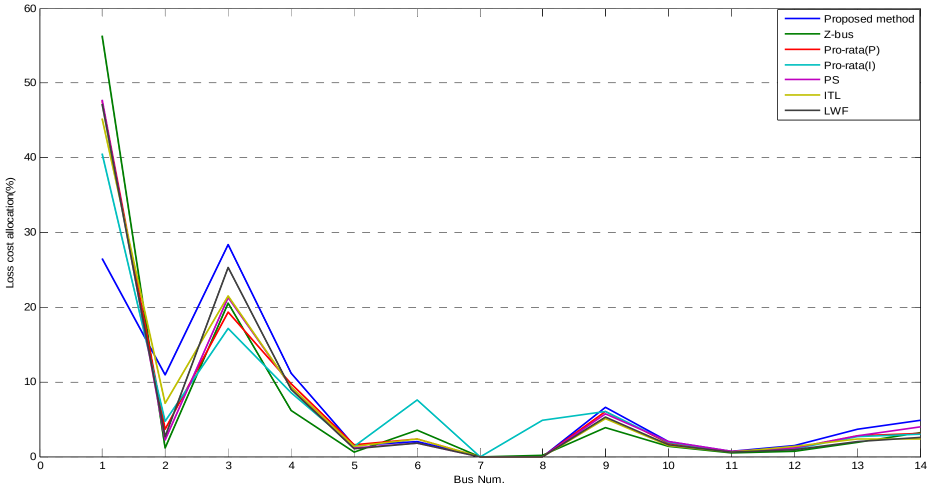

Fig. 2 Loss Allocation of different methods against proposed method

IV. Conclusion

In this paper a different method for transmission loss allocation problem has been developed. The method is based on decomposing the function transmission loss and current projection concept. In this method, the first loss function is decomposed into two components, load losses and no load losses. Therefore by using current projection concept, share on branch current due to loads and generators are determined. Finally after obtaining the branch current contributions, allocated loss of each bus is calculated. Numerical results and comparisons have demonstrated that the proposed method has a good performance, and it is consistent from the point of view of *fairness* and *transparency*. The ideas developed in this paper can be extended for a future development of a framework for cost allocation in a multi-owned transmission system.

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