# Nonlinear Thermal Hydraulic Model to Analyze Parallel Channel Density Wave Instabilities in Natural Circulation Boiling Water Reactor with Asymmetric Power Distribution

Sachin Kumar, Vivek Tiwari and Goutam Dutta

Abstract—The paper investigates parallel channel instabilities of natural circulation boiling water reactor. A thermal-hydraulic model is developed to simulate two-phase flow behavior in the natural circulation boiling water reactor (NCBWR) with the incorporation of ex-core components and recirculation loop such as steam separator, down-comer, lower-horizontal section and upper-horizontal section and then, numerical analysis is carried out for parallel channel instabilities of the reactor undergoing both in-phase and out-of-phase modes of oscillations. To analyze the relative effect on stability of the reactor due to inclusion of various ex-core components and recirculation loop, marginal stable point is obtained at a particular inlet enthalpy of the reactor core without the inclusion of ex-core components and recirculation loop and then with the inclusion of the same. Numerical simulations are also conducted to determine the relative dominance between two modes of oscillations i.e. in-phase and out-of-phase. Simulations are also carried out when the channels are subjected to asymmetric power distribution keeping the inlet enthalpy same.

*Keywords*—Asymmetric power distribution, Density wave oscillations, In-phase and out-of-phase modes of instabilities, Natural circulation boiling water reactor

### I. INTRODUCTION

**N**ONLINEAR analysis on density wave oscillations (DWOs) in boiling water reactors (BWRs) is an active research area for last several decades (evident from reviews conducted by Boure et al. [1], Leuba and Rey [2], D'Auria [3], Prasad et al. [4] and Nayak and Vijayan [5]). A lot of investigations have been carried out to determine the physical mechanisms of the instabilities [2], [6], [7]) and to obtain the marginal stability boundaries (MSBs) of the BWR [8], [9],

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[10], [11], [12], [13] and many renown researchers in order to predict its stability threshold in operational regime. The various linear and nonlinear numerical models, which are available in literature and used to analyze DWOs, are mainly based on the assumption of incompressible flow field in the two-phase flow region in the reactor core and provided a deep insight in determining the underlying mechanisms of DWOs and predicting MSB of BWR. DWOs, though can enhance or suppress the instability of the reactor under the influence of neutron dynamics via void-Doppler reactivity feedback effects (observed by Hennig [14], Ikeda et al. [15], [16] and Dutta and Doshi [17]), are mainly caused by thermal-hydraulics (TH) flow feedbacks as concluded in [1], [2], [8], [9], [10], [12], [13]. Lin et al. [18] analyzed a natural convection NC BWR considering a single channel system with a constant pressure drop BC and predicted type-I and type-II instabilities. Leuba et al. [2], [19] qualitatively explained the instability mechanisms of in-phase and out-of-phase modes of oscillations in a BWR and emphasized on the BCs practically observed at different modes of instabilities. Aritomi et al. [20] and Munoz-Cobe et al. [21], [22] extended the analysis from a single channel to multiple channels system and predicted out-of-phase modes of oscillations. Investigations carried out by the above 2 mentioned researchers demonstrated that the reactors can undergo in-phase or out-of-phase modes of oscillations purely because of thermal excitations, i.e., even in the absence of neutronic feedbacks, at an operating condition susceptible to DWOs. Lee and Pan [23] analyzed both in-phase and out-ofphase modes of oscillations in an NCBWR loop considering double parallel channels in the reactor core with equal and unequal heating. A TH numerical solver, which was earlier developed by Dutta and Doshi [13] adopting a methodology proposed by Hancox and Banerjee [24], [25] to analyze inphase modes of oscillations for FC and NC BWRs with a single channel model considering the compressibility effect of flow dynamics in the reactor core, is extended at present to investigate multiple channels existing in the NCBWR core with a objective to simulate both in-phase and out-of-phase modes of instabilities. The TH model presently takes into account the connectivity of the reactor core with various excore and recirculation loop components by incorporating the model of various ex-core and recirculation components of the reactor loop such as steam separator, down-comer, lower

horizontal section and upper horizontal section including. Extensive numerical experiments are performed to determine the effects of inclusion of various ex-core components. To analyze the relative effect on stability of the reactor due to inclusion of various ex-core components and recirculation loop, marginal stable point is obtained at a particular inlet enthalpy of the reactor core without the inclusion of ex-core components and recirculation loop and then with the inclusion of the same. Numerical simulations are also conducted to determine the relative dominance between two modes of oscillations i.e. in-phase and out-of-phase. Simulations are also carried out when the channels are subjected to asymmetric power distribution keeping the inlet enthalpy same.

# II. THERMAL-HYDRAULICS MODEL FORMULATION

The TH model is developed by assuming that there is homogeneous two-phase flow. It considers mass, momentum and energy conservation equations and solves them together in time-domain considering the compressibility effect of twophase flow dynamics in the core. The TH solver also models the steam separator, down-comer, lower horizontal section and upper horizontal section to take into account the effect of excore and recirculation loop components along with the reactor core.

# A. Mathematical model to simulate transients in the reactor core

The fundamental one-dimensional governing equations used to simulate two-phase flow dynamics in the reactor core are as follows:

Mass Conservation Equation:

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial t}(\rho A u) = 0 \tag{1}$$

Momentum Conservation Equation:

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial t}(\rho A u^2) = -A\frac{\partial p}{\partial z} - \tau_{w} - \rho Ag\frac{dH}{dz}$$
(2)

Energy Conservation Equation:

$$\frac{\partial}{\partial t}(\rho A e) + \frac{\partial}{\partial t}(\rho A u e_f) = q_w^{"} P_H$$
(3)

Where 
$$e = e_f - \frac{p}{\rho}$$
 and  $e_f = h + \frac{u^2}{2} + gH$ 

These equations can be written in compact form as follows:

$$\frac{\partial}{\partial t}[\underline{R}] + \frac{\partial}{\partial z}[\underline{S}] = [\underline{T}]$$
(4)
Where

Where,

$$\underline{R} = \begin{pmatrix} \rho A \\ \rho A u \\ \rho A e \end{pmatrix}, \underline{S} = \begin{pmatrix} \rho A u \\ A(\rho u^{2} + p) \\ \rho A u e_{f} \end{pmatrix}$$
$$, \underline{T} = \begin{pmatrix} 0 \\ p \frac{\partial A}{\partial z} - \tau_{w} P_{w} - \rho A g \frac{dH}{dz} \\ q_{w}^{"} P_{H} \end{pmatrix}$$

These conservative form of governing equations are first converted into the following primitive form:

$$\frac{\partial}{\partial t}[\underline{U}] + \underline{\underline{A}}(\underline{U})\frac{\partial}{\partial z}[\underline{U}] = [\underline{D}(\underline{U})]$$
(5)

where  $\underline{U}$  is a vector of unknown dependent variables

 $[W, h, p]^T$ ,  $\underline{\underline{A}}$  is a square matrix of coefficients which are functions of  $\underline{U}$ , and  $\underline{D}$  is a vector containing allowances for mass, momentum, and energy transfer across the system boundaries and between phases.

The eigen values of matrix  $\underline{\underline{A}}$  determine the mathematical class of Eqs. (5) and it is found that all eigen values of  $\underline{\underline{A}}$  are real (u, u + a, u - a); and hence they are classified as hyperbolic equations. Next, the set of Eq. (5) are transformed into a characteristic form and it can be written as:

$$\underline{\underline{B}}\frac{\partial}{\partial t}[\underline{U}] + \underline{\underline{A}}\underline{\underline{B}}\frac{\partial}{\partial z}[\underline{U}] = [\underline{C}]$$
(6)

Where  $\underline{\Lambda}$  is a diagonal matrix of eigenvalues of  $\underline{A}$ . After coefficient and source term linearization, the system of Eq. (6) is discretized with a characteristics-dependent implicit finitedifference scheme where the spatial derivative terms are approximated by backward or forward difference depending on the sign of the characteristics. For the present case of subsonic flow (u < a), the spatial derivatives for first two basic equations, which are characterized by  $\Lambda_{11} = u > 0$  and  $\Lambda_{22} = u + a > 0$  respectively, are approximated by backward difference equations and the third basic equation, which is characterized by  $\Lambda_{33} = u - a < 0$ , is approximated by forward difference equation. The resultant discretized equations are then combined together and used for numerical solution depending on the boundary conditions imposed to simulate the multiple parallel channels in the reactor core when being connected with the ex-core components of the reactor loop. The procedure adopted helps in treating the boundary conditions naturally.

# B. Steady state solution of the reactor core

Steady state equations are obtained by neglecting all terms containing  $\frac{\partial}{\partial t}$  in (1), (2) and (3) and the resultant discretized equations are as follows:

$$\rho_{i+1}A_{i+1}u_{i+1} = \rho_{i}A_{i}u$$
(7a)
$$p_{i} - p_{i+1} = \frac{1}{2} \left( \frac{1}{A_{i}} + \frac{1}{A_{i+1}} \right) [(\rho A u^{2})_{i+1} - (\rho A u^{2})_{i}]$$
(7b)
$$+ \frac{1}{2} [\{\rho(F+g)\}_{i} + \{\rho(F+g)\}_{i+1}](z_{i+1} - z_{i})$$
(*e*<sub>f</sub>)<sub>i+1</sub> - (*e*<sub>f</sub>)<sub>i</sub> =  $\frac{1}{2} \left( \left( \frac{q_{w}}{\rho A u} \right)_{i} + \left( \frac{q_{w}}{\rho A u} \right)_{i+1} \right) (z_{i+1} - z_{i})$ 
(7c)

The above set of equations along with thermodynamic equations of state is solved numerically by using a shooting method along with forward marching scheme. The results obtained from the present steady state model provides the solution at initial equilibrium condition, i.e., at t = 0 of the transient problem.

C. Mathematical model to simulate ex-core components and recirculation loop dynamics

The outlets of vertical channels existing in the reactor core are connected to a upper plenum and which in turn is linked to steam separator. Upper plenum receives the total mass flow rate exiting from the channels in the form of liquid and vapor mixture and supplies to steam separator. Water and steam in upper plenum are assumed to be mixed completely and in thermal equilibrium and there is no loss of momentum. Steam separator isolates the two-phase mixture into liquid and vapor phases and supplies the separated vapor and liquid water to the steam turbine and upper horizontal section respectively. In the upper horizontal section liquid is assumed to be saturated. This saturated water gets mixed with the sub-cooled feed water in the mixing chamber and then mixed sub-cooled water goes to down-comer. The water then goes into the lower plenum by passing through lower horizontal section and by making a 180 degree turn it moves up through the channels of the reactor core where the coolant water is heated up by the nuclear fuel elements.

The following assumptions are made while formulating the ex-core components and recirculation loop:

- Sub-cooled and incompressible flow of water.
- Saturated water in the upper horizontal section.

• No exchange of energy in the upper horizontal section, down-comer and lower horizontal section.

• Time invariant coolant enthalpy in the lower plenum, i.e., at the inlet of vertical channels.

Since the flow is assumed to be incompressible in ex-core components, the relation between mass flow rate and pressure drop across upper horizontal section, down-comer and lower horizontal section can be obtained by integrating the momentum conservation equation along each of its length respectively. After integration the final form of equation for upper horizontal section can be written as:

$$\left(\frac{L}{A}\right)_{uhs}\frac{\partial W_{uhs}(t)}{\partial t} = \left[(p_1 - p_2)\right]_{t=0} \left(1 - \frac{\partial W_{uhs}^2(t)}{\partial W_{uhs}^2(t=0)}\right) (8a)$$

+[ $(p_1-p_2)-(p_1-p_2)_{t=0}$ ]

Similarly for down-comer the final form of can be written as:

$$\left(\frac{L}{A}\right)_{dc} \frac{\partial W_{dc}(t)}{\partial t} = [(p_2 - p_3) + \rho g(z_2 - z_3)]_{t=0}$$

$$\left(1 - \frac{\partial W_{dc}^2(t)}{\partial W_{dc}^2(t=0)}\right) + [(p_2 - p_3) - (p_2 - p_3)_{t=0}]$$
(8b)

And similarly for lower horizontal section the final form of equation can be written as:

$$\left(\frac{L}{A}\right)_{lhs} \frac{\partial W_{lhs}(t)}{\partial t} = [(p_3 - p_4)]_{t=0} \left(1 - \frac{\partial W_{lhs}^2(t)}{\partial W_{lhs}^2(t=0)}\right) (8c) + [(p_3 - p_4) - (p_3 - p_4)_{t=0}]$$

Where  $p_I$  is the exit pressure of the channels or pressure of upper plenum and it is constant with respect to time. Since it is assumed that there is no momentum loss between upper plenum and steam separator, steam separator has also pressure  $p_I$  and hence the end of the upper horizontal section which is connected with steam separator has also pressure  $p_I$ .

Similarly  $p_2$  is the pressure at other end of the upper horizontal section and one end of the down-comer which is connected with the mixing chamber as no momentum loss is assumed between the upper horizontal section, mixing chamber and down-comer.

 $P_3$  is the pressure at the junction where down-comer and lower horizontal section are connected and  $p_4$  is the pressure of lower plenum and hence it is the inlet pressure of the channels which is the function of time.

Since the upper horizontal section receives only the saturated water, therefore

$$W_{uhs} = (1 - x_{ex})W_{ex} \tag{9}$$

Where 
$$W_{ex} = \sum_{i=1}^{n} (W_{ex})_i$$
 and  $x_{ex} = \frac{\sum_{i=1}^{n} (W_{ex})_i (x_{ex})_i}{\sum_{i=1}^{n} (W_{ex})_i}$ 

Where n is the number of parallel channels.

From the mixing chamber the down-comer receives the total mass flow rate which is sent to the channels by passing through lower horizontal section and lower plenum respectively. Therefore,

$$W_{dc} = W_{lhs} = W_{in}(t) \tag{10}$$

Now the equations (8a), (8b) and (8c) are combined in one equation and can be written as:

$$\left(\frac{L}{A}\right)_{uhs} \frac{\partial W_{uhs}(t)}{\partial t} + \left(\left(\frac{L}{A}\right)_{dc} + \left(\frac{L}{A}\right)_{lhs}\right) \frac{\partial W_{in}(t)}{\partial t}$$
$$= \left[\left(p_1 - p_2\right)\right]_{t=0} \left(1 - \frac{\partial W_{uhs}^2(t)}{\partial W_{uhs}^2(t=0)}\right)$$
(11)

+[(
$$p_2 - p_4$$
) +  $\rho g(z_2 - z_3)$ ]<sub>t=0</sub> $\left(1 - \frac{\partial W_{in}^2(t)}{\partial W_{in}^2(t=0)}\right)$ 

# III. SOLUTION METHODOLOGY AND BOUNDARY CONDITIONS

Steady state and transient equations for all components of the NC reactor loop are to be integrated and solved together treating the BCs naturally to simulate in-phase and out-ofphase modes of oscillations. The present section provides a solution methodology to remove the uncertainties existing in the literature in connection with the treatment of BCs while simulating parallel channel instability for both the modes of oscillations with the TH model developed. The solution methodology is in accordance with the mathematical constraints suggested by Dutta and Doshi [26] in earlier research work where the requisite mathematical support was provided to avoid ill-posed BCs while dealing with a set of hyperbolic equations to model subsonic flow situation in the reactor core.

#### A. Steady state boundary conditions

The NC reactor loop comprising of a reactor core accommodating multiple channels and various ex-core components which are mutually linked to each other is subjected to the following BCs:

• Uniform and constant pressure in separator, upper plenum and at the exit of all channels

• Time invariant enthalpy of sub-cooled liquid at the inlet of the channels because of no energy exchange in the downcomer and lower horizontal section.

• Distribution of total mass flow rate, coming through the down-comer and lower horizontal section, to the individual channels of reactor core keeping total pressure drop across the channels same.

A module is designed, which takes care of steady state governing equations and above mentioned BCs.

# B. Transient boundary conditions to simulate in-phase mode of oscillations

TH flow feedbacks across the reactor core are influenced by the recirculation loop dynamics. The reactor core TH equations have to be solved along with the recirculation loop momentum equation simultaneously maintaining a set of BCs which are similar to steady state BCs except the following:

• The total mass flow rate flowing through the down-comer and lower horizontal section, which essentially will be distributed to the individual channels, is time dependent and varies with time depending on their instantaneous TH feedbacks which are mainly originated due to common, but time variant pressure drop BC across the channels. *C.* Transient boundary conditions to simulate out-of-phase mode of oscillations

In the literature, the out-of-phase DWOs has been generally described as a result of constant pressure drop boundary conditions for all parallel channels lying in the reactor core while maintaining the total core inlet mass flow rate constant. This kind of instability is characterized by self sustained out-of-phase oscillations in which the inlet mass flow rate increases in half of the reactor core, while it decreases in the remaining half. Let us consider a case of two identical parallel channels, each representing a half of the core with 180° phase difference, which are subjected to

• Fixed and common pressure drop boundary conditions across the channels and

· Constant total inlet mass flow rate

A question arises how an asymmetric inlet flow distribution (indicated by 180° phase difference) can take place for two similar channels of same geometrical shape and size under identical heat and other input conditions. It is to be noticed that one can't specify velocity and pressure together as inlet boundary conditions for problem under consideration since the governing equations are of hyperbolic in nature and it is a case of subsonic flow situation. It implies the system of PDEs, representing the TH flow fields, are over specified in terms of boundary conditions and the problem, as a result, becomes illposed for out-of-phase mode of oscillations. The problem, with these hypothetical boundary conditions, is solved and it is observed that (i) it allows only small variations in inlet mass flow rates for individual channels, and (ii) no convergence is obtained since the sum of the inlet mass flow rates of all individual channels is not equal to the specified total inlet mass flow rate. These results confirm the previous studies made by Munoz-Cobo et al. [27]. Therefore, with these observations, it can be concluded that the two parallel channels with identical input conditions, essentially, are undergoing inphase oscillations (rather than out-of-phase oscillations) with an over specified total inlet mass flow rate boundary condition when a constant and common pressure drop boundary condition is strictly maintained for both the channels. It is also to be noted that as far as in-phase instabilities are concerned, there is no restriction on the incoming mass flow rate at the entrance of each channel. In fact, parallel channels with inphase instabilities will lead to oscillations for all individual channels separately and eventually, will result in global oscillations in the total mass flow rate entering into the reactor core, and therefore, the total inlet mass flow rate for the parallel channels can't be held constant during the transients. Therefore, to follow the physics of the problem and to avoid over specification of BCs, out-of-phase oscillations are to be simulated with specified pressure and enthalpy as inlet BCs and with specified pressure as exit BC only for the channels lying azimuthally in first half of the core. Transient solution provides the inlet mass flow rate of these channels. Total inlet mass flow rate minus the inlet mass flow rate of the channels lying in the first half of the core provides the inlet mass flow rate for the channels lying in the other half of the core and it acts as first BC. Specified enthalpy is considered to be the other inlet BC and outlet pressure is to be specified as exit BC.

For the next time step, the channels lying in the second half of the core are subjected to constant inlet pressure as BC, whereas the channels lying in first half of the core are subjected to specified flow rate as inlet BC. For subsequent time/levels, the above mentioned steps are repeated alternately.

## IV. RESULTS AND DISCUSSION

First, simulations are done for two channels keeping the power per unit length of the channel same which is equal to 36.47 (KW/m) for both of the channels and then for one channel keeping the power per unit length of the channel to 36.47 (KW/m). In both the cases all other input parameters which are inlet enthalpy (954.00 (KJ/kg) and exit pressure of the channels (6.8947 M.Pa) are kept same. The results obtained from both the cases (fig 1-2) state that there is no difference in DWOs for in-phase mode of oscillations whether there are two channels or one channel when operated at same power. So in this paper for symmetric power distribution results are obtained for one channel for in-phase mode of oscillations and the same result will be valid for two channels also. Extensive simulations are done for in-phase mode of oscillations to get the marginal stable point (MSP) for each of the following cases.

• Only core is incorporated in the model.

• Core, steam separator and down-comer are incorporated in the model.

• Core, steam separator, down-comer and lower horizontal section are incorporated in the model.

• Core, steam separator, down-comer, lower horizontal section and upper horizontal section are incorporated in the model.

In all the cases the inlet enthalpy is kept constant which is equal to 776.00 (KJ/kg). The value of constant exit pressure is 6.8947 M.Pa. For each of the cases two marginal stable point is obtained. One is towards the type-I unstable zone and other is towards type-II unstable zone. These MSPs are obtained at different-2 input power per unit length of the channel for different-2 cases and the same are listed below in the table I.

TABLE I MSPs when simulations are done with various ex-core components for in-phase mode of oscillations

Power per unit length of the		Ex-core components
channel (KW/m)		
MSP towards	MSP	
Type-I	towards	
	Type-II	
28.00	44.25	Only core
25.00	46.60	Core with steam separator
		and down-comer
23.00	53.00	Core with steam separator,
		down-comer and lower
		horizontal section
35.00	41.4	Core with steam separator,
		down-comer, lower horizontal
		section and upper horizontal
		section

The location of MSPs for different-2 cases can be seen in fig 6. In fig 6 the points shown by '+' are the MSPs that lie towards the type-I unstable zone while the points shown by 'X' are the MSPs that lie towards the type-II unstable zone.

The distance between the two MSPs for the same case comes under stable zone. Inclusion of steam separator with down-comer results in the increment in the stable zone at that particular line. If lower horizontal section is also included then it results in further increment of the same. But the inclusion of the upper horizontal section along with all ex-core components results in the minimum stable zone at that particular line.

Similarly simulations are carried out for out-of-phase mode of oscillations for NCBWR having two parallel channels keeping the power per unit length of the channel same for the two channels. All other input parameters are kept same to that of in-phase mode of oscillations. MSPs are obtained for each of the cases and the data are summarized in the table II.

TABLE II MSPs when simulations are done with various ex-core components for out-of-phase mode of oscil lations

FOR OUT-OF-FRASE MODE OF OSCILLATIONS			
Power per unit length of the		Ex-core components	
channel (KW/m)			
MSP towards	MSP towards		
Type-I	Type-II		
32.00	52.55	Only core	
31.5	50.00	Core with steam separator	
		and down-comer	
29.6	49.50	Core with steam separator,	
		down-comer and lower	
		horizontal section	
28.8	45.00	Core with steam separator,	
		down-comer, lower	
		horizontal section and upper	
		horizontal section	

The location of MSPs for different-2 cases can be seen in fig 7. In fig 7 the points shown by '+' are the MSPs that lie towards the type-I unstable zone while the points shown by 'X' are the MSPs that lie towards the type-II unstable zone.

From the data listed in table I and table II it is directed that out-of-phase modes of oscillations dominates over in-phase mode of oscillations towards type-I unstable zone while there is not much dominance observed between the two modes towards type-II unstable zone.

At the same inlet enthalpy and the same average power per unit length of the channels simulations are carried out with asymmetric power distribution to channels. Channel one is given average power + 0.1fraction of the average power while channel two is given power - 0.1fraction of the average power. The graphs are plotted between inlet mass flow rate (kg/s) and time (s). Chaotic oscillations are taken place for in-phase mode of oscillations as in fig 3. But the reactor becomes unstable for out-of-phase mode of oscillations as in fig 4 when operated on the same set of input parameters for which MSPs are obtained. Also it is found that frequency of oscillation is less when there is type-I instability in comparison to type- II instability as in fig 5-6

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Fig. 8 Plot showing MSPs for out-of-phase mode of oscillations when various ex-core components are included

#### V.CONCLUSION

The present work emphasize on developing a non-linear thermal hydraulic model to analyze parallel channel density wave oscillations in the reactor core loop with asymmetric power distributions to parallel channels.

The following conclusion can be drawn from the present paper.

• The model is capable of analyzing density wave oscillations in both the modes of oscillations i.e. in-phase and out-phase.

• The model is also capable of simulating the reactor core dynamics when there is unequal heating in the parallel channels.

• Extensive simulations are done when there is inclusion of various ex-core components however in the present work geometry of the ex-core components are virtually taken.

• The model can take care of the BCs naturally and there is no need of developing different set of codes for simulating inphase and out-of-phase modes of oscillations.

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