World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:3, No:7, 2009

Three Dimensional Analysis of Sequential Quasi Isotropic Composite Disc for Rotating Machine Application

Amin Almasi

***Abstract*—**Composite laminates are relatively weak in out of plane loading, inter-laminar stress, stress concentration near the edge and stress singularities. This paper develops a new analytical formulation for laminated composite rotating disc fabricated from symmetric sequential quasi isotropic layers to predict three dimensional stress and deformation. This analysis is necessary to evaluate mechanical integrity of fiber reinforced multi-layer laminates used for high speed rotating applications such as high speed impellers. Three dimensional governing equations are written for rotating composite disc. Explicit solution is obtained with "Frobenius" expansion series. Based on analytical results, there are two separate zones of three dimensional stress fields in centre and edge of rotating disc. For thin discs, out of plane deformations and stresses are small in comparison with plane ones. For relatively thick discs deformation and stress fields are three dimensional.

***Keywords*—**Composite Disc, Rotating Machine.

I. INTRODUCTION

COMPOSITE materials are increasingly found new applications in rotating machines [1], [2]. One of the important topics is high speed impeller. Composite laminates offer extended strength in fiber directions. However they are relatively weak in out of plane loading. Three dimensional stress fields need to be obtained to evaluate mechanical integrity, reliability and safety assessments of composite rotating disc. Previous researches and papers on composite three dimensional and inter-laminar stresses were mainly concentrated on rectangular plates [3]-[5] and relatively little works have been reported for composite rotating discs. In addition near edge and singularity stresses in composites have taken considerable attention in recent years [6]-[9]. Stresses at near edge or singularity points present challenge to composite application in rotating machine component due to local concentrated stresses and possibility of crack initiation, fatigue, fracture and machine failure. Previous works [10] were using finite element method to prediction of the three dimensional stress and strain which need very special meshes and are very sensitive to type, topology and density of the meshes. Furthermore results cannot be readily generalized and hence do not support parametric studies. This paper develops a new analytical formulation for laminated composite rotating disc fabricated from symmetric sequential quasi isotropic layers to predict three dimensional stress and deformation. First, sequential quasi isotropic laminates as a sub-group of quasi isotropic laminates with more isotropic characteristics are introduced. Three dimensional governing equations are written for rotating disc in most general form. The key step to effective analysis is to take advantage of "First Order Theory" and mechanical characteristics of symmetric sequential quasi isotropic laminates to simplified three dimensional complex equations. Obtained partial deferential equations are solved with expansion series and three dimensional analytical results are obtained.

Amin Almasi is with the Tecnicas Reunidas S.A., Madrid 28050, SPAIN (phone: +34 91 409 8254; fax: +34 91 750 4165; e-mail: aalmasi@trsa.es amin_almassi@yahoo.com).

II. SEQUENTIAL QUASI ISOTROPIC LAMINATES

For optimum arrangement of laminated fiber composites especially for rotating disc applications, quasi-isotropic lamination was introduced. This arrangement [1],[2] has isotropic properties for stretching stiffness. It was introduced to obtain isotropic stretching stiffness for composite lamination assembly, uniform loading and minimum stress concentration where as using fiber laminates with extended strength in fiber direction. Fiber direction of quasi-isotropic for "N" layer laminations can be expressed as (1).

$$\theta_k = k\frac{\pi}{N} \qquad k = 1,2,\ldots,N \tag{1}$$

In this paper new arrangement "Sequential Quasi-Isotropic" as a sub-group of quasi isotropic is introduced. "Sequential Quasi-Isotropic" arrangement has odd number (equal or more than three) of layers as (2).

$$\begin{gathered} N = 2m+1 \;\rightarrow\; m = \frac{N-1}{2} \\ \left[-\theta_m/\ldots/-\theta_3/-\theta_2/-\theta_1/0/\theta_1/\theta_2/\theta_3/\ldots/\theta_m\right] \\ \theta_k = k\frac{\pi}{N} \quad k = 1,2,\ldots,m \end{gathered} \tag{2}$$

Flexural stiffness of sequential quasi isotropic laminates can be expressed as (3). According to (3), four elements of flexural stiffness matrix are same as isotropic materials. Sequential quasi isotropic materials have very close isotropic properties to isotropic materials, not only in stretching stiffness, but also in major elements of flexural stiffness matrix (as shown in (3), except B_{16} and B_{26}, all other flexural elements are same as isotropic materials). In the other words by using these arrangements in high speed disc or similar applications, most uniform loadings, minimum stress concentrations and minimum distortions can be obtained where as using uni-directional extended strength fiber composite layers.

$$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} = \begin{bmatrix} \frac{h^2}{2}U_1 & \frac{h^2}{2}U_4 & B_{16} \\ \frac{h^2}{2}U_4 & \frac{h^2}{2}U_1 & B_{26} \\ B_{16} & B_{26} & \frac{h^2}{4}(U_1 - U_4) \end{bmatrix} \tag{3}$$

III. Rotating Disc Formulation

Fig. 1 shows an example of laminated composite rotating disc fabricated from symmetric sequential quasi isotropic layers (Fig. 1 shows $N=3$, arrangement $[-60/0/60]_s$).

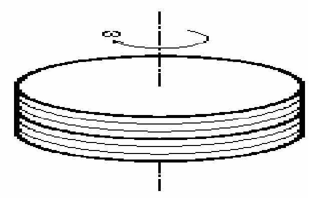

Fig. 1 Rotating disc fabricated from symmetric sequential quasi isotropic layers ($N=3$, arrangement $[-60/0/60]_s$).

For steady state high speed rotating disc fabricated from symmetric sequential quasi isotropic layers, three dimensional deformation functions can be expressed in "Tyler Series" about "z" according to "First Order Theory". With respect to symmetric lamination and isotropic properties of stretching and flexural stiffness properties, (4) can be obtained.

$$\begin{aligned} u_r(r,z) &= u(r) + z\psi_r(r) \\ u_\theta(r,z) &= 0 \\ u_z(r,z) &= z\psi_z(r) \end{aligned} \tag{4}$$

By three dimensional "Hook´s Law" and integration of equations, resultants can be expressed as (5).

$$\begin{aligned} N_r &= \overline{A}_{11}u' + \overline{A}_{12}\frac{u}{r} + \overline{A}_{13}\psi_z + \overline{B}_{11}\psi_r' + \overline{B}_{12}\frac{\psi_r}{r} - N_r^T \\ N_\theta &= \overline{A}_{12}u' + \overline{A}_{22}\frac{u}{r} + \overline{A}_{23}\psi_z + \overline{B}_{12}\psi_r' + \overline{B}_{22}\frac{\psi_r}{r} - N_\theta^T \\ N_z &= \overline{A}_{13}u' + \overline{A}_{23}\frac{u}{r} + \overline{A}_{33}\psi_z + \overline{B}_{13}\psi_r' + \overline{B}_{23}\frac{\psi_r}{r} - N_z^T \\ Q_r &= K_5^2\overline{A}_{55}(w' + \psi_r) + K_5^2\overline{B}_{55}\psi_z' \\ R_r &= K_5^2\overline{B}_{55}(w' + \psi_r) + K_5^2\overline{D}_{55}\psi_z' \\ M_r &= \overline{B}_{11}u' + \overline{B}_{12}\frac{u}{r} + \overline{B}_{13}\psi_z + \overline{D}_{11}\psi_r' + \overline{D}_{12}\frac{\psi_r}{r} - M_r^T \\ M_\theta &= \overline{B}_{12}u' + \overline{B}_{22}\frac{u}{r} + \overline{B}_{23}\psi_z + \overline{D}_{12}\psi_r' + \overline{D}_{22}\frac{\psi_r}{r} - M_\theta^T \end{aligned} \tag{5}$$

Governing equations can be written for half of symmetric laminated disc as (6) and (7).

$$\frac{\partial\sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{\partial\sigma_{rz}}{\partial z} = -\rho\omega^2 r \tag{6}$$

$$\frac{\partial\sigma_{rz}}{\partial r} + \frac{\partial\sigma_z}{\partial z} + \frac{1}{r}\sigma_{rz} = 0 \tag{7}$$

Multiplying (6) by "u_r" and integration from zero to "$h/2$", lead to governing equations for stretching and flexural as (8) and (9) respectively.

$$\frac{dN_r}{dr} + \frac{1}{r}(N_r - N_\theta) = -\rho\frac{h}{2}\omega^2 r \tag{8}$$

$$\frac{dM_r}{dr} - Q_r + \frac{1}{r}(M_r - M_\theta) = -\rho\frac{h^2}{8}\omega^2 r \tag{9}$$

Using same method (multiplying by "u_z" and integration) for (7) leads to (10) and (11). Equations (10) and (11) are governing equations for shear and out of plane direction respectively. Inter-laminar vertical stress between two halves of symmetric disc (out of plane stress) can be noted as (12).

$$\frac{dQ_r}{dr} + \frac{1}{r}Q_r - P(r) = 0 \tag{10}$$

$$\frac{dR_r}{dr} + \frac{1}{r}R_r - N_z = 0 \tag{11}$$

where

$$P(r) = \sigma_z(r, z=0) \tag{12}$$

By substitution of resultants as per (5) in equations (8), (9) and (11), three partial differential equations as (13) will be obtained for laminated rotating disc.

$$\begin{aligned} & a_1u'' + a_2\frac{1}{r}u' + a_3\frac{1}{r^2}u + a_4\psi_r'' + a_5\frac{1}{r}\psi_r' + a_6\frac{1}{r^2}\psi_r + a_7\psi_z' = a_8 r \\ & b_1u'' + b_2\frac{1}{r}u' + b_3\frac{1}{r^2}u + b_4\psi_r'' + b_5\frac{1}{r}\psi' + b_6\frac{1}{r^2}\psi_r + b_7\psi_r + b_8\psi_z' \\ & + b_9\frac{1}{r}\psi_z = b_{10}r \\ & d_1u' + d_2\frac{1}{r}u + d_3\psi_r' + d_4\frac{1}{r}\psi_r + d_5\psi_z'' + d_6\frac{1}{r}\psi_z' + d_7\psi_z = d_8 \end{aligned} \tag{13}$$

Boundary conditions are as (14).

$$\begin{aligned} r = 0 \quad &\rightarrow \quad u = 0 \;, \psi_r = 0 \;, Q_r = 0 \\ r = a \quad &\rightarrow \quad N_r = 0 \;, M_r = 0 \;, Q_r = 0 \end{aligned} \tag{14}$$

IV. Analytical Solution

Explicit analytical solution of (13) can be expressed in exponential series. According to "Frobenius" theory [11], [12], explicit solutions are written as (15).

$$u(r)=\sum_{n=0}^{\infty}A_n\ r^{n+s}$$
$$\psi_r(r)=\sum_{n=0}^{\infty}B_n\ r^{n+s} \tag{15}$$
$$\psi_z(r)=\sum_{n=0}^{\infty}C_n\ r^{n+s+1}$$

By substitution of (15) in (13), equations for "s" characteristic values are obtained as (16).

$$P_1(s)\,A_0+P_2(s)\,B_0=0$$
$$P_3(s)\,A_0+P_4(s)\,B_0=0 \tag{16}$$
$$P_5(s)\,A_0+P_6(s)\,B_0+P_7(s)\,C_0=0$$

where

$$P_1(s)=a_1s^2+(a_2-a_1)s+a_3\quad,\quad P_2(s)=a_4s^2+(a_5-a_4)s+a_6$$
$$P_3(s)=b_1s^2+(b_2-b_1)s+b_3\quad,\quad P_4(s)=b_4s^2+(b_5-b_4)s+b_6$$
$$P_5(s)=d_1s+d_2\quad,\quad P_6(s)=d_3s+d_4$$
$$P_7(s)=d_5s^2+(d_5+d_6)s+d_6$$

Characteristics equations for "s" values are as (17). It leads to six values for "s" which only three values are acceptable with respect to boundary conditions (14). By this method, three sets of general solutions can be obtained in form of (15).

$$[K_5^2\bar{D}_{55}(s^2+2s+1)]\quad[(\bar{A}_{11}s^2-\bar{A}_{22})(\bar{D}_{11}s^2-\bar{D}_{22})-(\bar{B}_{11}s^2-\bar{B}_{22})^2]=0 \tag{17}$$

Particular solutions can also be obtained in the same manner. For loading of constant rotation, particular result corresponds to "$s=3$". For uniform thermal loading "$s=1$".For both general and particular solutions, (A_n), (B_n) and (C_n) are calculated from recurrence relations obtained by substitution of (15) in (13).

V. Results and Discussion

Fig. 2, Fig. 3 and Fig. 4 show out of plane stress "*P(r)*" (as (12)) for rotating disc fabricated from ′′*T300/5208*′′ material composite layers with arrangement ($[-60/0/60]_s$) for diameter to thickness ratio (*D/h* = *(2a)/h*) equal by 4, 8 and 16 respectively.

Out of plane stress "*P(r)*" is dimensionless with proper stress values to study three dimensional stress fields. There are two out of plane zones in rotating discs, one in edge and other in centre. For relatively thick discs (Fig. 2 for diameter to thickness ratio: 4 and Fig. 3 for diameter to thickness ratio: 8) stress fields are three dimensional, however, for thin disc (Fig. 4 for diameter to thickness ratio: 16) there are relatively two distinct three dimensional zones in centre and edge of disc. For edge zone, out of plane stress is varying and try to balance force.

Fig. 5 presents schematic three dimensional deformation of rotating disc fabricated from symmetric sequential quasi isotropic layers (*N=3*). This schematic non-scale deformation shape is plotted based on analytical results of presented method and formulations.

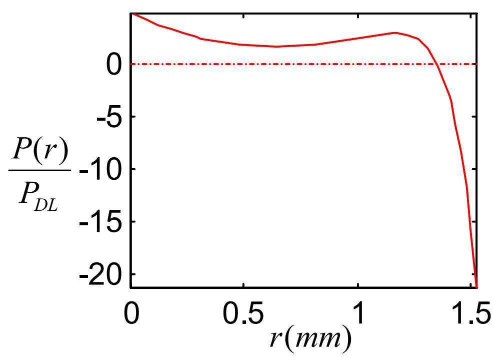

Fig. 2 Out of plane stress P(r) vs. radius for composite disc, diameter to thickness ratio: 4, layer arrangement: $[-60/0/60]_s$.

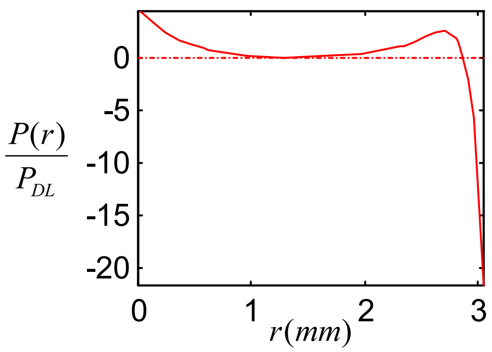

Fig. 3 Out of plane stress P(r) vs. radius for composite disc, diameter to thickness ratio: 8, layer arrangement: $[-60/0/60]_s$.

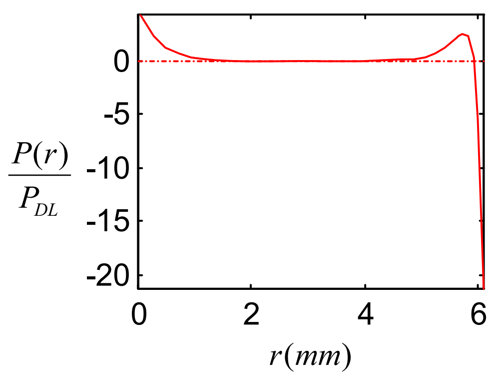

Fig. 4 Out of plane stress P(r) vs. radius for composite disc, diameter to thickness ratio: 16, layer arrangement: $[-60/0/60]_s$.

Simulations and numerical results show that for relatively thin discs, presented three dimensional solution leads to deformations and stress field very similar to plane analysis. It shows robustness of proposed method. This modern tool of analysis enables rotating equipment engineers to study three dimensional and especially out of plane stresses and deformations. Out of plane loading, inter-laminar stress and deformation, stress concentration near the edge and stress singularities can be investigated by this analytical method. This can provide valuable knowledge to develop laminated composite materials for high speed impellers and other high speed rotating machine applications.

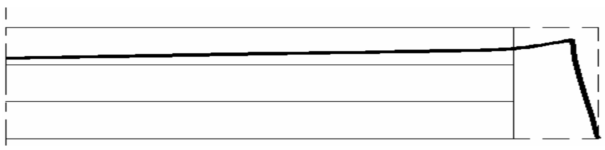

Fig. 5 Schematic three dimensional deformation of rotating disc fabricated from symmetric sequential quasi isotropic layers (Disc is symmetric, each half has three layer N=3, half of disc is shown).

VI. Conclusions

Three dimensional analysis of laminated composite rotating disc fabricated from symmetric sequential quasi isotropic layers are presented in this paper for high speed impeller applications. Explicit formulations are derived and analytical solutions are obtained by expansion series. Based on analytical results, there are two "Three Dimensional" stress fields, in centre and edge of rotating discs. For thin discs, out of plane deformations and stresses are small in comparison with plane ones. For relatively thick disc, deformations and stress fields are completely three dimensional.

References

[1] M. W. Hyer, "Stress Analysis of Fiber- Reinforced Composite Materials", Mc GrawHill, 1998.
[2] S. W. Tsai and H. T. Hahn, "Introduction to Composite Materials", Technomic Publishing Co. Inc., 1980.
[3] V. P. Revenko, "Three Dimensional Stress Sate of an Orthotropic Rectangular Prism under a Transvers Force Applied at its End", Int. Appl. Mech., Vol. 41, No. 4, pp. 367-373, 2005.
[4] C. Mittelstedt and W. Becker, "Interlaminar Stress Concentration in Layered Structure Part I – A Selective Literature Survey on the Free Edge Effects Since 1967", J. Comp. Materials, Vol. 38, No. 12, pp. 1037-1062, 2004.
[5] C. Mittelstedt and W. Becker, "Interlaminar Stress Concentration in Layered Structure Part II – Closed Form Analysis of Stress at Laminated Rectangular Wedges with Arbitrary Non-orthotropic Layup", J. Comp. Materials, Vol. 38, No. 12, pp. 1063-1090, 2004.
[6] S. E. Mikhailov and I. V. Namestnikova, "Stress Singularity Analysis in Space Juncation of Thin Plates", J. Eng. Math., 37, pp.327-341, 2000.
[7] Yu. V. Kokhanenco and V. M. Bystrov, "Edge Effects in a Laminated Composite with Longitudinally Compressed Laminas", Int. Appl. Mech., Vol.42, No. 8, pp. 922-927, 2006.
[8] V. M. Bystrov and Yu. V. Kokhanenco, "Edge Effect in a Fibrous Composite with Near Surface Fibers Subjected to Uniform Loading", Int. Appl. Mech., Vol.43, No. 11, pp. 1195-1201, 2007.
[9] A. Soyucok and Z. Soyucok, "Stress Distributed in Multilayered Composites Near Loading Edges", Int. Appl. Mech. Vol. 44, No. 4, pp. 471-480, 2008.
[10] Yu. V. Kokhanenco, "Numerical Solution to the Buckling Problem for Sandwich Plate under Uniaxial Compression", Int. Appl. Mech., Vol. 42, No. 9, pp. 1045-1051, 2006.
[11] S. J. Farlow, "Partial Differential Equations for Scientists and Engineers", Dover Books, 1993.
[12] E. A. Coddington, "An Introduction to Ordinery Differential Equations", Dover Books, 1995.

Amin Almasi is lead rotating equipment engineer in Technicas Reunidas S.A., Spain, Madrid. He is Chartered Professional Engineer (CPEng). He holds M.S. and B.S. in mechanical engineering. He specializes in rotating machines including reciprocating, centrifugal and screw compressors, gas and steam turbines, process pumps, engines, electric machines, condition monitoring and reliability. Before jointing TR, he worked as rotating equipment engineer in Fluor.

Mr Almasi is active member of IEEE, Engineers Australia, IMechE, ASME (American Society of Mechanical Engineering), CSME (Canadian Society of Mechanical Engineering), CMVI (Canadian Machinery Vibration Institute), SPE (Society of Petroleum Engineers), Vibration Institute, SMRP (Society for Maintenance and Reliability Professionals) and IDGTE (Institute of Diesel and Gas Turbine Engineers). He has authored several papers and articles dealing with rotating machines, condition monitoring, offshore and reliability.