Learning FCM by Tabu Search

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Abstract—Fuzzy Cognitive Maps (FCMs) is a causal graph, which shows the relations between essential components in complex systems. Experts who are familiar with the system components and their relations can generate a related FCM. There is a big gap when human experts cannot produce FCM or even there is no expert to produce the related FCM. Therefore, a new mechanism must be used to bridge this gap. In this paper, a novel learning method is proposed to construct causal graph based on historical data and by using metaheuristic such Tabu Search (TS). The efficiency of the proposed method is shown via comparison of its results of some numerical examples with those of some other methods.

Keywords—Fuzzy Cognitive Map (FCM), Learning, Meta-heuristic, Genetic Algorithm, Tabu search.

I. INTRODUCTION

Fuzzy Cognitive Maps were initially introduced by Robert Axelrod in 1976 and applied in political science [1]. Also it was used in numerous areas of application such as analysis of electrical circuits [2], medicine [3], supervisory systems [4, 5, 6], organization and strategy planning [7, 8], analysis of business performance indicators [9], software project management [10,11], Information retrievals[12], modeling of plant control [13], system dynamics and complex systems [14, 15, 16, 17, 18, 19, 20, 21] and modeling virtual world [22], etc.

This model contains components and their corresponding relations, which may be positive, negative, or neutral. A cognitive map is a directed graph that its nodes correspond to relevant concepts and the edges state the relation between these two nodes by a sign. A positive sign implies a positive relation; moreover, any increase in its source value leads to increase in its target value. A negative sign presents negative relation and any increase or decrease in its source value leads to reverse effect to its target value. In a cognitive map if there is no edge between two nodes it means that, there is no relation between them.

In 1988, Kosko introduced a new extension concept for Cognitive Map and named it fuzzy cognitive maps (FCM) [23, 24, 25, 26]. It named fuzzy cognitive maps. In a simple fuzzy cognitive map, the relation between two nodes is determined by taking a value in interval [-1, 1]. While -1 corresponds to the strongest negative, +1 corresponds to strongest positive one. The other values express different levels of influence. This model can be presented by a square matrix called connection matrix. The value of relation between two nodes is set in their corresponding cell. In the connection matrix, row and column is associated with a source node and a target node, respectively. A simple FCM with five nodes and ten weighted arcs is depicted in Fig.1.

![Fig. 1 A simple Fuzzy Cognitive Map (FCM)](image)

A group of experts can be utilized to improve the results. All experts are asked to determine the relevant factors in a brainstorm meeting. They discuss about main characteristics of the system, number and kind of concepts, which should be contained in the FCM. Then, they determine the structure and the interconnections of the network using fuzzy conditional statements. Each expert may draw his own individual FCM, which can be different from the others. In order to deal with these diagrams, the assigned weights by each expert can be considered and a new FCM will be constructed by all experts’ expertise. Thus, this constructed FCM will represent the knowledge and experience of all related experts. [27, 28].

FCMs can produced by expert manually or generate by other source of information computationally. Experts developed a FCM or a mental model manually based on their knowledge in related area. At first, they identify key domain issues or concepts. Secondly, they identify the causal relationships among these concepts and thirdly, they estimate causal relationships strengths. This achieved graph (FCM) shows not only the components and their relations but also the strengths.

In fuzzy diagrams, the influence of a concept on the others is considered as “negative”, “positive” or “neutral”. All relations are expressed in fuzzy terms, e.g. very weak, weak, medium, strong and very strong. In a simple FCM, all fuzzy variables are mapped into interval [-1, 1]. All the suggested linguistic variables, are considered and an overall linguistic weight is
obtained, with the defuzzification method of Centre of Gravity (COG) [29, 30], is transformed to a numerical weight belonging to the interval [-1, 1].

In general, the manual procedures for developing FCM have occurred, when at least there is one expert who has expertise in the area under studied. In some situations, a FCM could not construct manually such as:

1) There is no expert to define a FCM.
2) The experts’ knowledge is different with each other and they draw different FCM.
3) There are large amount of concepts and connections between them, which could not be drawn without mistakes.

The above situation shows that in many cases, to develop a FCM manually becomes very difficult and experts’ intervention could not resolve the problem. Therefore, a systematic way should be found in order to bridge this gap. For example designing a new method could eliminate the existing weakness. The related knowledge can be extracted by analyzing past information about the given systems. This paper is organized as follows: section 2 states some basic concepts and definitions of Fuzzy Cognitive Map (FCM) and history of FCM Automatic construction. The proposed learning model is presented in section 3 and in section 4 the experimental evaluation and discussion of the achieved results and model effectiveness is discussed. Finally, Section 5 covers conclusions and future research directions.

II. THEORETICAL BACKGROUND

A. Automated FCM or causal graph Construction

When the experts are not able to express their expertise or even there is no expert in the area under studied to add some expression based on her/his expertise, therefore a new way should be defined. For these reasons, the development of computational methods for learning FCM is necessary [31]. In this method, not only causal relations between nodes, but also the strength on each edge must be achieved based on historical data. The required knowledge is extracted from historical data by means and new computational procedures. Many algorithms for learning FCM model structure have been recently proposed. In general, four main categories of these algorithms are used:

1) Association rules algorithms
2) Distance based algorithms (classification)
3) Hebbian algorithm (soft computing)
4) Genetic algorithm (soft computing)

The application of association rules in FCM(s) used by Lee et al in 2002 for the first time [32]. They published a paper about FCM(s) and their relation with web mining inference amplification. In their paper, a fuzzy cognitive map (FCM) used to amplify inference results of Web mining as a dramatic usage of the Internet for a wide variety of daily management activities. They stated that causal knowledge is similar to IF_THEN rules with many differences. The causal knowledge seems natural and more understandable but richer in interpretation. Their propose model was composed of three major phases. At first, extract association rules from related historical data, secondly, transform into FCM causal knowledge based, and thirdly inference amplification. In their model, all rules discovered by association rule function and eliminate rules redundancy and search for the directly and indirectly rules. The other category in learning connection matrix of FCM is distance-based algorithm. In 1998 M. Schneider and el worked on constructing fuzzy cognitive maps automatically. [31] Their method found not only the degree of similarity between any two variables (represented by numerical vectors), but also the relations between variables. They used the fuzzy expert system tool (FEST) which determined the causality among variables.

D. Kardaras et al. presented another similar method in 1999. [33] In their paper assumed that a numerical vector (V) could represent every concept in an FCM, whereas each element (v) of the vector represents a measurement of the concept. Experts can determine an upper threshold (a_u) and a lower threshold (a_l) for every vector, so that:

∀ v (v ≥ a_u) ⇒ (μ_v = 1)  \( (1) \)

∀ v (v ≤ a_l) ⇒ (μ_v = 0)  \( (2) \)

where (μ_v) is the membership degree of element (v). For all the elements (v) between the thresholds, a formula is applied to project all elements into the interval (0, 1) proportionally. Therefore, every concept is a fuzzy set. Both the polarity and the strength of the relationship between two concepts are based on the concept of the similarity between two vectors. More specifically, in the case where two vectors (v_1) and (v_2) are monotonically increasing (direct relation), the distance (d_i) between two elements (v_i) of the vectors is defined as:

\[ d_i = |X_i(v_1) - X_2(v_2)| \]  \( (3) \)

where \( X_i \) and \( X_2 \) is the degree of membership for the (i) element of the vectors (v_1) and (v_2) respectively. In the case of monotonically decreasing vectors (reverse relation) the distance of the two elements (v_i) is defined as:

\[ d_i = |X_i(v_1) - (1 - X_2(v_2))| \]  \( (4) \)

Let AD be the average distance the vectors then

\[ AD = \left( \frac{\sum_{i=1,n} |d_i|}{n} \right) \]  \( (5) \)

Once the average distance is calculated then the similarity (S) of the two vectors is defined as:

\[ S = 1 - AD \]  \( (6) \)
The similarity is calculated twice, once based on direct relation and second based on the reverse relation. The higher similarity determines the polarity (1 or 2) and the strength of the relationships between the two fuzzy sets \((v_1)\) and \((v_2)\). Soft computing like neural network and genetic algorithm help data mining to discover appropriate knowledge in the form of Graph or Fuzzy Cognitive map (FCM) from historical data. Many scientists work on this area and investigated that FCM and its related connection matrix are learned and discovered by historical data.

Soft computing approach such as neural networks and genetic algorithm can be used to discover appropriate knowledge from historical data in the form of graph or FCM. Many researchers worked on these areas by investigating FCM learning methods using historical data.

Kosko proposed a new model by use of simple Differential Hebbian Learning law (DHL) in 1994, but he used this model to learning FCMs without any applications [34]. This learning process modified weights of edges existing in a FCM in order to find the desired connection matrix. In general, when the corresponding concept changes, the value of the related edges for that nodes will be modified too.

In 2002, Vazquez introduced a new extension to DHL algorithm presented by Kosko. He used a new idea to update edge values in a new formula. [35] Another method of learning FCMs based on the first approach (Hebbian algorithm), was introduced by Papageorgiou et al. in 2003. He developed another extension to Hebbian algorithm, called Nonlinear Hebbian Learning (NHL) [36]. Active Hebbian Algorithm (AHL) introduced by Papageorgiou et al. in 2004. In the recent method, experts not only determined the desired set of concepts, initial structure and the interconnections of the FCM structure, but also identified the sequence of activation concepts [37]. Another category in learning connection matrix of FCM is application of genetic algorithms or evolutionary algorithms. Koulouriotis et al. applied the Genetic Strategy (GS) to learn FCM structure in 2001 [38]. In mentioned model, they focused on the development of an ES-based transformation function, size of FCM model, type of learning algorithm. The focus of this model is to determine cause-effect relationships (causality) and their strength.

Other related papers were also published by Parsopoulos et al. in 2003. They tried to apply Particle Swarm Optimization (PSO) method, which belongs to the class of Swarm Intelligence algorithms, to learn FCM structure [39, 40]. Khan and Chong worked on learning initial state vector of FCM in 2003. They performed a goal-oriented analysis of FCM and their learning method did not aim to compute the connection matrix, and their model focused on finding initial state vector for FCM [41]. In 2005, Stach et al. applied real-coded genetic algorithm (RCGA) to develop FCM model from a set of historical data in 2005 [28]. In 2005, Parsopoulos et al combined these two categories and published a paper about using evolutionary algorithms to train Fuzzy Cognitive Maps. In their model, they investigated a coupling of differential evolution algorithm and unsupervised Hebbian learning algorithm [29]. Other work to train a FCM was done by Konar in 2005. He worked on reasoning and unsupervised learning in a FCM. In that paper, a new model was introduced for unsupervised learning and reasoning on a special type of cognitive maps realized with Petri nets [42]. In 2007 M.Ghazanfari et al. published a paper about using Simulated Annealing and genetic algorithm in FCM learning.[43] In that paper, they show that SA algorithm is better than GA in FCM with more nodes and introduced a new method to learn connection matrix rapidly. In this research, the other heuristic algorithms are used to learn FCM matrix.

Table 1 shows a comparison between some existing methods:

Table 1: Overview of Some Learning Approaches Applied to FCMs

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Learning Goal</th>
<th>Type of Data</th>
<th>Transformation Function</th>
<th>No of Node</th>
<th>Learning Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>DHL</td>
<td>Connection matrix</td>
<td>No</td>
<td>Single</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>BDA</td>
<td>Connection matrix</td>
<td>No</td>
<td>Single</td>
<td>Binary</td>
<td>5, 7, 9</td>
</tr>
<tr>
<td>NHL</td>
<td>Connection matrix</td>
<td>Yes &amp; No</td>
<td>Single</td>
<td>Continuous</td>
<td>5</td>
</tr>
<tr>
<td>AHL</td>
<td>Connection matrix</td>
<td>Yes &amp; No</td>
<td>Single</td>
<td>Continuous</td>
<td>8</td>
</tr>
<tr>
<td>GS</td>
<td>Connection matrix</td>
<td>No</td>
<td>Multiple</td>
<td>Continuous</td>
<td>7</td>
</tr>
<tr>
<td>PSO</td>
<td>Connection matrix</td>
<td>No</td>
<td>Multiple</td>
<td>Continuous</td>
<td>5</td>
</tr>
<tr>
<td>GA</td>
<td>Initial vector</td>
<td>N/A</td>
<td>N/A</td>
<td>Continuous</td>
<td>11</td>
</tr>
<tr>
<td>RCGA</td>
<td>Connection matrix</td>
<td>No</td>
<td>Single</td>
<td>Continuous</td>
<td>4, 6, 8, 10</td>
</tr>
<tr>
<td>GA</td>
<td>Connection matrix</td>
<td>No</td>
<td>Single</td>
<td>Continuous</td>
<td>Any Number</td>
</tr>
</tbody>
</table>

As mentioned before, a cause-effect relation is specified by a related Connection matrix. The elements of this matrix are the values of edges in the FCM. The aim of the proposed method is to find these elements. The relations between nodes and edges are calculated as:

\[
C_i(t+1) = f \left( \sum_{j=1}^{n} \epsilon_{ij} C_j(t) \right)
\]  

(7)

where \(\epsilon_{ij}\)'s are the elements of the matrix and \(f\) is a transform function which includes recurring relation on \(t \geq 0\) between \(C(t+1)\) and \(C(t)\) that can be presented by a logistic function like:

\[
f(x) = \frac{1}{1 + e^{-cx}}
\]

(8)

Eq. (7) and Eq. (8) can be expressed by Eq. (9):
Output, \( (t_{n+1}) = E \times Input, \( (t_n) \)

\[ (9) \]

Input, \((t_n)\) is input data for node i, Output, \((t_{n+1})\) is its corresponding output data and \(E\) is the Connection matrix of FCM. Eq.(9) implies that corresponding output for every node can be calculated. \(E\) (Related Connection Matrix) is a vital factor in Eq.(9) which should be determined in the FCM learning process. The proposed FCM learning method forms structure of a FCM and is able to generate state vector sequences that transform the input vectors into the output vectors. When all real input and output values of a FCM are in hand, the most important step is to find a new solution for the FCM and calculate the estimated output related to this new FCM.

\[ Output^{estimated}, \( (t_{n+1}) = E^{proposed} \times Input, \( (t_n) \) \]

\[ (10) \]

According to Eq. (10), Output, \(^{estimated}\) \((t_{n+1})\) is the estimated output and Input, \(^{real}\) \((t_n)\) is its corresponding input for the ith node. \(E^{proposed}\) is new proposed matrix. The real output is Output, \(^{real}\) \((t_{n+1})\) and Eq. (11) calculates the difference between real and estimated outputs:

\[ Error = Output^{estimated}, \( (t_{n+1}) - Output^{real}, \( (t_{n+1}) \) \]

\[ (11) \]

By using the later two equations, the objective is defined as minimizing the difference between real and estimated outputs. This objective is defined as:

\[ Total \_ Error = \sum_{i=1}^{N} \sum_{t=0}^{K-1} Output^{estimated}, \( (t_{n+1}) - Output^{real}, \( (t_{n+1}) \) \]

\[ (12) \]

Where \(N\) is the number of nodes and \(K\) is the iteration. \(Input, \( (t_n) \rightarrow Output, \( (t_{n+1}) \) \) \( \forall t = 0, \ldots, K - 1 \)

If \(Input, \( (t_n)\)\) defined as an initial vector, and Output, \((t_{n+1})\) as system response, \(K-1\) pairs in the form of \(\{initial vector, system response\}\) can be generated from the input data.

As mentioned before, there are many methods for automatic metaheuristic algorithms are used in FCM learning and compare with each other. Fig.2 shows the outline of the proposed method:

The proposed learning model uses TS to find the near optimum solution and some of them are used to escape the local minimum solution and to improve the optimum solution. The following sections provide details about these heuristic algorithms. Also, it is tried to demonstrate all essential elements of propose method, including structure of solution coding (chromosomes), generation of initial solution, Tabu list strategy, fitness function, stopping condition, genetic operators, and selection strategy.

**B. A proposed GA and TS methods for Learning FCM**

In this section, GA and TS algorithm for learning FCM are introduced. At first, we focused on GA parameters and new proper operators.

1) **Proposed genetic algorithms with new operators for learning FCM**

In this section, we introduce a different concept where we have a population of solutions and we would like to move from one population to another. Therefore, a group of solutions evolves towards the good area(s) in the search space. In trying to understand evolutionary mechanisms, Holland (1998) devised a new search mechanism, which he called a genetic algorithm. In its simple form, a genetic algorithm recursively applies the concepts of selection, crossover, and mutation to a randomly generated population of promising solutions with the best solution found being reported. [44, 45, 46] For designing the GA, six principle factors are considered as follows:

a) **Solution coding in GA (chromosome structure)**

The chromosome structure or a solution for FCM is formed as a matrix. In GA algorithms, the initial solution generates random.

b) **Fitness value in GA**

The fitness value is a criterion for the quality measurement of a chromosome or feasible solution. An off spring or new solution is accepted when its objective function value is minimum as compared with its parents.

c) **Mating pool selection strategy in GA**

For creating the new generation, it is necessary to select some chromosomes (mating pool) with the latest fitness in the current generation for recombining or creating chromosomes related to the new generation. In this paper, at first sort the
population from the best to the worst, assign the selection probability of each chromosome, and select them according to their ranking numbers.

d) Improved GA operators

In this paper, the chromosome structure is formed as a matrix. Thus, the GA linear operators cannot be used to a matrix type as the traditional forms. These operators should be improved proportional to the matrix type. Therefore, considering the nature of the matrix, each of three operators called crossover, mutation, and inversion are considered as follows:

e) Horizontal operator, vertical operator and Diametric operator in GA

In horizontal operator, at first two positions in row selected randomly and operation is exercised over this selected area. Fig.4 shows this operator. Vertical operator is similar to the horizontal operator but two positions selected in column randomly. Fig.5 shows this operator. In diametric operator, two numbers in the relevant matrix column or row limits and one of the directions of primary or secondary are selected randomly. Then, the operation is exercised over obtained diameters. For example in Fig.6, a random diameter is selected and then the inversion operator is used.

f) Operation on part of matrix in GA

In this case, two numbers in the relevant matrix column limits and two numbers in the relevant matrix row limits are selected randomly. Then, the operation is exercised over obtained district. For example in Fig.8, the block obtained from cross related to two elected chromosomes is selected randomly and then the crossover operator is used.

g) Stopping criteria in GA

The stopping criteria condition can be defined by two different ways:

Number of generations, In this case, the algorithm terminates, if the number of generations exceeds the specific number.

Best Solutions Found, In this case, the algorithm terminates, if the best solution find and the error function does not change after a period.

Glover in 1989 introduced tabu search (TS) as a method for escaping local optima. The goal is to obtain a list of forbidden (tabu) solutions in the neighborhood of a solution to avoid cycling between solutions while allowing a direction, which may degrade the solution although it may help in escaping from the local optimum. It is needed to specify how to generate solutions in the current solution’s neighborhood. Furthermore, a list of forbidden solutions need to updated after each step. When generating a solution in the neighborhood, this solution should not be in any of the directions listed in the tabu-list, although a direction in the tabu-list may be chosen with some probability if it results in a solution, which is better than the current one. In essence, the tabu-list aims at constraining or limiting the search scope in the neighborhood while still having a chance to select one of these directions. [44]
a) **TS algorithm**

Initialize the neighborhood length to 1 and the memory, M, to empty. Select an initial solution \( X_1 \) in X.

Initialize the best value \( F^{\text{best}} \) of \( F \) and the corresponding solution \( X^{\text{best}} : F^{\text{best}} = F(x_1) \) and \( X^{\text{best}} = X_1 \).

\[ i = 1 \]

**repeat**

\[ i = i + 1 \]

Consider \( X^{\text{neighbour}} \) at random in the neighborhood.

**if** \( F(x^{\text{neighbour}}) < F^{\text{best}} \) **then** \( X^{\text{best}} \leftarrow x^{\text{neighbour}} \)

\[ F^{\text{best}} \leftarrow F(x^{\text{neighbour}}) \]

**if** \( F(x^{\text{neighbour}}) < F(x) \) **then** \( x \leftarrow x^{\text{neighbour}} \)

**end if**

**else**

update \( M \) with this new element

**end if**

until loop condition is satisfied

**return** \( F^{\text{best}} \) and \( X^{\text{best}} \)

b) **Solution coding in TS**

The solution coding for TS is equal to GA which mentioned before.

c) **Initial solution in TS**

An initial solution is a starting solution (point) that will be used in the search process and considered as a random solution.

d) **Neighboring solutions in TS**

Neighboring solutions are the set of feasible solutions that can be generated from the current solution. Each feasible solution can be directly reached from current solution by a move (as genetic operations—mutation or inversion) and resulted neighboring solution.

e) **Stopping criteria**

The stopping criteria condition can be defined when Best Solutions found or when time is limited. In the first case, the algorithm terminates, if the best solution find and the error function does not change after a period. In the second case, the algorithm terminates if the time is finished.

IV. **COMPUTATIONAL RESULTS**

The aim of the experiments is to assess quality of the proposed methods for learning FCMs. In general, the goal of learning FCM is to find FCM connection matrix that generates the same state vector sequence as the input data for a given initial state vector. Now, an important parameter is considered. This variable measures similarity between the input data, and data generated by simulating the candidate FCM with the same initial state vector as the input data. The criterion is defined as a normalized average error between corresponding concept values at each iteration between the two state vector sequences:

\[
\text{error} = \frac{1}{(K - 1) \times N} \sum_{t=1}^{K-1} \sum_{i=1}^{N} (\text{Output}_{i+1}^{\text{estimated}}(t_{n+1}) - \text{Output}_{i}^{\text{real}}(t_{n+1}))^2 \tag{13}
\]

As mentioned before, in above formula \( \text{Output}_{i}^{\text{real}}(t_{n+1}) \) is the value of a node at iteration \( t \) in the input, \( \text{Output}_{i}^{\text{estimated}}(t_{n+1}) \) is the estimated value of that node at iteration \( t \) from simulation of the candidate FCM, \( K \) is the input data length and \( N \) is the number of nodes. All experiments were performed with logistic transformation function, this function is parameterized as:

\[
F(x) = \frac{l}{1 + e^{-0.2x}} \tag{14}
\]

In our experiment, for comparing the metaheuristic results, some test problems were solved by using GA and TS on a PC Pentium IV, 1.6 GHz. The metaheuristic algorithms were developed using Visual Basic 6. Three algorithms, GA and TS ran with different Node numbers: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15 and for every run, the Error and time consuming saved. Each considered FCM, in terms of the number of nodes, was simulated 10 times with the three algorithms, which totaled in \( 12 \times 10 \times 2 \) experiments. The obtained results are shown in table 2.

Considering the results of Tables (2), the presented metaheuristic algorithms are able to find and report the near-optimal and promising solutions in a reasonable computational time. This indicates the success of the proposed algorithms. In general, we can conclude the following results:

All heuristic algorithms found the near-optimal solutions
and the results of these experiments show that these algorithms gradually converge into a high-quality candidate FCM. Three examples of FCM learning experiments based on GA and TS are plotted in Fig.8-1, 8-2.

Fig. 9 shows the error of GA and TS algorithms. The error of TS algorithm is less than GA and it is good for learning FCM with little nodes:

VI. CONCLUSION

In this paper, we have developed new methods for learning FCMs. It has been shown that TS not only can improve the speed of learning process in some situations, but also can improve the quality of learning FCMs with more nodes. The error of TS algorithm is less than GA and it is a good method in learning FCM with little nodes. The quality of learning method based on GA deteriorates with the increasing size of the maps but TS over come this difficulty. Therefore, The TS algorithm found related solutions in less computational times in some situations. The produced results could provide some guidelines for other learning methods. The future work will concern on the improvement of the proposed learning method.

One of interesting and open issues is using the other heuristics methods for learning FCMs and comparing them with the others.

VI. REFERENCES


