

# Generation of Artificial Earthquake Accelerogram Compatible with Spectrum using the Wavelet Packet Transform and Nero-Fuzzy Networks

Peyman Shadman Heidari, Mohammad Khorasani

**Abstract**—The principal purpose of this article is to present a new method based on Adaptive Neural Network Fuzzy Inference System (ANFIS) to generate additional artificial earthquake accelerograms from presented data, which are compatible with specified response spectra. The proposed method uses the learning abilities of ANFIS to develop the knowledge of the inverse mapping from response spectrum to earthquake records. In addition, wavelet packet transform is used to decompose specified earthquake records and then ANFISs are trained to relate the response spectrum of records to their wavelet packet coefficients. Finally, an interpretive example is presented which uses an ensemble of recorded accelerograms to demonstrate the effectiveness of the proposed method.

**Keywords**—Adaptive Neural Network Fuzzy Inference System, Wavelet Packet Transform, Response Spectrum.

## I. INTRODUCTION

EARTHQUAKE response spectra are often used in the analysis and seismic design of the structures. The practice of using response spectra has been popular with engineers, where seismic threat is postulated in terms of smoothed spectral shape. In some cases, it is desirable to develop an artificial earthquake accelerogram compatible with a given design spectrum. The generation of accelerograms from response spectra is an inverse problem which does not have a unique solution. The inherent capability of ANN to learn inverse mapping from examples was first exploited by [1] to develop a new method for generating spectrum compatible accelerograms, which was later extended to generate multiple accelerograms from a given response spectrum by pre-classifying the accelerograms based on duration and using probabilistic neural network with stochastic neurons [2].

Recently, [3-4-5-6-7] developed innovative methodologies for the generation of artificial earthquake accelerograms using neural networks.

In this paper, a new method is proposed to generate a spectrum compatible earthquake accelerogram using the wavelet packet transform and ANFIS networks. The proposed method is validated, using accelerograms to train the ANFIS networks. The performance of the trained ANFIS network is estimated by generating accelerogram for response spectrum.

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## II. WAVELET PACKET TRANSFORM

The wavelet packet method is a generalization of wavelet decomposition that offers a richer range of possibilities for signal analysis. In wavelet analysis, a signal is split into an approximation and a detail. The approximation is then itself split into a second-level approximation and detail, and the process is repeated. For n-level decomposition, there are n+1 possible ways to decompose or encode the signal that shown in figure 1.

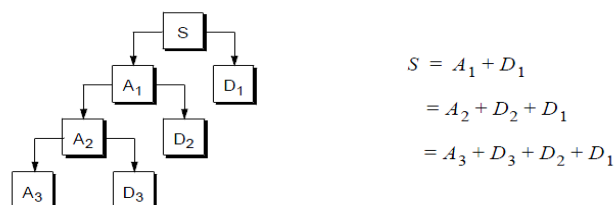


Fig. 1 Wavelet Analysis [Matlab, 2011]

In wavelet packet analysis, the details as well as the approximations can be split. This yields  $2n$  different ways to encode the signal. This is the wavelet packet decomposition tree that shown in figure 2.

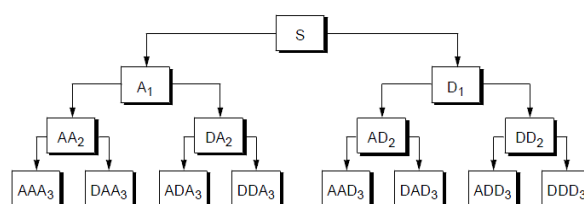


Fig. 2 Wavelet Packet Analysis [Matlab, 2011]

For instance, wavelet packet analysis allows the signal  $S$  to be represented as  $A1 + AAD3 + DAD3 + DD2$ . This is an example of a representation that is not possible with ordinary wavelet analysis [Matlab2011].

## III. ANFIS STRUCTURE

Fuzzy logic and fuzzy inference systems, primary suggested by Zadeh (Zadeh 1965), present a solution for making decisions based on unclear, imprecise or missing data. Fuzzy logic introduces knowledge using IF-THEN rules in the form of "if  $X$  and  $Y$  then  $Z$ ".

Generally, there are two types of fuzzy inference systems, namely Mamdani FIS (Mamadani and Assilian, 1976) and Takagi-Sugeno (ST) FIS (Takagi and Sugeno, 1983) that in this paper TSK model is used.

TSK (Takagi-Sugeno-Kang method of fuzzy inference) fuzzy inference system with two inputs  $x$  and  $y$  and one output  $F$  and a rule base with two fuzzy if-then rules as follows:

Rule 1: If  $x$  is  $A_1$  and  $y$  is  $B_1$  then  $f_1 = p_1.x + q_1.y + r_1$ ,  
Rule 2: If  $x$  is  $A_2$  and  $y$  is  $B_2$  then  $f_2 = p_2.x + q_2.y + r_2$ ,

Where  $A_1, A_2$  and  $B_1, B_2$  are fuzzy sets of input premise variables  $x$  and  $y$  respectively; and  $p_1, q_1, r_1$  and  $p_2, q_2, r_2$  are parameters of the output variables.

The neuro-adaptive learning method works similarly to that of neural networks. Neuro-adaptive learning techniques present a method for the fuzzy modeling procedure to learn information about a data set. Adaptive Neural Network Fuzzy inference Systems (ANFIS) is an architecture that is functionally equivalent to a TSK fuzzy rule base whose parameters are tuned using a learning algorithm in existence of input-output data sets. The parameters associated with the membership functions changes through the learning process.

The architecture of ANFIS is presented in Figure (3), wherein circle nodes are fixed nodes and square nodes are adaptive nodes whose parameters are changed throughout training process. The ANFIS structure is composed of the following parts:

**Layer 1:** Every node  $i$  in this layer is an adaptive node with a node function:

$$\begin{aligned} Q_{Ai}^1 &= \mu_{Ai}(X) \\ Q_{Bi}^1 &= \mu_{Bi}(Y) \end{aligned} \quad (1)$$

Where,  $A_i$  and  $B_i$  are the linguistic variables, and  $x$  and  $y$  are inputs to node  $i$ ,  $Q_{Ai}^1$  and  $Q_{Bi}^1$  are the memberships of  $A_i$  and  $B_i$  which is usually defined by a bell-shape function with maximum and minimum values equal to 1 and 0 as follows:

$$\mu_{A_i}(x) = \exp\left[-\left(\frac{x-c_i}{a_i}\right)^2\right], \mu_{B_i}(y) = \exp\left[-\left(\frac{y-c_i}{a_i}\right)^2\right] \quad i=1,2 \quad (2)$$

Where,  $a_i$  is the standard deviation (SD) and  $c_i$  is the center of the above Gaussian membership function.

**Layer 2:** Each node in this layer is a fixed node that calculates the firing strength  $w_i$  of a rule. The output of each node is the product of all the incoming signals to it as follows:

$$w_i = \mu_{A_i}(x) \times \mu_{B_i}(y), \quad i=1,2 \quad (3)$$

**Layer 3:** Every node in this layer is a fixed node. Each  $i$ th node calculates the ratio of the  $i$ th rule's firing strength to the sum of firing strengths of all the rules. The output from the  $i$ th node is the normalized firing strength given by:

$$\bar{w}_i = \frac{w_i}{w_1 + w_2} \quad (4)$$

**Layer 4:** Every node  $i$  in this layer is a square node with a node function given by:

$$\bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \quad (5)$$

Where  $w_i$  is the output of Layer 3 and  $\{p_i, q_i, r_i\}$  is the consequent parameter set.

**Layer 5:** This layer comprises of only one fixed node that calculates the overall output as the summation of all incoming signals, i.e.

$$O_1^5 = \text{overall output} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}, \quad i=1,2 \quad (6)$$

A hybrid-learning algorithm is used in ANFIS wherein the parameters of membership functions of input variables in antecedent part of fuzzy rules are optimized by a steepest descent algorithm while the linear parameters of the output variable in consequent part are optimized by least square method. If all the parameters defined are fixed, the final output of network is as follows:

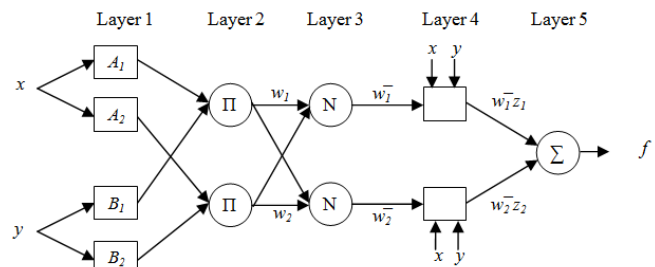


Fig. 3 ANFIS Structure

$$\begin{aligned} f &= \frac{w_1}{w_1 + w_2} f_1 + \frac{w_2}{w_1 + w_2} f_2 \\ &= \bar{w}_1 f_1 + \bar{w}_2 f_2 \\ &= \bar{w}_1 (p_1 x + q_1 y + r_1) + \bar{w}_2 (p_2 x + q_2 y + r_2) \\ &= (\bar{w}_1 x) p_1 + (\bar{w}_1 y) q_1 + (\bar{w}_1) r_1 + (\bar{w}_2 x) p_2 + (\bar{w}_2 y) q_2 + (\bar{w}_2) r_2 \end{aligned}$$

That  $f$  is linear in the consequent parameters ( $p_1, q_1, r_1, p_2, q_2, r_2$ ). Also ANFIS employs two modes of learning. First, a forward pass is made using current premise parameters to optimize rule consequent parameters using least square estimation based on output error. This is possible since outputs are a linear function of consequent parameters. Second, a backward pass is made to alter premise parameters using gradient-based learning. This process of learning is named Hybrid Learning. The backward pass employs learning in a similar way as to the back-propagation in neural networks.

For each pass, each rule antecedent parameter is changed according to:

$$\Delta \alpha = -\eta \frac{\partial E}{\partial \alpha} \quad (7)$$

and

$$\eta = \frac{k}{\sqrt{\sum_{\alpha} \left(\frac{\partial E}{\partial \alpha}\right)^2}} \quad (8)$$

Where  $E$  is the output error,  $\eta$  is the learning rate parameter and  $k$  is a parameter which is automatically varied during

learning process to adapt to the learning rate.  $k$  is increased if four consecutive learning epochs reduce output error and is decreased if two consecutive learning epochs result in non-monotonic changes in error.  $\partial E / \partial \alpha$  is calculated using the chain rule [Jang, 1993].

#### IV. PROPOSED METHODOLOGY

The main objective of this study is to present a new way based on Adaptive Neural Network Fuzzy inference Systems (ANFIS) and wavelet packet transform to generate artificial accelerogram which has a response spectrum close to a specified response spectrum used as the input of the ANFIS networks. Further, the accelerogram generated from a given response spectrum should also have the characteristics like the group of accelerograms used in the training of the ANFIS network. The recorded earthquake accelerograms used for the training of the ANFIS networks were categorized into similar duration earthquake records and were used for the ANFIS network training.

The suggested method is based on expanding an ANFIS network which takes discretized ordinates of the pseudo-velocity response spectrum of accelerogram as input, and the output of the ANFIS network produce the wavelet packet coefficients  $c_{ij}$  at level  $j$  of the wavelet packet transform of the earthquake accelerograms. Figure 4 shows the proposed method for generating accelerogram compatible with response or design spectrum.

The input layer of ANFIS network  $i$  has the pseudo-velocity response spectrum of accelerogram:

$$PSV(\omega_l, \xi) = \omega_l \max |x(t)|, l = 1, 2, \dots, n, \xi = 5\% \quad (9)$$

$$\ddot{x}(t) + 2\xi\omega_l\dot{x}(t) + \omega_l^2 x(t) = -a_g(t) \quad (10)$$

Where  $\omega_l$ ,  $\xi$  and  $a_g(t)$  are the fundamental frequency and the damping coefficient of the single degree of freedom system and the earthquake ground acceleration, respectively.

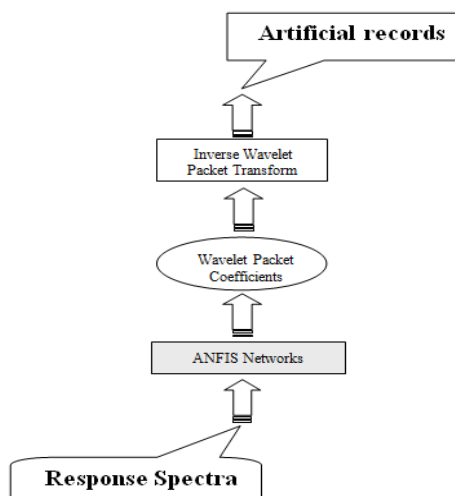


Fig. 4 Proposed method for generating accelerogram

Network output layer in ANFIS,  $C_j^i(k)$ , namely the  $k$ th of wavelet coefficients in the  $i$  level and  $j$  packet is for earthquake:

$$C_j^i(k) = \int_{-\infty}^{\infty} a_g(t) \psi(t) dt \quad (11)$$

Where  $\psi(t)$  is the Daubechies mother wavelet offers fast execution algorithms and produces no redundancies in WP transform, DB10 Wavelets are used in this study [Daubechies, 1992].

The method for training and using the ANFIS network is shown in Fig. 4. In this paper the records were decomposed into level 4 by using WP transform and then 16 wavelet packets were produced for training.

#### V. AN ILLUSTRATIVE EXAMPLE

The proposed method for generating an accelerogram compatible with response or design spectrum in this study has been applied to a sample of Iranian recorded earthquake accelerograms for training the ANFIS networks. All of the records were discretized at 0.02 s and duration of the strong ground motion was different. After considering records, a series of zeros are padded to the records to gain desired length (40 sec).

For the duration group (40 sec) the accelerograms using the wavelet packet transform are decomposed to level 4 and in this level have sixteen sets of wavelet packet coefficients, and then sixteen ANFIS networks were trained with pseudo-velocity response spectra and the wavelet packet coefficients of the earthquake accelerograms in the training group.

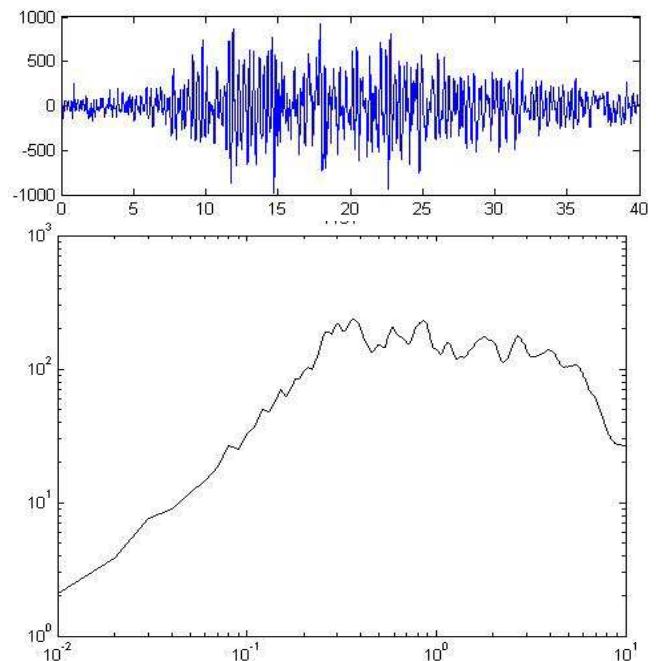


Fig. 5 A test of ANFIS networks from the training group with 40 second duration in Torbat heidariye station in 1979

After training the ANFIS networks, the trained ANFIS networks were tested with the records from the training group. Figures 5 shows a test of the trained ANFIS networks from the training group with a long duration (40 sec), with comparison of the actual and generated accelerogram, and their pseudo-velocity response spectra clearly displays that the trained ANFIS networks has learnt the training cases very well.

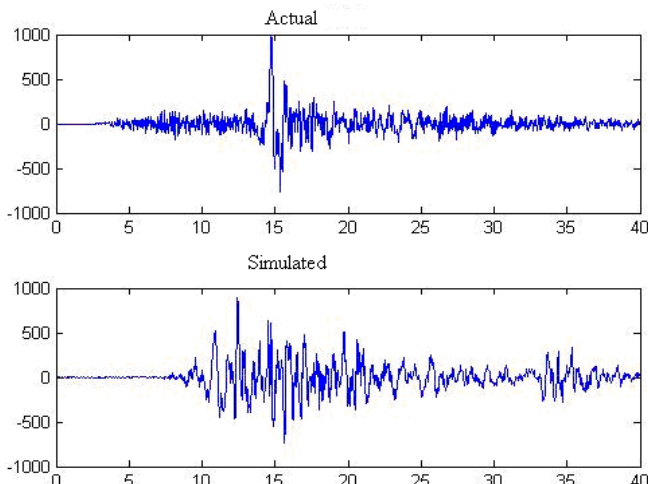


Fig. 6 A test of ANFIS networks with the new record with 40 second duration in Kushkolya station in 1999.

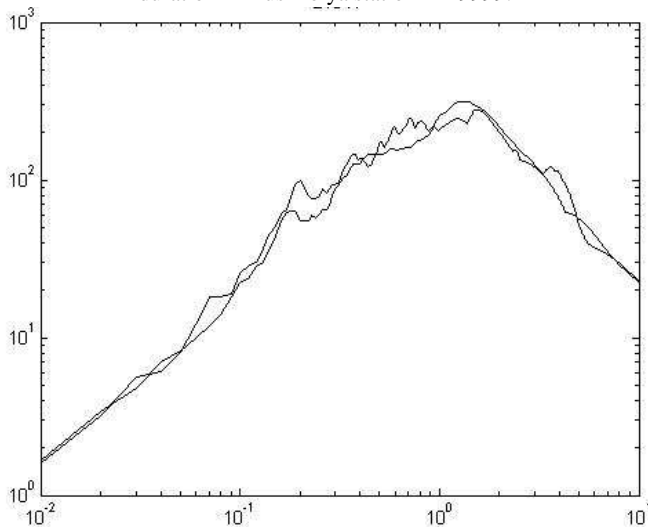


Fig. 7 Comparison of the response spectra of actual record with the simulated record in Kushkolya station in 1999

And also Figures 6 and 7 show a test of the trained ANFIS networks that the input spectrum does not include in the training group (40 sec), with comparison of the actual and Simulated records, and their response spectra clearly displays that the trained ANFIS networks has learnt the training cases very well.

## VI. CONCLUSIONS

A new method of using wavelet packet transform and Adaptive Neural Network Fuzzy inference Systems (ANFIS) in this study for the generation of an artificial earthquake record compatible with response spectrum is extended.

The proposed method is based on developing a ANFIS network which takes the response spectrum of records as input, and as produces the wavelet packet coefficients at level  $j$  of the wavelet packet transform of the earthquake records, the output of the ANFIS networks, then the generated record using inverse wavelet packet transform is obtained.

In an interpretive example, the proposed method was applied to a sample Iranian recorded earthquake accelerograms. For examination of the trained ANFIS networks, when given a response spectrum as input, the generated ANFIS networks either generate records very similar to the one from its training set; one which has a response spectrum close to input, or it synthesizes a new and realistic looking record. It is shown that both the time domain characteristics and the response spectra of the generated records are similar to the original recorded accelerograms.

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