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Improved asymptotic stability analysis for Lur'e systems with neutral type and time-varying delays

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Abstract—This paper investigates the problem of absolute stability and robust stability of a class of Lur'e systems with neutral type and time-varying delays. By using Lyapunov direct method and linear matrix inequality technique, new delay-dependent stability criteria are obtained and formulated in terms of linear matrix inequalities (LMIs) which are easy to check the stability of the considered systems. To obtain less conservative stability conditions, an operator is defined to construct the Lyapunov functional. Also, the free weighting matrices approach combining a matrix inequality technique is used to reduce the entailed conservativeness. Numerical examples are given to indicate significant improvements over some existing results.

Keywords—Lur'e system; Linear matrix inequalities; Lyapunov; Stability.

I. Introduction

TIME delay phenomenon is frequently encountered in many practical systems, such as biological systems, chemical systems, hydraulic systems, nuclear reactor and electrical networks. The Lur'e system, which contains the nonlinear element satisfying certain sector constraints, is an important class of the nonlinear systems. Since Lur'e and Postnikov first proposed the concept of absolute stability as early as the 1940s, the problem of stability of Lur'e systems have been widely studied for several decades, see for example [2-11], and the references therein.

As is well known, Lur'e system with neutral type and time-varying delays being a special case of nolinear system exists in many dynamic systems, a number of stability conditions have been developed for this type of systems in the past[6, 7, 10]. In [8], delay-independent stability criteria that are based on stability conditions for uncertain Lur'e systems with time delay are presented. Also, there are more concerns on derivation of delay-dependent stability conditions [6, 9], because delay-dependent stability criteria are often less conservative than delay-independent ones when the size of the time-delay is small. However, in each case above, the time delays considered are constant. In [10-11], new delay-dependent stability criteria for Lur'e systems with time-varying delays are obtained. Some methods such as using a free weighting matrix and a bounding technique for cross terms that were developed for stability conditions for linear time delay systems are applied to absolute stability condition for Lur'e systems with time delay. In [10], free weighting matrices for the Newton-Leibniz formula are used. By applying more general Lyapunov-Krasovskii functional, new less conservative delay-dependent stability conditions without using model transformation that often induces additional dynamics [12] are presented in [11]. However, the stability problem for Lur'e systems with neutral type and time-varying delays has not been fully investigated, which motivates this paper.

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In this paper, the problem of absolute and robust stability analysis for Lur'e systems with neutral type and time-varying delays is discussed. Since model transformation and bounding techniques for cross terms appearing in the derivative of corresponding Lyapunov functional may introduce additional conservativeness [13], neither model transformation nor bounding technique for cross terms is applied in analyzing the considered systems which may yield less conservative stability conditions. To obtain less conservative stability conditions, an operator $\wp(x_t)$ is defined to construct the Lyapunov functional. Also, the free-weighting matrix approach [14] is employed to further reduce the entailed conservativeness. Numerical examples illustrate the effectiveness and improvement of the obtained results.

II. Problem statement

Consider the linear neutral systems with multiple time delay described by following state equation

$$\begin{aligned}&\dot{x}(t) - C\dot{x}(t-\tau(t)) = Ax(t) + Bx(t-h(t)) + Df(\sigma(t)),\\ &\sigma(t) = Gx(t) + Hx(t-\tau(t)),\\ &x(t_0+\theta) = \phi(\theta), \quad \forall\theta \in [-\tau^*, 0],\end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $\sigma(t) \in \Re^m$ is the output vector, $f(\sigma(t)) \in \Re^m$ is the nonlinear function in feedback path, which is denoted as f for simplicity in the sequel, $\phi(\cdot)$ is a differentiable vector-valued initial function, $\tau > 0$ is a constant neutral delay, the discrete delay is $h(t)$ a time-varying function that satisfies.

$$\begin{aligned}&0 \le h(t) \le h < \infty, \dot{h}(t) \le h_D < 1,\\ &\tau(t) \le \tau < \infty, \dot{\tau}(t) \le \tau_D < 1,\end{aligned} \tag{2}$$

where τ, τ_D, h, h_D are constants, $\tau^* = \max(\tau, h)$. $A \in \Re^{n\times n}$, $B \in \Re^{n\times n}$, $C \in \Re^{n\times n}$ and $D \in \Re^{n\times m}$ are known constant matrices.

The form of the nonlinear feedback path is formulated as

$$\begin{aligned}&f(\sigma(t)) = \begin{bmatrix} f_1(\sigma_1(t)) & f_2(\sigma_2(t)) & \cdots & f_m(\sigma_m(t)) \end{bmatrix}^T,\\ &G = \begin{bmatrix} g_1 & g_2 & \cdots & g_m \end{bmatrix}^T, H = \begin{bmatrix} h_1 & h_2 & \cdots & h_m \end{bmatrix}^T,\\ &\sigma(t) = \begin{bmatrix} \sigma_1(t) & \sigma_2(t) & \cdots & \sigma_m(t) \end{bmatrix}^T,\end{aligned} \tag{3}$$

$f(\cdot)$ satisfies a sector condition with $f_j(\cdot)$, $j = 1, 2, \cdots, m$, belonging to sectors $[k_j^-, k_j^+]$, i.e.

$$f_j(\cdot) \in K_{[k_j^-, k_j^+]} = \{f_j(\cdot) \,|f_j(0) = 0, (f_j(\sigma_j) - k_j^- \sigma_j) \cdot (f_j(\sigma_j) - k_j^+ \sigma_j) \le 0, \sigma_j \ne 0\} \tag{4}$$

Remark 1. The constants k_j^- and k_j^+ are allowed to be positive, negative or zero. Hence, the resulting condition is more general than the usual sector condition in [8, 10].

The purpose of this paper is to formulate a practically computable criterion to check the stability of system described by (1)-(4).

III. MAIN RESULT

For the convenience of presentation, we denote

$$\begin{aligned} K_1 &= diag\{|k_1^-|, |k_2^-|, \cdots, |k_m^-|\}, \\ K_2 &= diag\{k_1^+, k_2^+, \cdots, k_m^+\}, \\ K_3 &= diag\{k_1^-, k_2^-, \cdots, k_m^-\}, \\ K_4 &= diag\{k_1^- k_1^+, k_2^- k_2^+, \cdots, k_m^- k_m^+\}. \end{aligned}$$

We define an operator $\wp : C([-\tau, 0], \Re^n) \to \Re^n$ as: $\wp(x_t) = x(t) - Cx(t - \tau(t))$.

Remark 2. It can be easily seen that the operator $\wp(x_t)$ defined in this paper is quite different from the one studied in [6, 21, 22]. In this paper, the neutral delay $\tau(t)$ is a time-varying function, but the operator $\wp(x_t)$ is always defined when the neutral delay is a known constant delay ([5, 21, 22]). In fact, if we let $\tau(t)$ is a known constant delay, the operator $\wp(x_t)$ is degenerated into the one considered in [21].

In order to obtain our main results, the following Lemmas and Assumption are needed:

Lemma 1.(Schur Complement). Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $\Sigma_2 = \Sigma_2^T > 0$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ holds if and only if:

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, or \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 2([16]). Assume that $\alpha \in \Re^{n_a}$, $\beta \in \Re^{n_b}$ and $N \in \Re^{n_a \times n_b}$. Then, for any matrices $X \in \Re^{n_a \times n_b}$, $Y \in \Re^{n_a \times n_b}$ and $Z \in \Re^{n_a \times n_b}$, the following inequality holds:

$-2\alpha^T N \beta \le \begin{bmatrix} \alpha \\ \beta \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, if $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$.

Lemma 3. Let $x(t) \in \Re^n$ be a vector-valued function with first-order continuous derivative entries, then for any matrices $F = [\, F_1 \;\; F_2 \;\; F_3 \;\; F_4 \;\; F_5 \;\; F_6 \,]$, $Z > 0$ and a scalar $h > 0$, the following inequality holds

$$-\int_{t-h(t)}^{t} \dot{x}^T(\alpha) Z \dot{x}(\alpha) d\alpha \le \xi^T(t) \Xi_1 \xi + h\xi^T(t) F^T Z^{-1} F \xi(t)$$

where

$$\xi(t) = [\, x^T(t) \;\; \dot{x}^T(t) \;\; x^T(t - h(t)) \;\; x^T(t - \tau(t)) \;\; \dot{x}^T(t - \tau(t)) \;\; f^T(\sigma(t)) \,]^T$$

$$\Xi_1 = \begin{bmatrix} F_1 + F_1^T & F_2 & -F_1^T + F_3 & F_4 & F_5 & F_6 \\ * & 0 & -F_2^T & 0 & 0 & 0 \\ * & * & -F_3^T - F_3 & -F_4 & -F_5 & -F_6 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}.$$

Proof: For any matrix $N = [\, N_1 \;\; N_2 \;\; N_3 \;\; N_4 \;\; N_5 \;\; N_6 \,]$, we have

$$\begin{aligned} 0 &= 2[x^T(t) N_1^T + \dot{x}^T(t) N_2^T + x^T(t - h(t)) N_3^T \\ &\quad + x^T(t - \tau(t)) N_4^T + \dot{x}^T(t - \tau(t)) N_5^T + f^T(\sigma(t)) N_6^T] \\ &\quad \cdot [x(t) - x(t - h(t)) - \textstyle\int_{t-h(t)}^{t} \dot{x}(s) ds] \\ &= 2\xi^T(t) N^T [\, I \;\; 0 \;\; -I \;\; 0 \;\; 0 \;\; 0 \,] \xi(t) \\ &\quad - 2 \textstyle\int_{t-h(t)}^{t} \xi^T(t) N^T \dot{x}(s) ds \end{aligned} \tag{5}$$

Employing the matrix inequality $\begin{bmatrix} Z & F \\ F^T & F^T Z^{-1} F \end{bmatrix} \ge 0$ and using Lemma 2, we have

$$\begin{aligned} -2 \textstyle\int_{t-h(t)}^{t} \xi^T(t) N^T \dot{x}(s) ds &\le \textstyle\int_{t-h(t)}^{t} \dot{x}^T(\alpha) Z \dot{x}(\alpha) d\alpha \\ &\quad + 2\xi^T(t) \cdot [F^T - N^T] [\, I \;\; 0 \;\; -I \;\; 0 \;\; 0 \;\; 0 \,] \\ &\quad \cdot \xi(t) + h\xi^T(t) F^T Z^{-1} F \xi(t). \end{aligned} \tag{6}$$

Substituting (5) into (6), we have

$$-\int_{t-h(t)}^{t} \dot{x}^T(\alpha) Z \dot{x}(\alpha) d\alpha \le 2\xi^T(t) \Xi_1 \xi(t) + h\xi^T(t) F^T Z^{-1} F \xi(t). \tag{7}$$

where Ξ_1 is the same as defined in Lemma 3. This ends the proof of Lemma 3. ■

Assumption 1. All the eigenvalues of matrix C are inside the unit circle.

3.1 Stability issue

In this section, based on Lyapunov method and linear matrix inequality techniques, following stability criteria are derived.

Theorem 1. Under Assumption 1, the system (1) is asymptotically stable in the sector bounded by (4), if there exist real matrices P_i (i=2,3,$\cdots$7), $F = [\, F_1 \;\; F_2 \;\; F_3 \;\; F_4 \;\; F_5 \;\; F_6 \,]$, M_{12}, M_{14}, M_{24} and S_{12}, symmetric positive definite matrices Z, R, Q, S_{11}, S_{22} and P_1, symmetric nonnegative definite matrices M_{11}, M_{22}, M_{33}, M_{44}, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ and $T = diag\{t_1, t_2, \cdots, t_m\}$ satisfying the following matrix inequalities:

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & hF_1^T \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \Sigma_{26} & hF_2^T \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & \Sigma_{36} & hF_3^T \\ * & * & * & \Sigma_{44} & \Sigma_{45} & \Sigma_{46} & hF_4^T \\ * & * & * & * & \Sigma_{55} & \Sigma_{56} & hF_5^T \\ * & * & * & * & * & \Sigma_{66} & hF_6^T \\ * & * & * & * & * & * & -hZ \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \tag{9}$$

$$\begin{bmatrix} M_{11} & M_{12} & -QC - G^T \Lambda K_1 H & M_{14} \\ * & M_{22} & C^T QC - H^T \Lambda H & M_{24} \\ * & * & M_{33} & H^T \Lambda \\ * & * & * & M_{44} \end{bmatrix} \ge 0, \tag{10}$$

where $\Sigma_{11} = P_2^T A + A^T P_2 + F_1^T + F_1 + QA + A^T Q + R + S_{11} + \tau_D M_{11} - 2G^T TK_4 G$, $\Sigma_{12} = P_1 - P_2^T + A^T P_3 + S_{12} + F_2 + G^T \Lambda K_1 G$, $\Sigma_{13} = P_2^T B + A^T P_4 - F_1^T + F_3 + QB$, $\Sigma_{14} = A^T P_5 + F_4 + \tau_D M_{12} - A^T QC - 2G^T TK_4 G$, $\Sigma_{15} = P_2^T C + A^T P_6 + F_5$, $\Sigma_{16} = P_2^T D + A^T P_7 + QD + A^T G^T \Lambda + F_6 + \tau_D M_{14} + G^T TK_2 + G^T TK_3$, $\Sigma_{22} = -P_3 - P_3^T + S_{22} + hZ$, $\Sigma_{23} = P_3^T B - P_4 - F_2^T$, $\Sigma_{24} = G^T \Lambda K_1 H - P_5$, $\Sigma_{25} = P_3^T C - P_6$, $\Sigma_{26} = P_3^T D - P_7$, $\Sigma_{33} = P_4^T B + B^T P_4 - (1 - h_D)R - F_3^T - F_3$, $\Sigma_{34} = B^T P_5 - B^T QC - F_4$, $\Sigma_{35} = P_4^T C + B^T P_6 - F_5$, $\Sigma_{36} = P_4^T D + B^T P_7 + B^T G^T \Lambda - F_6$, $\Sigma_{44} = \tau_D M_{22} - (1 - \tau_D)S_{11} - 2H^T TK_4 H$, $\Sigma_{45} = P_5^T C - (1 - \tau_D)S_{12}$, $\Sigma_{46} = P_5^T D - C^T QD + \tau_D M_{24} + H^T TK_2 + H^T TK_3$, $\Sigma_{55} = P_6^T C + C^T P_6 + \tau_D M_{33} - (1 - \tau_D)S_{22}$, $\Sigma_{56} = P_6^T D + C^T P_7 + C^T G^T \Lambda$, $\Sigma_{66} = P_7^T D + D^T P_7 + D^T G^T \Lambda + \Lambda GD + \tau_D M_{44} - 2T$.

Proof: We choose the following Lyapunov-Krasovskii functional candidate as follows:

$$V = V_1 + V_2 + V_3 + V_4 + V_5 \tag{11}$$

where
$V_1 = \xi^T(t)EP\xi(t) + \wp^T(x_t)Q\wp(x_t)$,
$V_2 = \int_{t-h(t)}^{t} x^T(s)Rx(s)ds$,
$V_3 = \int_{t-\tau(t)}^{t} \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds$,
$V_4 = \int_{t-h}^{t} (s - t + h)\dot{x}^T(s)Z\dot{x}(s)ds$,
$V_5 = 2\sum_{i=1}^{m} \lambda_i \int_0^{\sigma_i} (f_i(s) + |k_i^-| s)ds$,
$E = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$,
$P = \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 & 0 \\ P_2 & P_3 & P_4 & P_5 & P_6 & P_7 \end{bmatrix}$,
where $P_1 > 0$, $R > 0$, $Z > 0$, $\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0$, $P_i > 0 (i = 2, 3, \cdots, 7)$ and $\lambda_i \geq 0 (i = 2, 3, \cdots, m)$ are solutions of (8)-(9).

The derivative of along the trajectory of system (1) is given by

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + \dot{V}_5. \tag{12}$$

From (1), we have

$$\begin{aligned}\dot{V}_1 &= 2\xi^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} + 2\wp^T(x_t)Q\dot{\wp}(x_t) \\ &= 2\xi^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ \begin{pmatrix} Ax(t) - \dot{x}(t) + Bx(t - h(t)) \\ +C\dot{x}(t - \tau(t)) + Df(\sigma(t)) \end{pmatrix} \end{bmatrix} \\ &\quad + 2[x(t) - Cx(t - \tau(t))]^T Q[Ax(t) + Bx(t - h(t)) \\ &\quad + Df(\sigma(t)) + \dot{\tau}(t)C\dot{x}(t - \tau(t))] \\ &= \xi^T(t)(\Gamma + \Gamma^T + \Pi + \Pi^T)\xi(t) + 2\dot{\tau}(t)[x^T(t) \\ &\quad \cdot QC\dot{x}(t - \tau(t)) - x^T(t - \tau(t))C^T QC\dot{x}(t - \tau(t))],\end{aligned} \tag{13}$$

where

$$\Gamma = \begin{bmatrix} P_1^T & P_2^T \\ 0 & P_3^T \\ 0 & P_4^T \\ 0 & P_5^T \\ 0 & P_6^T \\ 0 & P_7^T \end{bmatrix} \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ A & -I & B & 0 & C & D \end{bmatrix} \text{ and } \Pi = \begin{bmatrix} I \\ 0 \\ 0 \\ C^T \\ 0 \\ 0 \end{bmatrix} Q \begin{bmatrix} A & 0 & B & 0 & 0 & D \end{bmatrix}.$$

$$\dot{V}_2 \leq x^T(t)Rx(t) - (1 - h_D)x^T(t - h(t))Rx(t - h(t)) \tag{14}$$

$$\begin{aligned}\dot{V}_3 &\leq \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\ &\quad - (1 - \tau_D) \begin{bmatrix} x^T(t - \tau(t)) & \dot{x}^T(t - \tau(t)) \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} \begin{bmatrix} x(t - \tau(t)) \\ \dot{x}(t - \tau(t)) \end{bmatrix} \\ &= x^T(t)S_{11}x(t) + 2x^T(t)S_{12}\dot{x}(t) + \dot{x}^T(t)S_{22}\dot{x}(t) \\ &\quad - (1 - \tau_D)x^T(t - \tau(t))S_{11}x(t - \tau(t)) \\ &\quad - 2(1 - \tau_D)x^T(t - \tau(t))S_{12}\dot{x}(t - \tau(t)) \\ &\quad - (1 - \tau_D)\dot{x}^T(t - \tau(t))S_{22}\dot{x}(t - \tau(t))\end{aligned} \tag{15}$$

$$\begin{aligned}\dot{V}_4 &\leq h\dot{x}^T(t)Z\dot{x}(t) - \int_{t-h(t)}^{t} \dot{x}^T(s)Z\dot{x}(s)ds \\ &\leq h\dot{x}^T(t)Z\dot{x}(t) + \xi^T(t)\Xi_1\xi(t) + h\xi^T(t)F^T Z^{-1}F\xi(t)\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{V}_5 &= 2\sum_{i=1}^{m} \lambda_i[f_i(\sigma_i) + |k_i^-| (g_i^T x(t) + h_i^T x(t - \tau(t)))] \\ &\quad \cdot [g_i^T \dot{x}(t) + h_i^T \dot{x}(t - \tau(t))] \\ &= 2f^T(\sigma(t))\Lambda G[Ax(t) + Bx(t - h(t)) + C\dot{x}(t - \tau(t)) \\ &\quad + Df(\sigma(t))] + 2x^T(t)G^T \Lambda K_1 G\dot{x}(t) + 2\dot{x}(t)^T G^T \Lambda \\ &\quad \cdot K_1 Hx(t - \tau(t)) + 2\dot{\tau}(t)[x^T(t)G^T \Lambda K_1 H\dot{x}(t - \tau(t)) \\ &\quad + x^T(t - \tau(t))H^T \Lambda H\dot{x}(t - \tau(t)) \\ &\quad + \dot{x}^T(t - \tau(t))H^T \Lambda f(\sigma(t))].\end{aligned} \tag{17}$$

Using the condition (2), it holds

$$\begin{aligned}&2\dot{\tau}(t)[x^T(t)QC\dot{x}(t - \tau(t)) - x^T(t - \tau(t))C^T QC\dot{x}(t - \tau(t)) \\ &\quad + x^T(t)G^T \Lambda K_1 H\dot{x}(t - \tau(t)) + x^T(t - \tau(t))H^T \Lambda H \\ &\quad \cdot \dot{x}(t - \tau(t)) + x^T(t - \tau(t))H^T \Lambda f(\sigma(t))] \\ &\leq (\tau_D - \dot{\tau}(t))[x^T(t)M_{11}x(t) + 2x^T(t)M_{12}x(t - \tau(t)) \\ &\quad + 2x^T(t)M_{14}f(\sigma(t)) + x^T(t - \tau(t))M_{22}x(t - \tau(t)) \\ &\quad + 2x^T(t - \tau(t))M_{24}f(\sigma(t)) + \dot{x}^T(t - \tau(t))M_{33} \\ &\quad \cdot \dot{x}(t - \tau(t)) + f^T(\sigma(t))M_{44}f(\sigma(t))] + 2\dot{\tau}(t)x^T(t) \\ &\quad \cdot [QC + G^T \Lambda K_1 H]\dot{x}(t - \tau(t)) + 2\dot{\tau}(t)x^T(t - \tau(t)) \\ &\quad \cdot [H^T \Lambda H - C^T QC]\dot{x}(t - \tau(t)) + 2\dot{\tau}(t)\dot{x}^T(t - \tau(t)) \\ &\quad \cdot H^T \Lambda f(\sigma(t))\end{aligned} \tag{18}$$

where M_{11}, M_{12}, M_{14}, M_{22}, M_{24}, M_{33} and M_{44} are solutions of (10). Since the nonlinearities $f_j(\cdot)$ satisfy the constraint (4), for any $T = diag\{t_1, t_2, \cdots, t_m\} \geq 0$, we have

$$\begin{aligned}0 &\leq -2\sum_{j=1}^{m} t_j(f_j(\sigma_j) - k_j^-\sigma_j)(f_j(\sigma_j) - k_j^+\sigma_j)\\ &= -2x^T(t)G^TTK_4Gx(t) - 4x^T(t)G^TTK_4Gx(t-\tau(t))\\ &\quad + 2x^T(t)[G^TTK_2 + G^TTK_3]f(\sigma(t)) - 2x^T(t-\tau(t))\\ &\quad \cdot H^TTK_4Hx(t-\tau(t)) + 2x^T(t-\tau(t))[H^TTK_2\\ &\quad + H^TTK_3]f(\sigma(t)) - 2f^T(\sigma(t))Tf(\sigma(t))\end{aligned} \tag{19}$$

Calculating the derivative of V along the solution of system (1) and adding the terms on the right of (19) into $\dot{V}$ yield

$$\dot{V} \leq \xi^T(t)\Phi\xi(t) + h\xi^T(t)F^TZ^{-1}F\xi(t) - \dot{\tau}(t)\zeta^T(t) \cdot \begin{bmatrix} M_{11} & M_{12} & -QC - G^T\Lambda K_1 H & M_{14}\\ * & M_{22} & C^TQC - H^T\Lambda H & M_{24}\\ * & * & M_{33} & H^T\Lambda\\ * & * & * & M_{44}\end{bmatrix}\zeta(t) \tag{20}$$

where

$$\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & \dot{x}^T(t-\tau(t)) & f^T(\sigma(t))\end{bmatrix}^T,$$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16}\\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26}\\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36}\\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46}\\ * & * & * & * & \Phi_{55} & \Phi_{56}\\ * & * & * & * & * & \Phi_{66}\end{bmatrix}$$

where $\Phi_{ij} = \Sigma_{ij}$ for $i,j = 1,2,\cdots,7$, Σ_{ij} $(i,j = 1,2,\cdots,7)$ are the same as defined in the Theorem 1.

A sufficient condition for asymptotically stability of system (1) is that the operator $\wp(x_t)$ is stability and there exist real matrices P_i (i=2,3,$\cdots$7), $F = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6\end{bmatrix}$, M_{12}, M_{14}, M_{24} and S_{12}, symmetric positive definite matrices Z, R, Q, S_{11}, S_{22} and P_1, symmetric nonnegative definite matrices M_{11}, M_{22}, M_{33}, M_{44}, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ and $T = diag\{t_1, t_2, \cdots, t_m\}$ such that

$$\dot{V} \leq \xi^T(t)\Phi\xi(t) + h\xi^T(t)F^TZ^{-1}F\xi(t) < 0 \tag{21}$$

for all $\xi(t) \neq 0$. Thus, using Lemma 1, it is easy to check that LMIs (8)-(10) is equivalent to LMI (21). It is well known that Assumption 1 guarantees the stability of different system $x(t) - Cx(t-\tau(t)) = 0$. Therefore, system described by (1)-(4) is asymptotically stable according to Theorem 8.1 in [1]. This completes the proof. ■

When the nonlinear function satisfying:

$$f_j(\cdot) \in K_{[0,\infty]} = \{f_j(\cdot)\,|f_j(0) = 0, \sigma_j f_j(\sigma_j) > 0, \sigma_j \neq 0\} \tag{22}$$

which is located in a finite sector $[0,\infty]$,Theorem 2 is presented as follows.

Theorem 2. Under Assumption 1, the system (1) is asymptotically stable in the sector bounded by (22), if there exist real matrices M_{12}, M_{14}, M_{24}, S_{12}, P_i (i=2,3,$\cdots$7) and $F = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6\end{bmatrix}$, symmetric positive definite matrices Z, R, Q, S_{11}, S_{22} and P_1, symmetric nonnegative definite matrices M_{11}, M_{22}, M_{33}, M_{44}, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ and $T = diag\{t_1, t_2, \cdots, t_m\}$ satisfying the following matrix inequality:

$$\begin{bmatrix} \mathrm{E}_{11} & \Sigma_{12} & \Sigma_{13} & \mathrm{E}_{14} & \Sigma_{15} & \mathrm{E}_{16} & hF_1^T\\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \Sigma_{26}7 & hF_2^T\\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & \Sigma_{36} & hF_3^T\\ * & * & * & \mathrm{E}_{44} & \Sigma_{45} & \mathrm{E}_{46} & hF_4^T\\ * & * & * & * & \Sigma_{55} & \Sigma_{56} & hF_5^T\\ * & * & * & * & * & \mathrm{E}_{66} & hF_6^T\\ * & * & * & * & * & * & -hZ\end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} S_{11} & S_{12}\\ * & S_{22}\end{bmatrix} > 0, \tag{24}$$

$$\begin{bmatrix} M_{11} & M_{12} & -QC - G^T\Lambda K_1 H & M_{14}\\ * & M_{22} & C^TQC - H^T\Lambda H & M_{24}\\ * & * & M_{33} & H^T\Lambda\\ * & * & * & M_{44}\end{bmatrix} \geq 0, \tag{25}$$

where $\Sigma_{ij}, (i,j = 1,2,\cdots,6)$ are the same as defined in the Theorem 1 and $\mathrm{E}_{11} = P_2^TA + A^TP_2 + F_1^T + F_1 + QA + A^TQ + R + S_{11} + \tau_D M_{11}$, $\mathrm{E}_{14} = A^TP_5 + F_4 + \tau_D M_{12} - A^TQC$, $\mathrm{E}_{16} = P_2^TD + A^TP_7 + QD + A^TG^T\Lambda + F_6 + \tau_D M_{14} + G^TT$, $\mathrm{E}_{44} = \tau_D M_{22} - (1-\tau_D)S_{11}$, $\mathrm{E}_{46} = P_5^TD - C^TQD$, $\mathrm{E}_{66} = P_7^TD + D^TP_7 + D^TG^T\Lambda + \Lambda GD + \tau_D M_{44} + H^TT$.

Proof: For neutral system (1) with nonlinearity located in the sector , that is the nonlinear function satisfying , we have

$$2\sum_{j=1}^{m} t_j f_j(\sigma_j(t))g_j^Tx(t) \geq 0, j = 1,2,\cdots,m \tag{26}$$

where $t_j(j = 1,2,\cdots,m)$, the proof of stability conditions (23)-(25) follow the similar line as that in Theorem 1 unless calculating the derivative of V along the solution of system (1) and adding the terms on the right of (19) is replaced by (26) into $\dot{V}$, respectively. And the results (23)-(25) are produced by the uniform approach to the later proof of Theorem 1. ■

3.2 Robust stability

In this section, we extend the obtained stability conditions to robust stability problem for uncertain Lur'e system with neutral type

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t-h(t))\\ &\quad + (D + \Delta D(t))f(\sigma(t)) + (C + \Delta C(t))\dot{x}(t-\tau(t)),\\ \sigma(t) &= Gx(t) + Hx(t-\tau(t)),\\ x(t_0+\theta) &= \phi(\theta), \quad \forall\theta \in [-\tau^*, 0],\end{aligned} \tag{27}$$

where A, B, C and D follow the same definitions of those as in system (1). $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$ and $\Delta D(t)$ are the parametric uncertainties in the system, , which are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) & \Delta C(t) & \Delta D(t)\end{bmatrix} = LK(t) \cdot \begin{bmatrix} E_a & E_b & E_c & E_d\end{bmatrix} \tag{28}$$

where $K(t)$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$K^T(t)K(t) \leq I, \tag{29}$$

and L, E_a, E_b, E_c and E_d are known real constant matrices which characterize how the uncertainty enters the nominal matrices A, B, C and D.

Before proceeding further, system (25) can be written as:

$$\begin{aligned}\dot{x}(t) - C\dot{x}(t-\tau) &= Ax(t) + Bx(t-h(t)) \\ &\quad + Df(\sigma(t)) + Lu, \\ z &= E_a x(t) + E_b x(t-h(t)) \\ &\quad + E_c\dot{x}(t-\tau) + E_d f(\sigma(t)),\end{aligned} \tag{30}$$

with the constraint: $u = K(t)z$.

We further have:

$$\begin{aligned}u^T u \le &[E_a x(t) + E_b x(t-h(t)) + E_c\dot{x}(t-\tau) + E_d(\sigma(t))]^T \\ &\cdot [E_a x(t) + E_b x(t-h(t)) + E_c\dot{x}(t-\tau) + E_d(\sigma(t))].\end{aligned} \tag{31}$$

Using the similar method in the proof of Lemma 3, we can obtain the following Lemma

Lemma 4. Let $x(t) \in \Re^n$ be a vector-valued function with first-order continuous derivative entries, then for any matrices $F = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 \end{bmatrix}$, $Z > 0$ and a scalar $h > 0$, the following inequality holds

$$\begin{aligned}-\int_{t-h(t)}^{t} \dot{x}^T(\alpha) Z \dot{x}(\alpha) d\alpha &\le q^T(t)\Xi_2 q(t) \\ &\quad + h\xi^T(t) F^T Z^{-1} F\xi(t)\end{aligned}$$

where $\xi(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t-h(t)) & x^T(t-\tau(t)) & \dot{x}^T(t-\tau(t)) & f^T(\sigma(t)) & u^T \end{bmatrix}^T$ and $\Xi_2 = \begin{bmatrix} F_1+F_1^T & F_2 & -F_1^T+F_3 & F_4 & F_5 & F_6 & F_7 \\ * & 0 & -F_2^T & 0 & 0 & 0 & 0 \\ * & * & -F_3^T-F_3 & -F_4 & -F_5 & -F_6 & -F_7 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}$.

Base on Theorem 1, we can perform the robust stability analysis for uncertain neutral system (27).

Theorem 3. Under Assumption 1, the system (27) is asymptotically stable in the sector bounded by (4), if there exist real matrices P_i (i=2,3,$\cdots$8), $F = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 \end{bmatrix}$, M_{12}, M_{14}, M_{24} and S_{12}, symmetric positive definite matrices Z, R, Q, S_{11}, S_{22} and P_1, symmetric nonnegative definite matrices M_{11}, M_{22}, M_{33}, M_{44}, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ and $T = diag\{t_1, t_2, \cdots, t_m\}$ satisfying the following matrix inequality:

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & -hZ & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \tag{33}$$

$$\begin{bmatrix} M_{11} & M_{12} & -QC - G^T\Lambda K_1 H & M_{14} \\ * & M_{22} & C^TQC - H^T\Lambda H & M_{24} \\ * & * & M_{33} & H^T\Lambda \\ * & * & * & M_{44} \end{bmatrix} \ge 0, \tag{34}$$

where

$$\Omega_{11} = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} & \hat{\Sigma}_{13} & \hat{\Sigma}_{14} & \hat{\Sigma}_{15} & \hat{\Sigma}_{16} & \hat{\Sigma}_{17} \\ * & \hat{\Sigma}_{22} & \hat{\Sigma}_{23} & \hat{\Sigma}_{24} & \hat{\Sigma}_{25} & \hat{\Sigma}_{26} & \hat{\Sigma}_{27} \\ * & * & \hat{\Sigma}_{33} & \hat{\Sigma}_{34} & \hat{\Sigma}_{35} & \hat{\Sigma}_{36} & \hat{\Sigma}_{37} \\ * & * & * & \hat{\Sigma}_{44} & \hat{\Sigma}_{45} & \hat{\Sigma}_{46} & \hat{\Sigma}_{47} \\ * & * & * & * & \hat{\Sigma}_{55} & \hat{\Sigma}_{56} & \hat{\Sigma}_{57} \\ * & * & * & * & * & \hat{\Sigma}_{66} & \hat{\Sigma}_{67} \\ * & * & * & * & * & * & \hat{\Sigma}_{77} \end{bmatrix}$$

and $\hat{\Sigma}_{i,j} = \Sigma_{i,j}$ (i,j=1,2,$\cdots$,6), $\Sigma_{i,j}$ (i,j=1,2,$\cdots$,6) are the same as defined in the Theorem 1, and $\hat{\Sigma}_{17} = P_2^T L - A^T P_8 + QL + F_7$, $\hat{\Sigma}_{27} = P_3^T L - P_8$, $\hat{\Sigma}_{37} = P_4^T L + B^T P_8 - F_7$, $\hat{\Sigma}_{47} = P_5^T L - C^T QL$, $\hat{\Sigma}_{57} = P_6^T L + C^T P_8$, $\hat{\Sigma}_{67} = P_7^T L + D^T P_8 + \Lambda GL$ and $\hat{\Sigma}_{77} = P_8^T L + L^T P_8 - \varepsilon I$.

Proof of Theorem 3. Applying the similar method in the proof of Theorem 1, we can obtain

$$\begin{aligned}\dot{V} \le &\, q^T(t)\hat{\Phi}q(t) + hq^T(t)F^T Z^{-1} F q(t) - \dot{\tau}(t)\zeta^T(t) \\ &\cdot \begin{bmatrix} M_{11} & M_{12} & -QC - G^T\Lambda K_1 H & M_{14} \\ * & M_{22} & C^TQC - H^T\Lambda H & M_{24} \\ * & * & M_{33} & H^T\Lambda \\ * & * & * & M_{44} \end{bmatrix} \zeta(t)\end{aligned} \tag{35}$$

where

$\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & \dot{x}^T(t-\tau(t)) & f^T(\sigma(t)) \end{bmatrix}^T$,

$$\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & \hat{\Phi}_{13} & \hat{\Phi}_{14} & \hat{\Phi}_{15} & \hat{\Phi}_{16} & \hat{\Sigma}17 \\ * & \hat{\Phi}_{22} & \hat{\Phi}_{23} & \hat{\Phi}_{24} & P_3^T C - P_6 & c\Phi_{26} & \hat{\Sigma}27 \\ * & * & \hat{\Phi}_{33} & \hat{\Phi}_{34} & \hat{\Phi}_{35} & \hat{\Phi}_{36} & \hat{\Sigma}37 \\ * & * & * & \hat{\Phi}_{44} & \hat{\Phi}_{45} & \hat{\Phi}_{46} & \hat{\Sigma}47 \\ * & * & * & * & \hat{\Phi}_{55} & \hat{\Phi}_{56} & \hat{\Sigma}57 \\ * & * & * & * & * & \hat{\Phi}_{66} & \hat{\Sigma}67 \\ * & * & * & * & * & * & \hat{\Sigma}77 \end{bmatrix}$$

where $\Omega_{12} = \begin{bmatrix} hF_1^T & hF_2 & hF_3 & hF_4 & hF_5 & hF_6 & hF_7 \end{bmatrix}^T$, $\Omega_{13} = \begin{bmatrix} \varepsilon E_a & 0 & \varepsilon E_b & 0 & \varepsilon E_c & \varepsilon E_d & 0 \end{bmatrix}^T$, $\hat{\Phi}_{ij} = \Sigma_{ij}$ (i,j=1,2,$\cdots$,6), Σ_{ij} (i,j=1,2,$\cdots$,6) are the same as defined in the Theorem 1, $\hat{\Phi}_{k7}$ (k=1,2,$\cdots$,6) are the same as defined in the Theorem 3 and $\hat{\Sigma}77 = P_8^T L + L^T P_8$.

A sufficient condition for asymptotically stability of system (25) is that there exist real matrices P_i (i=2,3,$\cdots$8), $F = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 \end{bmatrix}$, M_{12}, M_{14}, M_{24} and S_{12}, symmetric positive definite matrices Z, R, Q, S_{11}, S_{22} and P_1, symmetric nonnegative definite matrices M_{11}, M_{22}, M_{33}, M_{44}, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ and $T = diag\{t_1, t_2, \cdots, t_m\}$, such that

$$\dot{V} \le q^T(t)\hat{\Phi}q(t) + hq^T(t)F^T Z^{-1} F q(t) < 0 \tag{36}$$

for all $q(t) \ne 0$. Using the S-procedure in [1], one can see that this condition is implied by the existence of nonnegative scalar $\varepsilon > 0$ such that

$$\begin{aligned}&q^T(t)\Phi q(t) + hq^T(t)F^T Z^{-1} F q(t) + \varepsilon\{[E_a x(t) \\ &\quad + E_b x(t-h(t)) + E_c\dot{x}(t-\tau(t)) + E_d f(\sigma(t))]^T \\ &\quad \cdot [E_a x(t) + E_b x(t-h(t)) + E_c\dot{x}(t-\tau(t)) \\ &\quad + E_d f(\sigma(t))] - u^T u\} < 0\end{aligned} \tag{37}$$

for all $q(t) \neq 0$. By using Lemma 1, the matrix inequalities (30)-(31) imply (34). It is well known that Assumption 1 guarantees the stability of different system $x(t) - Cx(t-\tau) = 0$. Therefore, system (25) is robustly asymptotically stable according to Theorem 8.1 in [2]. This completes the proof.

While the nonlinear system (25) which is located in a sector $[0, \infty]$, condition for robust stability analysis of uncertain system (25) is obtained in Theorem 4.

Theorem 4. Under Assumption 1, the system (1) is asymptotically stable in the sector bounded by (21), if there exist real matrices P_i (i=2,3,$\cdots$8), $F = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 \end{bmatrix}$, M_{12}, M_{14}, M_{24} and S_{12}, symmetric positive definite matrices Z, R, Q, S_{11}, S_{22} and P_1, symmetric nonnegative definite matrices M_{11}, M_{22}, M_{33}, M_{44}, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ and $T = diag\{t_1, t_2, \cdots, t_m\}$ satisfying the following matrix inequality:

$$\begin{bmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} \\ * & -hZ & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \quad (38)$$

$$\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} > 0, \quad (39)$$

$$\begin{bmatrix} M_{11} & M_{12} & -QC - G^T \Lambda K_1 H & M_{14} \\ * & M_{22} & C^T QC - H^T \Lambda H & M_{24} \\ * & * & M_{33} & H^T \Lambda \\ * & * & * & M_{44} \end{bmatrix} \geq 0, \quad (40)$$

where

$$\hat{\Omega}_{11} = \begin{bmatrix} \hat{E}_{11} & \hat{E}_{12} & \hat{E}_{13} & \hat{E}_{14} & \hat{E}_{15} & \hat{E}_{16} & \hat{E}_{17} \\ * & \hat{E}_{22} & \hat{E}_{23} & \hat{E}_{24} & \hat{E}_{25} & \hat{E}_{26} & \hat{E}_{27} \\ * & * & \hat{E}_{33} & \hat{E}_{34} & \hat{E}_{35} & \hat{E}_{36} & \hat{E}_{37} \\ * & * & * & \hat{E}_{44} & \hat{E}_{45} & \hat{E}_{46} & \hat{E}_{47} \\ * & * & * & * & \hat{E}_{55} & \hat{E}_{56} & \hat{E}_{57} \\ * & * & * & * & * & \hat{E}_{66} & \hat{E}_{67} \\ * & * & * & * & * & * & \hat{E}_{77} \end{bmatrix}$$

and $\hat{\Omega}_{12} = \begin{bmatrix} hF_1^T & hF_2 & hF_3 & hF_4 & hF_5 & hF_6 & hF_7 \end{bmatrix}^T$, $\hat{\Omega}_{13} = \begin{bmatrix} \varepsilon E_a & 0 & \varepsilon E_b & 0 & \varepsilon E_c & \varepsilon E_d & 0 \end{bmatrix}^T$, $\hat{E}_{ij} = E_{ij}$ (i,j=1,2,$\cdots$,6), E_{ij} (i,j=1,2,$\cdots$,6) are the same as defined in the Theorem 2, and $\hat{E}_{17} = P_2^T L - A^T P_8 + QL + F_7$, $\hat{E}_{27} = P_3^T L - P_8$, $\hat{E}_{37} = P_4^T L + B^T P_8 - F_7$, $\hat{E}_{47} = P_5^T L - C^T QL$, $\hat{E}_{57} = P_6^T L + C^T P_8$, $\hat{E}_{67} = P_7^T L + D^T P_8 + \Lambda GL$ and $\hat{E}_{77} = P_8^T L + L^T P_8 - \varepsilon I$.

IV. Numerical examples

The following numerical examples are presented to illustrate the usefulness of the proposed theoretical results given in Section 2 and 3.

Example 1. Consider the lur'e system with neutral type with following parameters:

$A = \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}$,

TABLE I
The maximum upper bound delays h of Example 1 with $\tau_D = 0.1$

h_D	0.00	0.20	0.40	0.60	0.80
h in [10]	4.7407	2.2424	1.5434	0.9888	0.7228
h in this paper	Any	Any	Any	1.9991	1.0106

TABLE II
The maximum upper bound delays h of Example 1 with $\tau_D = 0.5$

h_D	0.00	0.20	0.40	0.60	0.80
h in [10]	3.0562	1.8344	1.3029	0.7793	0.6282
h in this paper	Any	Any	Any	1.7376	0.9017

$G = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}^T$ and $H = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T$.

Where the system matrices considered here are of the same parameters as that in Example 2 of [15] and Example 1 of [10]. Since the nonlinearity is different, the method in [15] could not be used here. Using the Theorem 1 in this paper, the maximum upper bound delays h for stability of system (25) are listed in the following tables for different value of h_D. In Table 1, we let $\tau_D = 0.1$; in Table 2, we let $\tau_D = 0.5$; in Table 3, we let $\tau_D = 0.9$. It is clear to see that the results in this paper are much less conservative than those in [10].

Example 2. Since system of retard type can be viewed as a special case of neutral system, consider the system (25) with the same parameters as in [10, 17, 18].

$A = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -0.2 & -0.5 \\ 0.5 & -0.2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}$, $G = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}^T$, $H = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T$, $h_D = 0$, $L = E_a = E_b = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $E_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $E_d = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$, $K(t) = \begin{bmatrix} \sin(\omega t) & 0 \\ 0 & \sin(\omega t) \end{bmatrix}$.

Using Theorems 3 or 4, the maximum upper bound delay h of the system (25) with nonlinearity satisfying (4) or (21) in this paper and methods in [10, 17, 18] is listed in Table 4. It shows that the method in this paper produces better result than [17, 18].

Example 3. Consider the system described by (27) with $A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}$, $G = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}^T$, $H = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T$, $L = E_a = E_b = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $E_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $E_d = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$, $K(t) = \begin{bmatrix} \sin(\omega t) & 0 \\ 0 & \sin(\omega t) \end{bmatrix}$.

Using Theorem 2 or 4, the maximum upper bound delay h of the system (25) with nonlinearity satisfying (4) is listed in Table 5. And methods in [10, 19, 20] are compared with our results in the Table. It shows that the method in this paper is more effective.

V. Conclusion

In This paper, the problem of absolute stability and robust stability of a class of Lur'e systems with neutral type is

TABLE III
THE MAXIMUM UPPER BOUND DELAYS h OF EXAMPLE 1 WITH $\tau_D = 0.9$

h_D	0.00	0.20	0.40	0.60	0.80
h in [10]	1.7376	0.1085	0.1019	0.0983	0.0967
h in this paper	1.1285	0.8996	0.6378	0.5679	0.3389

TABLE IV
THE MAXIMUM UPPER BOUND DELAYS h OF EXAMPLE 2

h_D	$f_j(\cdot) \in K[0, 0.5]$	$f_j(\cdot) \in K[0, 10000]$	$f_j(\cdot) \in K[0, \infty]$
Nian [17]	0.3053	—	—
Yang et al. [18]	—	—	2.055
He [9]	99899999	9989993	—
Gao et al. [10]	Any	Any	Any
h in this paper	Any	Any	Any

TABLE V
THE MAXIMUM UPPER BOUND DELAYS h OF EXAMPLE 3

h_D	$f_j(\cdot) \in K[0, 0.5]$	$f_j(\cdot) \in K[0, 10000]$	$f_j(\cdot) \in K[0, \infty]$
Yu et al. [19]	2.312	—	2.055
Han [20]	2.449	—	—
Gao et al. [10]	Any	Any	4.158
h in this paper	Any	Any	4.426

investigated. Sufficient conditions are given in terms of linear matrix inequalities which can be easily solved by LMI Toolbox in Matlab. Numerical examples are given to indicate significant improvements over some existing results.

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