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A weighted least square algorithm for low-delay FIR filters with piecewise variable stopbands

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Abstract—Variable digital filters are useful for various signal processing and communication applications where the frequency characteristics, such as fractional delays and cutoff frequencies, can be varied. In this paper, we propose a design method of variable FIR digital filters with an approximate linear phase characteristic in the passband. The proposed variable FIR filters have some large attenuation in stopband and their large attenuation can be varied by spectrum parameters. In the proposed design method, a quasi-equiripple characteristic can be obtained by using an iterative weighted least square method. The usefulness of the proposed design method is verified through some examples.

Keywords—Weighted Least Squares Approximation, Variable FIR Filters, Low-Delay, Quasi-Equiripple

I. INTRODUCTION

Variable digital filters (VDFs) are digital filters with controllable spectral characteristics such as variable cutoff frequency response, adjustable passband width, controllable fractional delay, etc. [1]. They have many applications in different areas of signal processing for communications, acoustics, images, measurements, and so on [2]-[8].

In the field of measurement signal processing, a digital filter with a large stopband attenuation and a linear phase characteristic is required to perform high-precision measurements. While an exactly linear phase FIR filter possesses many good properties, its group delay could be unacceptably large. This causes a fall of processing speed since the hardware required for filtering and a computational cost become a large. In order to realize high-speed processing, there is a way that the large stopband attenuation is given only in the frequency band including a principal noise. However, in this way, the filter must be redesigned whenever the frequency band including the principal noise changes.

In [3]-[5], authors considered to design the exactly linear phase variable FIR filters in which a piecewise large attenuation band of the stopband is variable. As a result, the filter, has about the same measurement accuracy as the case of a filter with a large attenuation in the overall stopband, was implemented by few degrees.

By the way, an exactly linear phase FIR filter has high redundancy because the linear phase characteristic does not be needed in the transition bands and the stopbands. Thus, more high-speed signal processing can be expected by using filters with an approximate linear phase characteristics only in the passband. However, as far as authors know there is no report on the design method of an approximate linear phase FIR filter that a piecewise large attenuation band of the stopband is variable. For convenience, we shall call the filters having approximate linear phase characteristics in passband the ”low-delay” digital filters.

In this paper, we propose a design method of a low-delay FIR filter that one or more piecewise large attenuation bands in the stopband are variable. In the proposed design method, we use the weighted least square method and combine the iterative technique to obtain a quasi-equiripple magnitude characteristic. The effectiveness of the proposed method and the proposed VDF is confirmed through numerical examples.

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II. DESIGN PROBLEM

A. Preliminaries

Let the desired frequency response of the VDF with a variable magnitude response as shown in Fig. 1 be

$$D(\omega,\delta_1,\cdots,\delta_K)=\begin{cases} A(\omega)e^{j\theta(\omega)} & 0\le\omega\le\omega_p \\ 0 & \omega_s\le\omega\le\delta_1 \\ 0 & \delta_1<\omega\le\delta_1+\tau_1 \\ 0 & \delta_1+\tau_1<\omega\le\delta_2 \\ 0 & \delta_2<\omega\le\delta_2+\tau_2 \\ \vdots & \vdots \\ 0 & \delta_K<\omega\le\delta_K+\tau_K \\ 0 & \delta_K+\tau_K<\omega\le\pi \end{cases} \quad (1)$$

Here, $A(\omega)$ and $\theta(\omega)$ are the desired amplitude and phase responses, ω_p and ω_s are the normalized angular frequency in the bassband and stopband, $\delta_1\in[\delta_{1,1}\ \delta_{1,m1}]\ \cdots\ \delta_K\in[\delta_{K,1}\ \delta_{K,mK}]$ are the normalized angular frequency on the left of the frequency band with large attenuation, and $\tau_1,\cdots,\tau_K$ denote the band-width to be large attenuation. In this paper, $\delta_1,\cdots,\delta_K$ are called as the spectrum parameter.

In order to realize some large attenuation, we consider the wighting parameter as follows.

$$W(\omega,\delta_1,\cdots,\delta_K)=\begin{cases} w_0 & 0\le\omega\le\omega_p \\ 1 & \omega_s\le\omega\le\delta_1 \\ w_1 & \delta_1<\omega\le\delta_1+\tau_1 \\ 1 & \delta_1+\tau_1<\omega\le\delta_2 \\ w_2 & \delta_2<\omega\le\delta_2+\tau_2 \\ \vdots & \vdots \\ w_K & \delta_K<\omega\le\delta_K+\tau_K \\ 1 & \delta_K+\tau_K<\omega\le\pi \end{cases} \quad (2)$$

where, $w_0,w_1,\cdots,w_K$ are the positive real value.

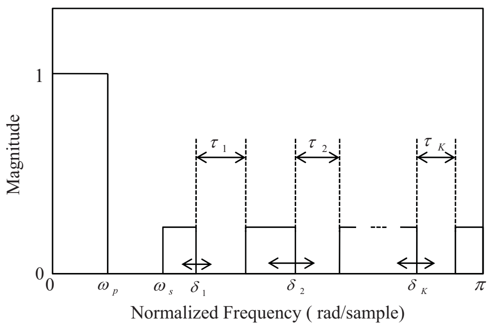


Fig. 1. Desired magnitude responses of the proposed variable FIR filters

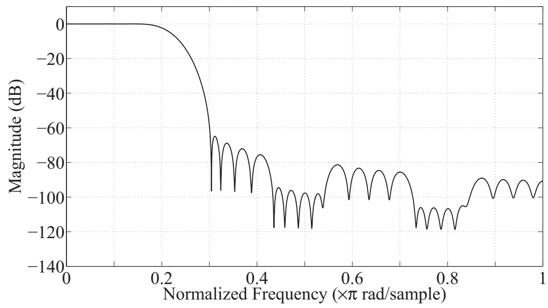


Fig. 2. Magnitude response of a filter by WLS

B. Weighted Least Square Approximation

The frequency response of the variable FIR filters can be shown as

$$H\left(\omega, \delta_1, ..., \delta_K\right) = \sum_{n=0}^{N} h\left(n, \delta_1, ..., \delta_K\right) e^{-jn\omega} \tag{3}$$

where $h\left(n, \delta_1, ..., \delta_K\right)$ is the real filter coefficients and is defined as the L_Kth-order polynomial with the spectrum parameters as follows.

$$\begin{aligned} &h(n, \delta_1, \cdots, \delta_K) \\ &\quad = \sum_{l_1=0}^{L_1} \cdots \sum_{l_K=0}^{L_K} g(n, l_1, \cdots, l_K) \delta_1^{l_1} \cdots \delta_K^{l_K} \end{aligned} \tag{4}$$

Bellow, for simplicity, we will restrict our discussion to the case of $K = 2$.

Then, the approximation error between the desired frequency and designed frequency responses is

$$e(\omega, \delta_1, \delta_2) = D(\omega, \delta_1, \delta_2) - H(\omega, \delta_1, \delta_2) \tag{5}$$

Considered the discrete evaluation points $\Delta_1 = [\delta_{1,1} \ \cdots \ \delta_{1,m1}]$ for δ_1, $\Delta_2 = [\delta_{2,1} \ \cdots \ \delta_{1,m2}]$ for δ_2, and $\omega_t (t = 1, 2, \cdots, M)$ for ω, the evaluation function of WLS can be shown as

$$\boldsymbol{V} = \sum_{l=1}^{m1} \sum_{k=1}^{m2} \sum_{t=1}^{M} W(\omega_t, \delta_{1,l}, \delta_{2,k}) |e(\omega_t, \delta_1, \delta_2)|^2 \tag{6}$$

Here we define the following matrices.

$$\begin{aligned} \boldsymbol{g} = [&g(0,0,0) \ g(0,0,1) \ \cdots \\ &g(n, l_1, l_2) \ \cdots \ g(N, L_1, L_2)]^T \end{aligned} \tag{7}$$

$$\boldsymbol{U} = [U_{1,1} \ U_{1,2} \ \cdots \ U_{l,k} \ \cdots \ U_{m1,m2}]^T \tag{8}$$

$$\begin{aligned} U_{l,k} = &\left[\begin{matrix} u_{l,k}(\omega_0, 0,0,0) & u_{l,k}(\omega_0, 0,0,1) & \cdots \\ u_{l,k}(\omega_1, 0,0,0) & u_{l,k}(\omega_1, 0,0,1) & \cdots \\ \vdots & \vdots & \vdots \\ u_{l,k}(\omega_M, 0,0,0) & u_{l,k}(\omega_M, 0,0,1) & \cdots \end{matrix} \right. \\ &\left. \begin{matrix} u_{l,k}(\omega_0, n, l_1, l_2) & \cdots & u_{l,k}(\omega_0, N, L_1, L_2) \\ u_{l,k}(\omega_1, n, l_1, l_2) & \cdots & u_{l,k}(\omega_1, N, L_1, L_2) \\ \vdots & \vdots & \vdots \\ u_{l,k}(\omega_M, n, l_1, l_2) & \cdots & u_{l,k}(\omega_M, N, L_1, L_2) \end{matrix} \right] \end{aligned} \tag{9}$$

$$u_{l,k}(\omega_i, n, l_1, l_2) = \delta_{1,l}^{l_1} \delta_{2,k}^{l_2} e^{-jn\omega_i} \tag{10}$$

$$\boldsymbol{W} = \mathrm{diag}[W_{1,1} \ W_{1,2} \ \cdots \ W_{l,k} \ \cdots \ W_{m1,m2}] \tag{11}$$

$$\begin{aligned} W_{l,k} = [&W(\omega_0, \delta_{1,l}, \delta_{2,k}) \ W(\omega_1, \delta_{1,l}, \delta_{2,k}) \ \cdots \\ &\cdots \ W(\omega_M, \delta_{1,l}, \delta_{2,k})] \end{aligned} \tag{12}$$

$$\boldsymbol{D} = [D_{1,1} \ D_{1,2} \ \cdots \ D_{l,k} \ \cdots \ D_{m1,m2}]^T \tag{13}$$

$$\begin{aligned} D_{l,k} = [&D(\omega_0, \delta_{1,l}, \delta_{2,k}) \ D(\omega_1, \delta_{1,l}, \delta_{2,k}) \ \cdots \\ &\cdots \ D(\omega_M, \delta_{1,l}, \delta_{2,k})] \end{aligned} \tag{14}$$

Then, the least square solution on Eq.(6) can be obtained by

$$\begin{aligned} \boldsymbol{g} = &\Big(\mathrm{Re}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}\mathrm{Re}(\boldsymbol{U}) + \mathrm{Im}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}\mathrm{Im}(\boldsymbol{U})\Big)^{+} \\ &\Big(\mathrm{Re}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}\mathrm{Re}(\boldsymbol{D}) + \mathrm{Im}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}\mathrm{Im}(\boldsymbol{D})\Big) \end{aligned} \tag{15}$$

where $\mathrm{Re}(\ \cdot\ )$ and $\mathrm{Im}(\ \cdot\ )$ are the real and imaginary parts of $(\ \cdot\ )$, and $(\ \cdot\ )^{+}$ denotes the pseudo-inverse matrix of $(\ \cdot\ )$. Fig. 2 is an example of the filters which is obtained by solving eq. (15). It has been well known that the filters obtained by WLS method have a large magnitude ripple near the band edges.

C. Quasi-Equiripple Approximation

To apply the WLS method in designing our variable quasi-equiripple FIR filters, the weighting function is adjusted in every iteration and the WLS algorithm is used to obtain the coefficients. In this paper, the weighting function $\boldsymbol{W}$ in kth iteration step is updated as follows:

$$W^{[k]}(\omega, \delta_1, \delta_2) = W^{[k-1]}(\omega, \delta_1, \delta_2) \beta^{[k-1]}(\omega, \delta_1, \delta_2) \tag{16}$$

where

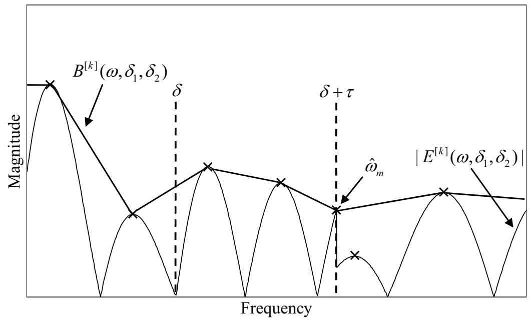
$$\beta^{[k]}(\omega, \delta_1, \delta_2) = \Big(B^{[k]}(\omega, \delta_1, \delta_2)/A^{[k]}\Big)^{\alpha} \tag{17}$$

$$A^{[k]} = \frac{1}{M} \sum_{t=1}^{M} B^{[k]}(\omega_t, \delta_1, \delta_2), \tag{18}$$

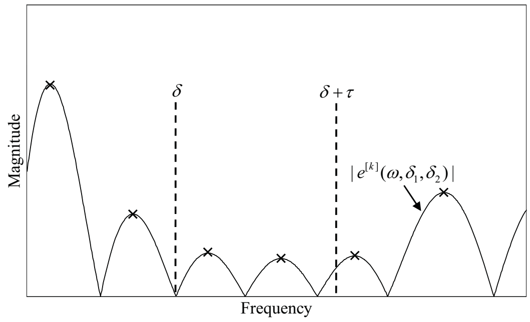
and the parameter α is the empirical convergence factor and the superscript $[\ \cdot\ ]$ denotes the number of the iterations.

In [11], the envelope function $B^{[k]}(\omega, \delta_1, \delta_2)$ is given as the function of straight line formed by joining together all the extremal points of the same frequency band of interest on the error function $|e^{[k]}(\omega, \delta_1, \delta_2)|$. However, to obtain the filters with some different stopband attenuation, we must minimize the following the weighted error function:

$$E^{[k]}(\omega, \delta_1, \delta_2) = W^{[k]}(\omega, \delta_1, \delta_2) e^{[k]}(\omega, \delta_1, \delta_2). \tag{19}$$



(a) $E^{[k]}(\omega, \delta_1, \delta_2)$



(b) $e^{[k]}(\omega, \delta_1, \delta_2)$

Fig. 3. Choice of extremal points

That is, $B^{[k]}(\omega, \delta_1, \delta_2)$ is to be given from the weighted error function $|E^{[k]}(\omega, \delta_1, \delta_2)|$.

In an ordinary filter design problem, a constant weighting function is used in the same frequency band. On the other hand, to obtain the filters with some different stopband attenuation, we need to use the different wighting functions even if it is the same frequency band. Therefore, the extremal points of the $|e^{[k]}(\omega, \delta_1, \delta_2)|$ and $|E^{[k]}(\omega, \delta_1, \delta_2)|$ are not necessarily the same and the magnitude of their ripples also may be different (see Fig. 3). If all the extremal points of the $|E^{[k]}(\omega, \delta_1, \delta_2)|$ are used for calculation of $B^{[k]}(\omega, \delta_1, \delta_2)$ as with [11], it may cause a significant problem in convergence.

To overcome this problem, in this paper we decide the extremal points, which use to calculate $B^{[k]}(\omega, \delta_1, \delta_2)$, as follows.

Let the mth extremal frequency point in the stopbands be $\hat{\omega}_m$. As shown in Fig. 3, $|E^{[k]}(\hat{\omega}_m, \delta_1, \delta_2)|$ at $\hat{\omega}_m = \delta+\tau$ is the extremal point but $|e^{[k]}(\hat{\omega}_m, \delta_1, \delta_2)|$ is not one. So, at $\hat{\omega}_m = \delta+\tau$, $|E^{[k]}(\hat{\omega}_m, \delta_1, \delta_2)|$ is compared with $|E^{[k]}(\hat{\omega}_{m+1}, \delta_1, \delta_2)|$, and only large one is selected as an extremal point to decide $B^{[k]}(\omega, \delta_1, \delta_2)$. Similarly, at $\hat{\omega}_m = \delta$, $|E^{[k]}(\hat{\omega}_m, \delta_1, \delta_2)|$ is compared with $|E^{[k]}(\hat{\omega}_{m-1}, \delta_1, \delta_2)|$, and only large one is selected as an extremal point. Using the extremal points $\bar{\omega}_i$ obtained by this way, $B^{[k]}(\omega, \delta_1, \delta_2)$ can be calculated by

$$B^{[k]}(\omega, \delta_1, \delta_2) = \frac{\omega - \bar{\omega}_i}{\bar{\omega}_{i+1} - \bar{\omega}_i}|E^{[k]}(\bar{\omega}_{i+1}, \delta_1, \delta_2)| + \frac{\bar{\omega}_{i+1} - \omega}{\bar{\omega}_{i+1} - \bar{\omega}_i}|E^{[k]}(\bar{\omega}_i, \delta_1, \delta_2)| \quad \text{for } \bar{\omega}_i < \omega < \bar{\omega}_{i+1}. \tag{20}$$

Then, the least square solution in kth iteration step can be obtained by

$$\boldsymbol{g}^{[k]} = \left(\mathrm{Re}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}^{[\mathrm{k}]}\mathrm{Re}(\boldsymbol{U}) + \mathrm{Im}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}^{[\mathrm{k}]}\mathrm{Im}(\boldsymbol{U})\right)^{+} \left(\mathrm{Re}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}^{[\mathrm{k}]}\mathrm{Re}(\boldsymbol{D}) + \mathrm{Im}(\boldsymbol{U}^{\mathrm{T}})\boldsymbol{W}^{[\mathrm{k}]}\mathrm{Im}(\boldsymbol{D})\right) \tag{21}$$

The design procedure of the proposed design method is summarized as follows.

[DESIGN PROCEDURE]

1. Set the filter order N, the spectral parameter polynomial's order (L_1, L_2), desired frequency response $\boldsymbol{D}$, initial weighting function $\boldsymbol{W}^{[0]}$, and the upper bound number J for the iterations. And , set to $k = 0$.
2. Calculate the least squared solution using eq.(21).
3. If $\dfrac{\|\boldsymbol{g}^{[k]} - \boldsymbol{g}^{[k-1]}\|}{\|\boldsymbol{g}^{[k]}\|} \leq \epsilon$ $(\epsilon \ll 1)$ or $k = J$, then terminates.
4. Find the extremal frequency points $\bar{\omega}_i$ of $|E^{[k]}(\omega, \delta_1, \delta_2)|$.
5. Calculate $B^{[k]}(\omega, \delta_1, \delta_2)$ using eq.(20).
6. Calculate $W^{[k+1]}(\omega, \delta_1, \delta_2)$ using eqs.(16)-(18) and go back to Step 2 as $k = k + 1$.

III. Simulation

In this section, the design examples of the VDF with two variable large attenuation are presented to illustrate the effectiveness of the proposed method. In all the following examples, $\alpha = 1.2$ and $\epsilon = 10^{-2}$ are used.

A. example 1

The design specifications are as follows:
Filter order: $N = 48$
Polynomial order: $L_1 = L_2 = 5$
Passband edge: $\omega_p = 0.15\pi$
Stopband edge: $\omega_s = 0.30\pi$
Desired frequency response:

$$D(\omega, \delta_1, \delta_2) = \begin{cases} e^{-j18\omega} & 0 \leq \omega \leq 0.15\pi \\ 0 & 0.3\pi \leq \omega \leq \pi \end{cases}$$

Band width: $\tau_1 = \tau_2 = 0.1\pi$
Weight value: $w_0 = 1, w_1 = w_2 = 10$

In this example, a total grid point $M = 425$ were used with 75 points in the passband and 350 points in the stopband. And, the discrete evaluation points for the spectrum parameter δ_1 and δ_2 were $\Delta_1 = [0.40\pi\ 0.41\pi\ 0.42\pi\ 0.43\pi\ 0.44\pi\ 0.45\pi]$ and $\Delta_2 = [0.65\pi\ 0.66\pi\ 0.67\pi\ 0.68\pi\ 0.69\pi\ 0.70\pi]$, respectively.

The design took 19 iterations for the above design specifications. The magnitude response and group delay response of the low-delay VDF with $(\delta_1, \delta_2) = (0.40\pi, 0.65\pi), (0.45\pi, 0.65\pi), (0.40\pi, 0.70\pi)$, and $(0.45\pi, 0.70\pi)$ are depicted with a solid line in from Fig. 4(a) to (d). For comparison, the exactly linear phase VDF with about same attenuation is designed by [5] and its magnitude response is depicted with a dotted line. The order of the exactly linear phase VDF is 48 and its group delay is 24 samples.

It is seen from Fig. 4 that large stopband attenuation is changed depending on each spectrum parameter δ_1 or δ_2 while the group delay response has the ripples at near 18 samples and it does not almost change even if the spectrum parameters are changed.

B. example 2

The design specifications are as follows:
Filter order: $N = 72$
Polynomial order: $L_1 = L_2 = 5$
Passband edge: $\omega_p = 0.20\pi$
Stopband edge: $\omega_s = 0.30\pi$
Desired frequency response:

$$D(\omega, \delta_1, \delta_2) = \begin{cases} e^{-j25.0\omega} & 0 \le \omega \le 0.20\pi \\ 0 & 0.30\pi \le \omega \le \pi \end{cases}$$

Band width: $\tau_1 = \tau_2 = 0.1\pi$
Weight value: $w_0 = 1, w_1 = w_2 = 10$

In this example, a total grid point $M = 630$ were used with 140 points in the passband and 490 points in the stopband. And, the discrete evaluation points for the spectrum parameter δ_1 and δ_2 were $\Delta_1 = [0.40\pi\ 0.41\pi\ 0.42\pi\ 0.43\pi\ 0.44\pi\ 0.45\pi]$ and $\Delta_2 = [0.70\pi\ 0.71\pi\ 0.72\pi\ 0.73\pi\ 0.74\pi\ 0.75\pi]$, respectively.

The design took 7 iterations for the above design specifications. The magnitude response and group delay response of the low-delay VDF with $(\delta_1, \delta_2) = (0.40\pi, 0.70\pi), (0.415\pi, 0.715\pi), (0.435\pi, 0.735\pi)$, and $(0.45\pi, 0.75\pi)$ are depicted with a solid line in from Fig. 5(a) to (d). For comparison, the exactly linear phase VDF with about same attenuation is designed by [5] and its magnitude response is depicted with a dotted line. The order of the exactly linear phase VDF is 68 and its group delay is 34 samples.

Like example 1, the large stopband attenuation is changed depending on each spectrum parameter δ_1 or δ_2 while the group delay response has the ripples at near 25 samples and it does not almost change even if the spectrum parameters are changed. Moreover, it is seen from Fig. 5(b) and (c) that the magnitude response of the obtained VDF is the quasi-equiripple even at the δ_1 or δ_2 not used for evaluation.

IV. Conclusion

In this paper, we proposed a design method of low-delay FIR filters with piecewise variable stopbands. The proposed method is based on a weighted least square algorithm and a quasi-equiripple magnitude response is obtained by using an iterative WLS technique. With the proposed method, the about same magnitude response as the exactly linear phase VDF is realizable with the lower delay.

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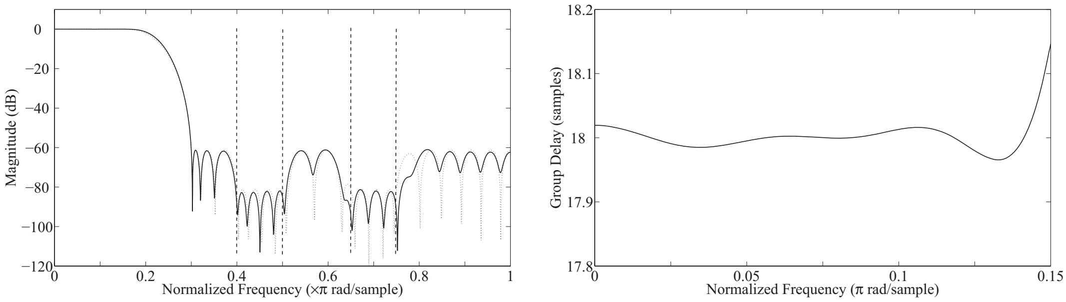
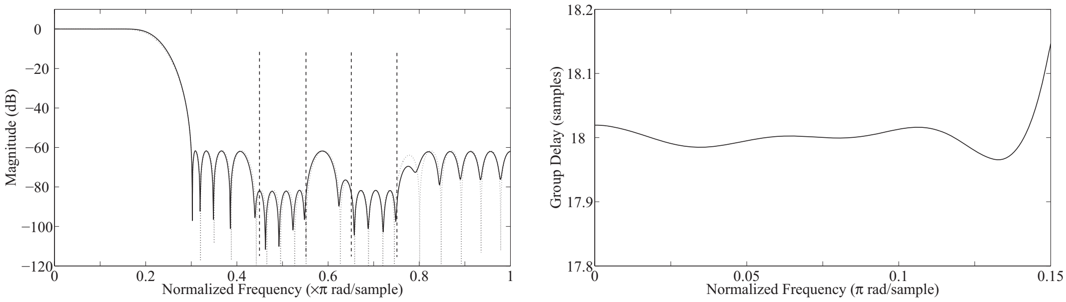
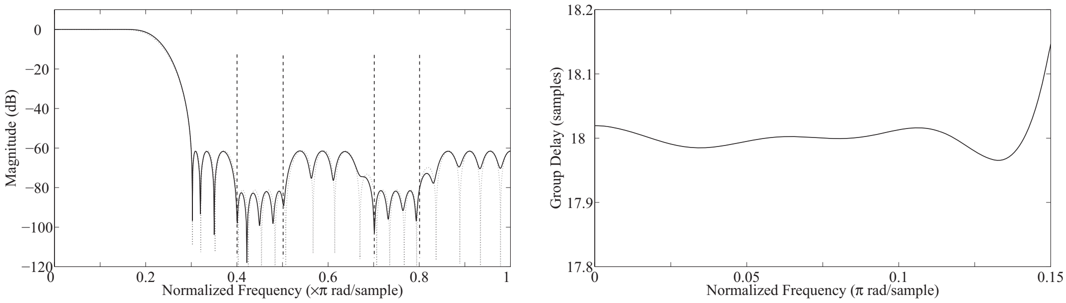


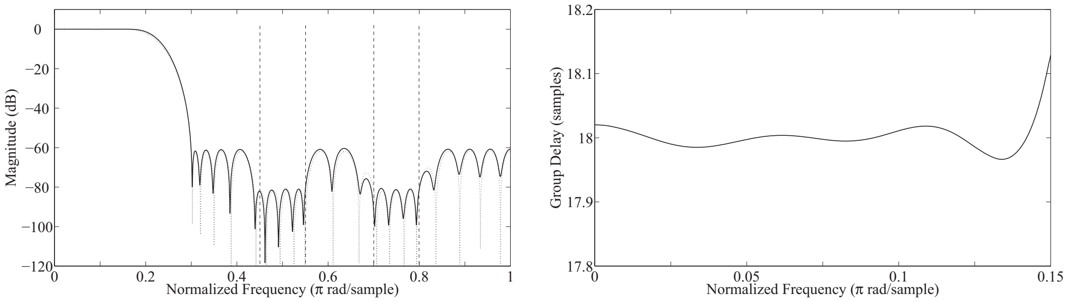
(a) $\delta_1 = 0.40\pi,\ \delta_2 = 0.65\pi$



(b) $\delta_1 = 0.45\pi,\ \delta_2 = 0.65\pi$

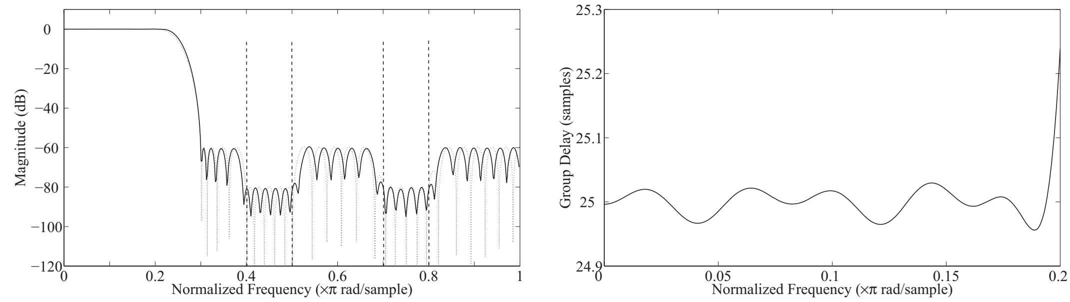


(c) $\delta_1 = 0.40\pi,\ \delta_2 = 0.70\pi$

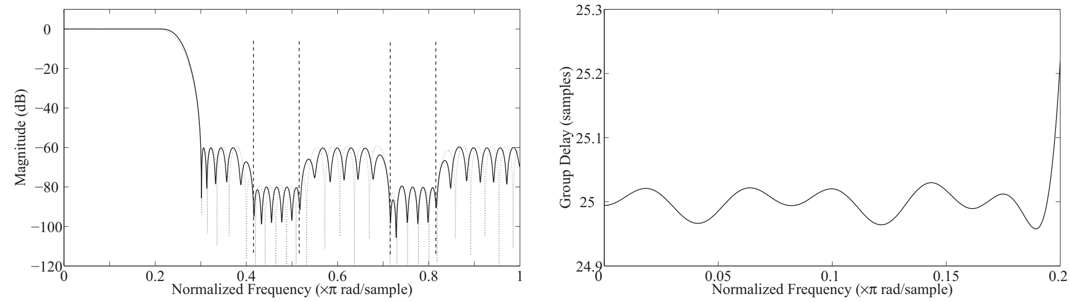


(d) $\delta_1 = 0.45\pi,\ \delta_2 = 0.70\pi$

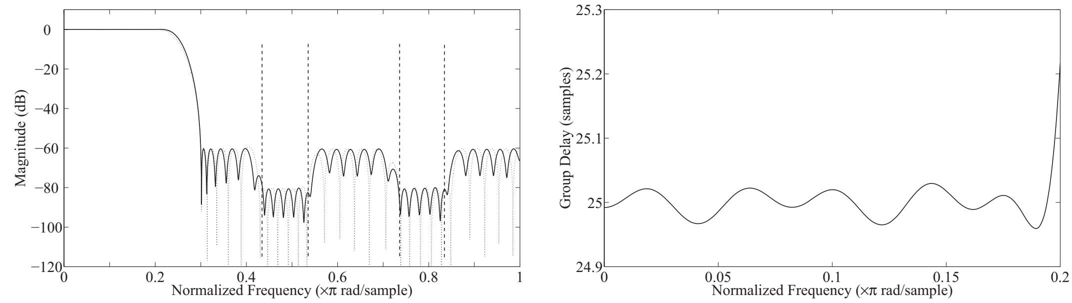
Fig. 4. Magnitude and group delay responses (example 1)



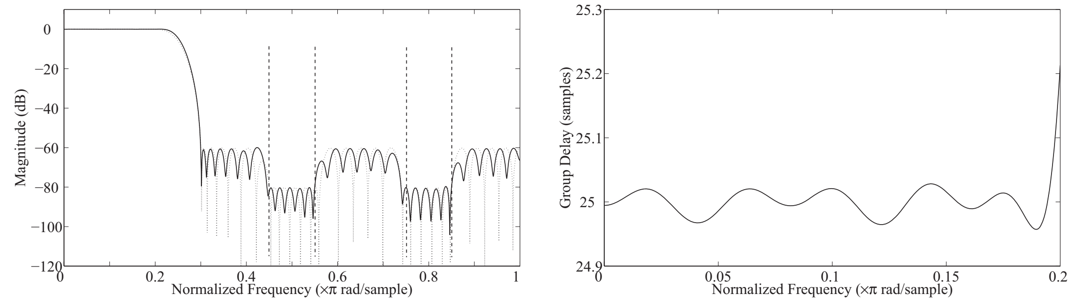
(a) $\delta_1 = 0.40\pi,\ \delta_2 = 0.70\pi$



(b) $\delta_1 = 0.415\pi,\ \delta_2 = 0.715\pi$



(c) $\delta_1 = 0.435\pi,\ \delta_2 = 0.735\pi$



(d) $\delta_1 = 0.45\pi,\ \delta_2 = 0.75\pi$

Fig. 5. Magnitude and group delay responses (example 2)