

# Robust Cerebellar Model Articulation Controller Design for Flight Control Systems

Y. J. Huang, T. C. Kuo, B. W. Hong, and B. C. Wu

**Abstract**—This paper presents a robust proportional-derivative (PD) based cerebellar model articulation controller (CMAC) for vertical take-off and landing flight control systems. Successful on-line training and recalling process of CMAC accompanying the PD controller is developed. The advantage of the proposed method is mainly the robust tracking performance against aerodynamic parametric variation and external wind gust. The effectiveness of the proposed algorithm is validated through the application of a vertical takeoff and landing aircraft control system.

**Keywords**—vertical takeoff and landing, cerebellar model articulation controller, proportional-derivative control.

## I. INTRODUCTION

ROBUST flight control is of interest to control engineers because environmental changes vary during flight. Controlling a vertical takeoff and landing (VTOL) aircraft is not simple. Numerous control designs for stabilization and trajectory tracking have been proposed for VTOL aircraft control systems [1-8].

In general, VTOL aircraft control systems are non-minimum phase. Various control methods were studied, and a method using an approximate input-output linearization approach was proposed [3]. Later, nonlinear state feedback was proposed for robust hovering control of a VTOL aircraft [4]. In [5], a two-step linearization was developed by applying a linear high gain approximation of backstepping to a pre-transformed VTOL aircraft model. A recent approach for trajectory control is a nonlinear output-feedback controller based on a global exponential observer, some global coordinate transformations, Lyapunov's direct method, and an extension of the backstepping technique [7]. In addition, a systematic and robust sliding mode controller which can arbitrarily place all closed-loop poles in sliding mode, was developed to achieve output tracking [8]. However, the prevailing control methods required the mathematical model of the VTOL aircraft control system to determine the control law.

PD control has been applied widely in industry because of its

simple structure [9, 10]. In general, PD control is used for steady-state tracking of step inputs or slow time-varying reference trajectories. However, PD control is not robust against system uncertainties and external disturbances because the proportional and derivative coefficients are usually fixed.

To improve system performance and enhance system robustness of PD control, adaptive algorithms and self-learning rules need to be developed. CMAC is an iterative learning controller that imitates the human cerebellum through iterative weight updating [11-13]. Learning behaviors and the convergence of the iterative learning in a CMAC structure have been proved in [14], making it useful in many applications.

In this paper, the proposed PD-based CMAC strategy is to achieve tracking control for VTOL aircrafts. At first, a normal PD controller is designed such that the error dynamics can be assigned in advance. Besides guaranteeing the stability and output accuracy, the PD controller also provides CMAC the training patterns. The CMAC is designed to enhance tracking ability and system robustness. These two controllers will cooperate with each other.

## II. SYSTEM DESCRIPTION

VTOL aircraft control systems are complex and highly nonlinear. During flight, the aerodynamic parameters vary considerably. For the sake of simplicity, a linearized VTOL model with a minimal number of states and inputs is often utilized to validate the design of new control methods. Figure 1 shows the typical coordinate system for a VTOL aircraft in the vertical plane.

Define the state vector  $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$ , where  $x_1$  is the horizontal velocity (knots),  $x_2$  is the vertical velocity (knots),  $x_3$  is the pitch rate (deg/s), and  $x_4$  is the pitch angle (deg). Consider a typical flight condition with the nominal airspeed of 135 knots [1, 2]. The simplified dynamic equations of this VTOL aircraft in the vertical plane can be described as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{v}(t, \mathbf{x}) \quad (1)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \quad (2)$$

where

$$\mathbf{A} = \begin{bmatrix} -0.0336 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.7070 & 1.3229 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

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$$\mathbf{B} = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.0447 & -7.5922 \\ -5.5200 & 4.9900 \\ 0 & 0 \end{bmatrix} \quad (4)$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (5)$$

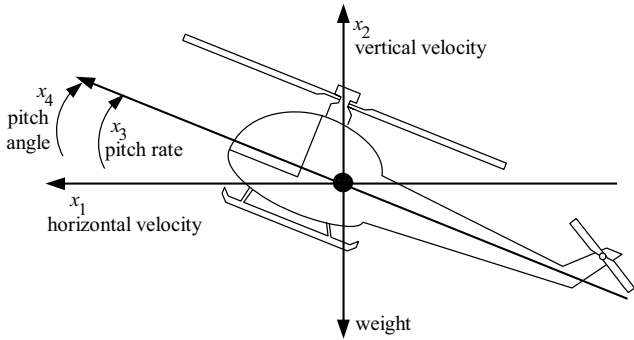


Fig. 1. VTOL aircraft in a vertical plane

In (1),  $\mathbf{u} = [u_1, u_2]^T$  is the control input, consisting of the collective pitch control  $u_1$  and the longitudinal cyclic pitch control  $u_2$ ;  $\mathbf{v}(t, \mathbf{x})$  is the lumped perturbations, consisting of the system uncertainty  $\Delta \mathbf{A} \mathbf{x}$  and wind gust; and  $\mathbf{y} = [y_1, y_2]^T$  is the output, i.e., the vertical and horizontal speed. The control input  $u_1$  collectively alters the pitch angle (angle of attack with respect to the air) of the main rotor blades to provide vertical movement, and the control input  $u_2$  tilts the main rotor disc by individually varying the pitch of the main rotor blades to provide horizontal movement. Notably,  $u_1$  and  $u_2$  have some cross-effect on the vertical and horizontal velocities, respectively. With airspeed ranging from 60 to 170 knots, significant changes occur in the elements  $\mathbf{A}_{32}$  and  $\mathbf{A}_{34}$ , where  $\mathbf{A}_{ij}$  denotes the  $i$ th row and  $j$ th column element of the matrix  $\mathbf{A}$ . Assume that  $|\Delta \mathbf{A}_{32}| \leq 0.2192$  and  $|\Delta \mathbf{A}_{34}| \leq 1.2031$ .

Let  $y_{1d}$  and  $y_{2d}$  denote the desired vertical and horizontal velocities, respectively. The tracking error vector is defined as  $\mathbf{e} = \mathbf{y}_d - \mathbf{y}$ , where  $\mathbf{y}_d = [y_{1d}, y_{2d}]^T$ . The aim is to develop a high performance VTOL control system with low sensitivity to plant parametric variation and external disturbances, and with a tracking error approaching zero. In the following section, the determination of the control input does not require the knowledge of (1) and (2). This mathematical model is used only for control system performance verification.

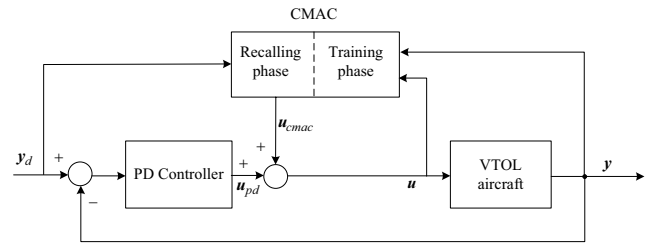


Fig. 2. VTOL aircraft control system

### III. VTOL AIRCRAFT CONTROL SYSTEM

The configuration of the proposed control scheme is shown in Fig. 2. The control law is defined as follows:

$$\mathbf{u} = \mathbf{u}_{pd} + \mathbf{u}_{cmac} \quad (6)$$

where  $\mathbf{u}_{pd}$  is the PD control input and  $\mathbf{u}_{cmac}$  is the CMAC control input. The PD controller is designed to first stabilize the states of the VTOL aircraft control system. In this stage, the robustness and tracking capability are not sufficiently satisfied. We further introduce  $\mathbf{u}_{cmac}$  to complete satisfactory tracking performance and system robustness. Adequate training patterns and training time in the learning process required by the CMAC will be provided by the PD controller. Consequently, the PD controller and the CMAC are in a harmonizing status during learning and controlling cycles.

#### A. PD control

A PD controller consists of a proportional control factor and a derivative control factor. Define the PD controller as

$$\mathbf{u}_{pd} = \mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} \quad (7)$$

where  $\mathbf{K}_p$  and  $\mathbf{K}_d$  are the proportional gain matrix and the derivative gain matrix, respectively. Properly chosen, a simple PD controller is capable of improving damping and reduces maximum overshoot, rising time, and settling time. However, the proportional and derivative gains are fixed in general. Low sensitivity to parametric variations and external disturbances cannot be guaranteed if a proper PD controller is used for VTOL aircrafts. In the following section, an intelligent CMAC approach is developed to incorporate the PD controller and bolster strong system robustness and stability.

#### B. CMAC design

##### (1) Recalling phase

CMAC performs like an on-line tuning look-up table. This method imitates the model of the human memory, and has a fast learning capability. Typically, CMAC includes recalling and training procedures. Figure 3 shows the structure of CMAC. The whole input space is quantized by the discrete reference states,  $z_1, z_2, \dots, z_{40}$ . Every reference state  $z_j$  is mapped into the output  $y_{z_j}$ . Let the output of the CMAC be defined a

$$y_{z_j} = \mathbf{a}_{z_j} \mathbf{w} \quad (8)$$

where  $\mathbf{w} = [w_1, w_2, \dots, w_{42}]^T$  is the memory weighting vector and  $\mathbf{a}_{z_j} = [a_{j,1}, a_{j,2}, \dots, a_{j,42}]$  is the associated memory row vector of  $z_j$ . The number of memory is 42 in the CMAC. For each state, the number of referred memory is 3. Thus, three elements in  $\mathbf{a}_{z_j}$  are 1, and all else are 0. Figure 4 shows the memory allocation of the CMAC with reference states and referred memories. For example, the reference state  $z_1$  maps three memory addresses,  $m_1 \sim m_3$ , and the reference state  $z_{40}$  maps three memory addresses,  $m_{40} \sim m_{42}$ . Suppose that the memory row vectors  $\mathbf{a}_{z_1} = [1, 1, 1, 0, \dots, 0] \in R^{42}$  and  $\mathbf{a}_{z_{40}} = [0, \dots, 0, 1, 1, 1] \in R^{42}$  are chosen. Then the corresponding memory weights are  $w_1 \sim w_3$  and  $w_{40} \sim w_{42}$ , respectively. Referring to Fig. 4, every two adjacent reference states overlap two memory addresses. Because two same memory addresses are activated, the outputs of two adjacent reference states will not differ much. In other words, the input quantization affects the learning accuracy. More reference states imply more accurate learning. Accordingly, more memory addresses will be required.

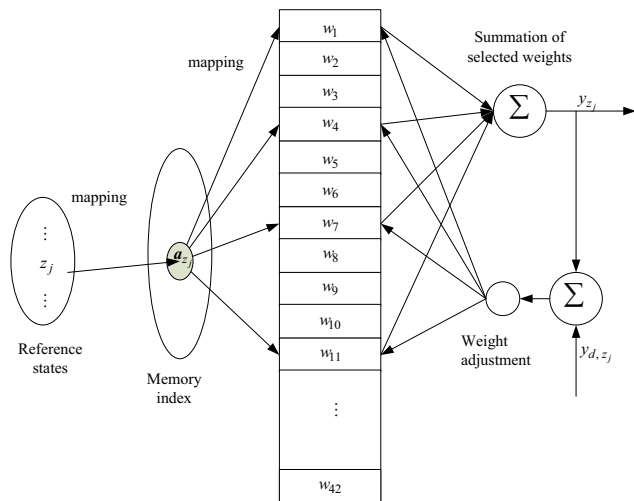


Fig. 3. Structure of the CMAC.

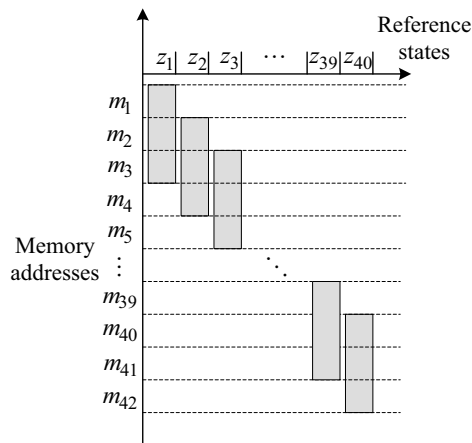


Fig. 4. Memory allocation of the CMAC

(2) Training phase

Referring to Fig. 2, the training object of the CMAC is to construct a learning process for mapping from velocities to control inputs. The vertical and horizontal axes each need one CMAC. The training patterns are the vertical and horizontal velocities of the VTOL aircraft. Responding to the desired velocities  $y_{1d}$  and  $y_{2d}$ , the CMAC output will approach the desired control input. In (8), the memory weighting vector  $\mathbf{w}$  is adjusted iteratively according to the training error. Only the selected weights are adjusted in response to the error in each learning period.

There are two weighting vectors,  $\mathbf{w}_i$ ,  $i = 1, 2$ , for vertical and horizontal axes, respectively. The on-line updating law is chosen to be

$$\mathbf{w}_i^{(k+1)} = \mathbf{w}_i^{(k)} + \Delta \mathbf{w}_i^{(k)}$$

$$= \mathbf{w}_i^{(k)} + \frac{\phi_i}{3} \mathbf{a}_i^T (u_i^{(k)} - u_{i,cmac\_training}^{(k)}), i = 1, 2, \quad (9)$$

where  $\mathbf{a}_i^{(k)}$  denotes the associate memory row vector of the  $k$  th training pattern for the  $i$  th CMAC. The constant  $\phi$  is the learning rate, and satisfies  $0 < \phi < 1$ . Generally, learning accuracy will be enhanced with a large number of training patterns.

C. Control system performance

Referring to Fig. 2, the PD controller is first designed to stabilize the VTOL aircraft control system. The initial memory weights of the CMAC are zero, i.e.,  $\mathbf{u}_{cmac}$  is zero. The control input comes only from the PD controller. Once the PD controller begins to work, a series of training patterns for the CMAC will be obtained, and the CMAC begins learning and merging the control.

In every sampling cycle, the desired vertical and horizontal velocities, i.e. the reference commands, are sent to the PD controller and the CMAC. The CMAC recalls the corresponding memories to determine the control input  $\mathbf{u}_{cmac}$ . With feedback of the tracking error, the PD controller determines the control input  $\mathbf{u}_{pd}$ . Then, a training process follows. The velocity vector  $\mathbf{y}$  is used as the training pattern. The control input vector  $\mathbf{u}$  is used as the desired output of the CMAC. Through the weights updating law (9), the updated memory weights of the CMAC will be applied in the next sampling cycle. If the learning result of the CMAC is accurate, the CMAC will support the PD controller to ensure system robustness as well as stability. The control algorithm is investigated as follows.

Consider that the VTOL aircraft is in forward flight and level altitude. Two cases are investigated:

**Case I.** Vertical velocity  $y_{1d} = 0.9(1 - e^{-2t})$  and horizontal velocity  $y_{2d} = 0$  (normalized);

**Case II.** Vertical velocity  $y_{1d} = 0$  and horizontal velocity

$$y_{2d} = 0.9(1 - e^{-2t}) \text{ (normalized).}$$

The initial condition is assumed to be  $x_1 = x_2 = 0$ . The PD controller is chosen to have  $\mathbf{K}_p = [50 \ 0; 0 \ 30]$  and  $\mathbf{K}_d = [0.2 \ 0; 0 \ 0.2]$ . The weights updating law (9) is applied, in which the learning rate is chosen as  $\phi_i = 0.1$ ,  $i = 1, 2$ . Assume that a wind gust, with a downward vertical constant acceleration of -1 (normalized), occurs and lasts from  $t = 8$  sec to 9 sec.

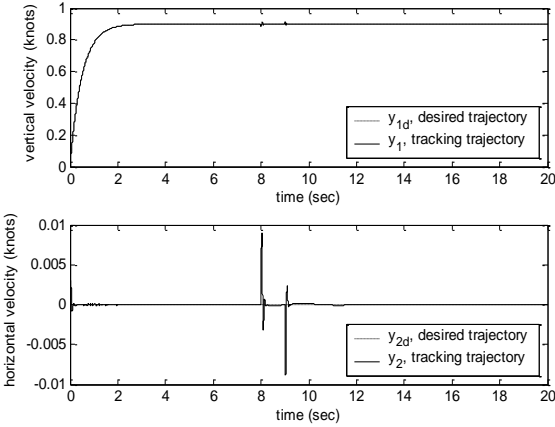


Fig. 5. Vertical and Horizontal velocities in Case I.

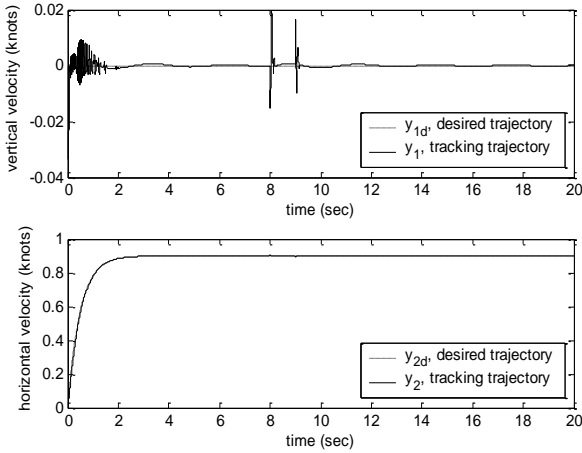


Fig. 6. Vertical and Horizontal velocities in Case II

Figures 5 and 6 show the tracking responses and the robust performance. Using the proposed control method, the tracking is fast and the system robustness is ensured. The pitch rates and pitch angles are small and smooth, as shown in Figs. 7 and 8. The pitch angle adaptively changes to maintain the tracking and the system robustness. The control activities of  $\mathbf{u}_{pd}$  and  $\mathbf{u}_{cmac}$  are shown in Figs. 9 to 12. Because the training process of the CMAC is successfully completed,  $\mathbf{u}_{cmac}$  dominates the control action, and  $\mathbf{u}_{pd}$  converges to zero. The proposed CMAC is capable of taking over most of the control need. Therefore, the PD controller can abdicate quickly. This implies that the control system will be stable and robust even when the PD controller is not well designed.

In summary, the PD controller stabilizes the VTOL aircraft control system and supplies training patterns to the CMAC. The CMAC ensures system robustness and contributes sufficient control to achieve speed tracking performance.

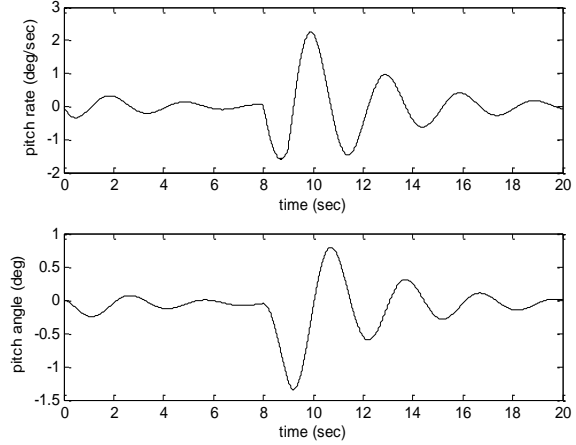


Fig. 7. Pitch rate and angle in Case I

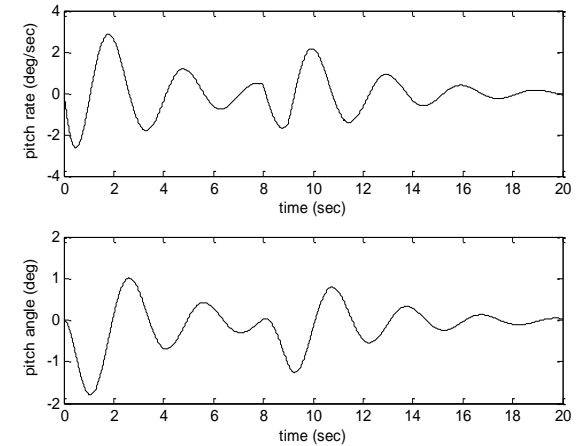


Fig. 8. Pitch rate and angle in Case II

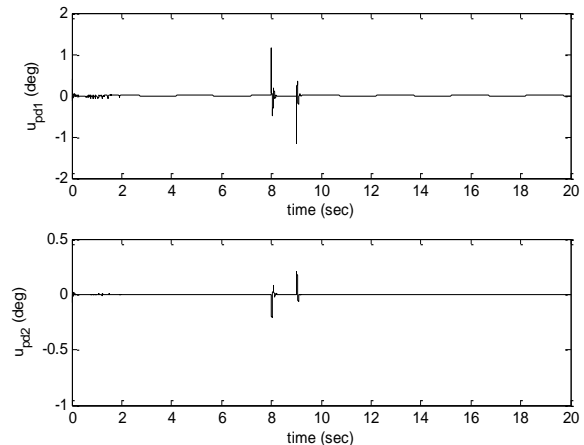


Fig. 9. Control inputs from PD controller in Case I

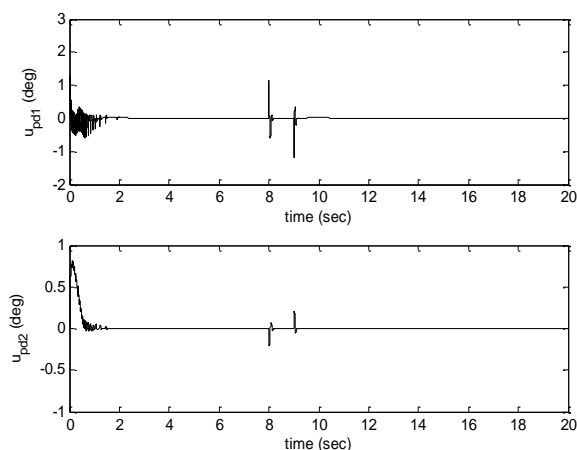


Fig. 10. Control inputs from PD controller in Case II

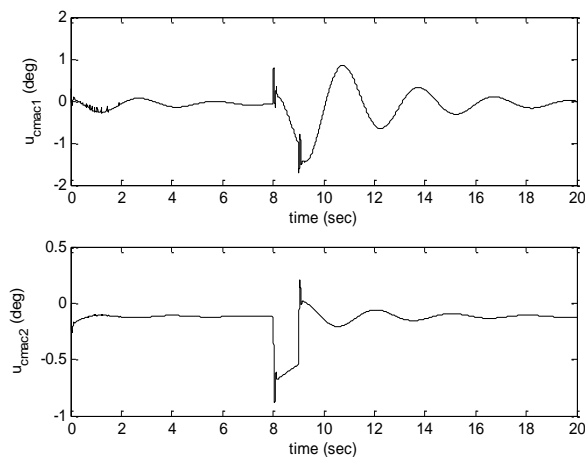


Fig. 11. Control inputs from CMAC controller in Case I

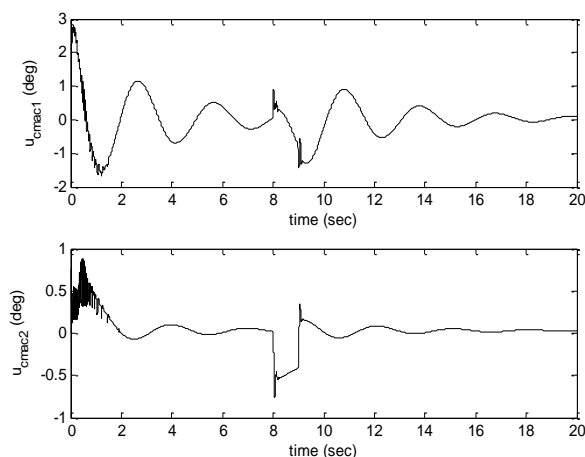


Fig. 12. Control inputs from CMAC controller in Case II

#### IV. CONCLUSION

PD control is a simple and effective control method. However, it does not ensure the robustness if used alone for uncertain systems. CMAC can be used for robust control. However, it requires training patterns for tuning some weighting factors. A novel CMAC used together with a PD controller design is

proposed in this paper. The PD controller provides the CMAC training patterns. The CMAC assists the PD controller to ensure the robustness. Even when the PD controller is not designed well, the CMAC is capable of doing a good job of robust control through on-line recalling and training procedures. Numerical examples validate the effectiveness of the proposed control method.

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