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On the Fast Convergence of DD-LMS DFE using a good strategy initialization

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***Abstract*—In wireless communication system, a Decision Feedback Equalizer (DFE) to cancel the intersymbol interference (ISI) is required. In this paper, an exact convergence analysis of the (DFE) adapted by the Least Mean Square (LMS) algorithm during the training phase is derived by taking into account the finite alphabet context of data transmission. This allows us to determine the shortest training sequence that allows to reach a given Mean Square Error (MSE). With the intention of avoiding the problem of ill-convergence, the paper proposes an initialization strategy for the blind decision directed (DD) algorithm. This then yields a semi-blind DFE with high speed and good convergence.

***Keywords*—Adaptive Decision Feedback Equalizer, Performance Analysis, Finite Alphabet Case, Ill-Convergence, Convergence speed.

I. Introduction

Wireless communications systems support a wide range of high-quality services that require high transmission rates. However, the propagation characteristics of wireless communication channels make it difficult to achieve high speed data transmission at low error rates because of the presence of ISI. To combat the effect of ISI, the classical technique of adaptive equalization is often suggested as a possible method.

In this paper we analyze Decision Feedback Equalizers that are usually considered as a good compromise between complexity and performance [8][4]. The European wireless local-area-network system (HIPERLAN) is a typical example of high speed wireless communication application that adopts DFE to overcome the adverse effect of multipath ISI [16].

However, the functioning of the DFE equalizer is in general disturbed by the problem of ill-convergence preventing a correct equalization of the channel[11]. This situation can be solved by using a training sequence. Indeed, in wireless communications cellular systems (GSM or UMTS), a short training sequence is included in each transmitted frame in order to help the equalizer to improve its tracking capability. The training sequence is employed in order to adapt the equalizer weights into an opened-eye condition using the LMS adaptive algorithm. Then the equalizer is changed to the DD mode, in which the effective information is transmitted. In order to have high convergence speed, the length of the training sequence must be minimized. In this paper, we determine the algorithm parameters that give the shortest training sequence and ensure a good equalization.

For this purpose, performance analysis of the DFE equalizer must be developed. In this paper, we propose a new approach using mathematical tools allowing an exact analysis of adaptive equalizers in the finite alphabet case [6]. In particular, an exact convergence analysis of the DFE equalizer during the training phase is deduced without any constraining or unrealistic hypothesis. Using this exact analysis, we can deduce quantitative and qualitative results for DFE design. In particular, we propose a good convergence strategy of the DFE equalizer after a very short training sequence.

In order to assess such results, this paper is organized as follows: in section 2, we describe the system model and the DFE equalizer. In section 3, we present an exact analysis approach tailored for digital transmission context. We use this approach to give an exact convergence analysis of the DFE equalizer during the training phase. In section 4, we study the problem of local minima and ill-convergence in order to analyze different behavior of the DFE which is be done in section 5. In fact, in this section, two situations are analyzed:

− when local minima of the error surface are not attainable, we determine the shortest training sequence that allows to reach a fixed bit error rate (BER).

− when local minima are reachable, we propose a good initialization guidelines that ensure desirable convergence after a very short training sequence.

II. Problem formulation and general hypothesis

We consider a data transmission system over a transversal channel and a DFE equalizer as depicted in figure 1. The channel is assumed to be invariant during the transmission and it is characterized by its impulse response $F = [f_1, ..., f_{L-1}]$. The received baseband signal sampled at the symbol rate at time n is:

$$c_n = a_n + \sum_{k=1}^{L-1} f_k a_{n-k} + b_n - \sum_{k=1}^{L-1} h_k \hat{a}_{n-k} \qquad (1)$$

Where L is the channel constraint length, $\{a_n\}$ is the transmitted data which remain to a finite alphabet set (for example $\{\pm 1\}$, $\{\pm 1, \pm j\}$) depending on the modulation, $\{\hat{a}_n\}$ is the received data after decision ($\hat{a}_n = sign(c_n)$), $H_n = [h_1, ..., h_{L-1}]$ is the parameter vector of the DFE equalizer and b_n is the additive noise assumed to be zero-mean noise independent of a_n with variance σ_b^2.

The adaptation of the DFE equalizer is assured by the Decision Directed (DD) algorithm [15]. During the training sequence we have :

$$\begin{cases} c_n &= a_n + F^T A_n + b_n - H_n^T A_n \\ e_n &= c_n - a_n \\ H_{n+1} &= H_n + \mu e_n A_n^* \end{cases} \qquad (2)$$

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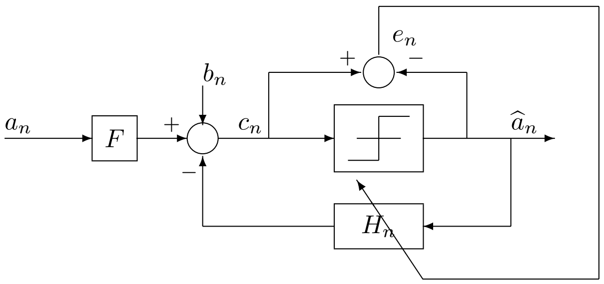

Fig. 1. The transmission system including a Decision Feedback Equalizer

where e_n is the decision error, $A_n = [a_{n-1}, ..., a_{n-L+1}]^T$ and μ is a positive step size of the (DD) algorithm.
The DFE is described by the mean behavior and the evolution of the covariance of the deviation vector $V_n = H_n - F$. So, the DFE behavior can be described by the following recursions:

$$\begin{aligned} E(V_{n+1}) &= E((I-\mu A_n^* A_n^T)V_n) \\ E(V_{n+1}V_{n+1}^H) &= \mu^2\sigma_b^2 E(A_n^* A_n^T) + \\ & E((I-\mu A_n^* A_n^T)V_n V_n^H (I-\mu A_n^* A_n^T)^H) \end{aligned} \quad (3)$$

Since A_n and V_n are dependent, it is difficult to solve equation (3) [1] [2] [3]. To overcome this problem, we here propose to apply an original approach based on the finite alphabet input [6] [7][13].

III. Analysis of the DFE : an exact approach

A. The finite alphabet approach formulation

In digital transmission context, the input signal a_n remains in a finite alphabet set such as QAM signal... Consequently, the observation vector A_n remains also in a finite alphabet set $A = \{W_1, W_2, ..., W_N\}$ with cardinality N. For example $N = 4$ and $A = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$ if $L = 1$ and a_n is a BPSK signal $\{\pm 1\}$. Since A_n is stationary, it can be modeled by a discrete-time Markov chain $\{\theta(n)\}$ with finite state space $\{1, 2, ..., N\}$[14]:

$$A_n = W_{\theta(n)}$$

This Markov chain is characterized by its probability transition matrix $P = [P_{ij}]$ and its stationary probability vector π_∞.
The finite alphabet approach consists, since there is N possibilities of $W_{\theta(n)}$, in splitting the vector $E(V_n)$ and the matrix $E(V_n V_n^H)$ in N components defined by :

$$\begin{aligned} q_j(n) &= E(V_n 1_{\theta(n)=j}) \\ Q_j(n) &= E(V_n V_n^H 1_{\theta(n)=j}) \end{aligned}$$

respectively where $1_{\theta(n)=j}$ is the indicator function. So, we have:

$$\begin{aligned} E(V_n) &= \sum_{j=1}^{N} q_j(n) \\ E(V_n V_n^H) &= \sum_{i=1}^{N} Q_j(n) \end{aligned} \quad (4)$$

The system is governed by the following recursions :

$$q_j(n+1) = \sum_{i=1}^{N} E(V_{n+1} 1_{\theta(n+1)=j} 1_{\theta(n)=i})$$
$$Q_j(n+1) = \sum_{i=1}^{N} E(V_{n+1} V_{n+1}^H 1_{\theta(n+1)=j} 1_{\theta(n)=i})$$

According to (3), we have :

$$q_j(n+1) = \sum_{i=1}^{N} E(M_n V_n 1_{\theta(n+1)=j} 1_{\theta(n)=i})$$
$$Q_j(n+1) = \mu^2\sigma_b^2 \sum_{i=1}^{N} E(A_n^* A_n^T 1_{\theta(n+1)=j} 1_{\theta(n)=i}) +$$
$$\sum_{i=1}^{N} E(M_n V_n V_n^H M_n^H 1_{\theta(n+1)=j} 1_{\theta(n)=i})$$

Where $M_n = (I - \mu A_n^* A_n^T)$
Since A_n remains in a finite alphabet set, M_n remains also in a finite alphabet set $\{M_i = I - \mu W_i^* W_i^T\}$, and then we obtain:

$$q_j(n+1) = \sum_{i=1}^{N} M_i E(V_n 1_{\theta(n+1)=j} 1_{\theta(n)=i})$$

$$Q_j(n+1) = \mu^2\sigma_b^2 \sum_{i=1}^{N} W_i^* W_i^T E(1_{\theta(n+1)=j} 1_{\theta(n)=i}) +$$

$$\sum_{i=1}^{N} M_i E(V_n V_n^H 1_{\theta(n+1)=j} 1_{\theta(n)=i}) M_i^H$$

Since the input is caracterized by a Markov chain, we have :

$$E(V_n 1_{\theta(n+1)=j} 1_{\theta(n)=i}) = P_{ij} E(V_n 1_{\theta(n)=i}) = P_{ij} q_i(n)$$

$$E(V_n V_n^H 1_{\theta(n+1)=j} 1_{\theta(n)=i}) = P_{ij} E(V_n V_n^H 1_{\theta(n)=i}) = P_{ij} Q_i(n)$$

$$E(1_{\theta(n+1)=j} 1_{\theta(n)=i}) = P_{ij} \pi_\infty$$

Hence,

$$\begin{aligned} q_j(n+1) &= \sum_{i=1}^{N} (I - \mu W_i^* W_i^T) q_i(n) P_{ij} \\ Q_j(n+1) &= \sum_{i=1}^{N} (M_i Q_i(n) M_i^H P_{ij}) + Z_j \end{aligned} \quad (5)$$

With $Z_j = \mu^2\sigma_b^2 \sum_{i=1}^{N} W_i^* W_i^T P_{ij} \pi_\infty$.
In order to rewrite (5) in linear form, we introduce the useful notations :

$$\begin{aligned} \tilde{q}(n) &= [q_1(n)^T, q_2(n)^T, ..., q_N(n)^T]^T \\ \tilde{Q}(n) &= [vec(Q_1(n))^T, vec(Q_2(n))^T, ..., vec(Q_N(n))^T]^T \\ \tilde{Z}(n) &= [vec(Z_1)^T, vec(Z_2)^T, ..., vec(Z_N)^T]^T \end{aligned}$$

Where $vec(.)$ is the operator that transforms matrix (m, n) in vector of length mn.

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Referring to [12] in order to use Kronecker product, the compact formulae is:

$$\begin{array}{l} \tilde{q}(n+1) = \Lambda \tilde{q}(n) \\ \tilde{Q}(n+1) = \Gamma \tilde{Q}(n) + \tilde{Z} \end{array} \quad (6)$$

Where

$$\begin{array}{l} \Lambda = (P^T \otimes I) diag(I - \mu W_i^* W_i^T) \\ \Gamma = (P^T \otimes I) diag((I - \mu W_i^* W_i^T) \otimes (I - \mu W_i^* W_i^T)) \end{array} \quad (7)$$

Γ and Λ contain all relevant informations about the DFE in the training phase.

Since these matrices are constants and depend only on the step size and statistical properties of the input signal, performances of the algorithm depend only on eigenvalues of these matrices.

B. Transient analysis

According to equation (6), the exact transient behavior of the DFE during the training phase is characterized by :

$$\begin{array}{l} \tilde{q}(n) = \Lambda^n \tilde{q}(0) \\ \tilde{Q}(n) = \Gamma^n \tilde{Q}(0) + (\Gamma - I)^{-1}(\Gamma^n - I)\tilde{Z} \end{array} \quad (8)$$

We can then determine exactly :

- The critical step size μ^c defined as the step from which the algorithm diverges. This is found when Γ has an eigenvalue λ higher than 1. So, we can determine μ^c as follows :

$$\mu^c = arg(\lambda_{max}(\mu) = 1)$$

- The optimal step size μ^{opt} defined as the step that gives the maximal speed convergence then the shortest training sequence. this is found when Γ has all eigenvalues small. So, we can determine μ^{opt} as follows :

$$\mu^{opt} = arg(min(\lambda_{max}(\mu)))$$

The quantity $\lambda_{max}(\mu^{opt})$ fixes the speed of convergence.

Simulation results

For simplicity, we consider in this simulation :

- A BPSK constellation $\{\pm 1\}$ characterized by the following transition matrix:

$$p = \left[\begin{array}{cc} 0.7 & 0.3 \\ 0.3 & 0.7 \end{array} \right]$$

It is important to note that the input sequence is correlated.

- A noise with power equal to unit.
- A non minimum phase channel with order 2.

In this case $\mu^c = 0.98$ and $\mu^{opt} = 0.5$. These values are exactly determined from figure 2.

IV. Design of DFE with short training sequence

In this section, we propose to design an optimal DFE equalizer :

- DFE with short training sequence in order to speed up the convergence, this is assured by a good choice of the algorithm parameters (step size...).
- DFE that Converges to global minima. In fact, the problem of ill-convergence can be avoided by using a good strategy initialization of the DFE parameters.

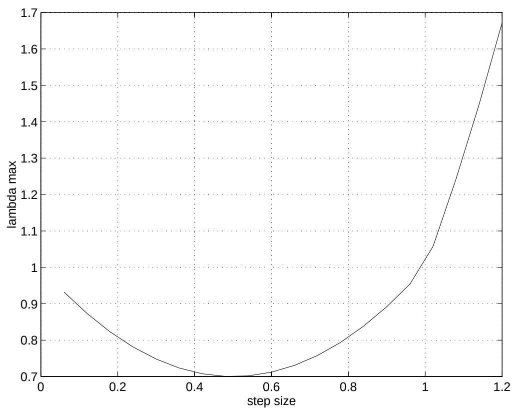

Fig. 2. Largest eigenvalue of Γ versus step size μ

A. Polytopes and local minima

In the context of channel equalization in the presence of finite alphabet signals, criteria of optimization applied to the DFEs, present a multimodal error surface, having several minima [5]. Among them, we distinguich, on the one hand, the global minimum corresponding to the desired minimum and, on the other hand, the local minima (undesirable minima) preventing the convergence of the algorithm.

The cost function error surfaces are obtained by concatenating polytopes which are related to the partition of the space of the parameter vector H_n. The interest of this partition is that each polytope cannot present more than one convergence point.

In the case of a quadratic criterion, these surfaces are piecewise quadratic and two situations can occur :

- Case 1 : error surface without local minima (figure 3): when these parabolas can present minima that do not belong to the associated polytopes, minima are said to be not attainables.

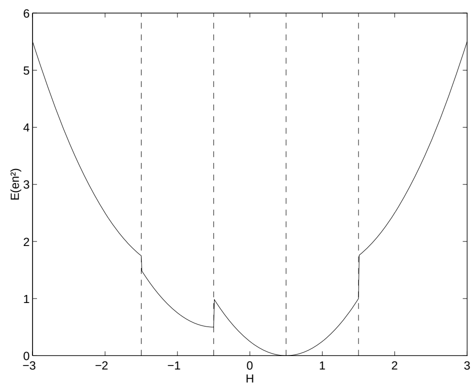

Fig. 3. Error surface of DD cost function for a first order equalizer and a minimum phase channel

- Case 2 : error surface with local minima (figure 4): when these parabolas present minima that belong to the associated polytopes, it is the worst case because minima are attainables.

Some kinds of models are used in order to describe channel behaviors, among them a statistical model using Markov chain

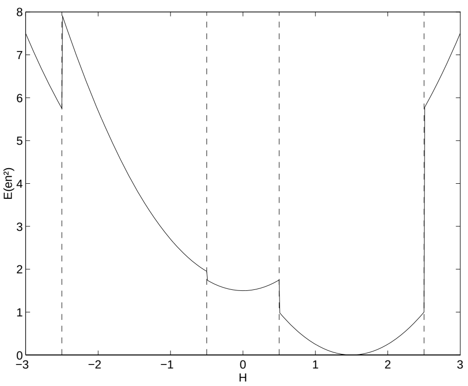

Fig. 4. Error surface of DD cost function for a first order equalizer and a non minimum phase channel

to describe transition from state to state related to the channel impulse response variations. The transition matrix depends on the transmission medium.
For example, to describe Land Mobile Satellite channel, we can consider three state Markov chain tries to account for different shadowing events. Each three possible channel impulse response are of order 2 [9].

B. Minimal length of the training sequence

In case 1, when the minima are not attainables , there is no problem of ill-convergence. So, we can determine the shortest training sequence that allows to reach a fixed $BER = BER_0$. It is important to note that there is a relationship between BER and MSE, so if the BER is fixed the MSE is also fixed [8][10].
Both the training sequence and the MSE depend on Γ, consequently, they depend on the alphabet, the step size and the transition matrix P.
From(2), we can deduce the MSE at the iteration k (Appendix A) as follows:

$$MSE(k) = \sigma_b^2 + \sum_{i=1}^{N} W_i^H \otimes W_i^T vec(Q_i(k)) \qquad (9)$$

The goal here is to determine the smallest value of k such as $MSE(k) = MSE_0$.
For simplicity, let us consider a particular equalization scheme with filter length $L = 1$ and i.i.d inputs belonging to a fixed alphabet which remains in $\{\pm 1\}$. The noise is with variance σ_b^2 equal to 1.
According to (7), Γ in this case is equal to :

$$\Gamma = (1-\mu)^2 \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

and $\tilde{Z} = \frac{\mu^2}{2}[1,1]^T$.
We must firstly determine μ such as $MSE = MSE_0$.
If we introduce the quantity Excess MSE ($EMSE$) defined as :

$$EMSE = \frac{MSE - \sigma_b^2}{\sigma_b^2}$$

According to (8) and (9), at the iteration k $EMSE$ is equal to:

$$EMSE(k) = \frac{-\mu}{\mu-2} + (1-\mu)^{2k}((a+b) + \frac{\mu}{\mu-2})$$

Where a and b are the components of the vector $\tilde{Q}(0)$.
If we fixe $MSE = MSE_0$, we have then :

$$\frac{EMSE_0 + \frac{\mu}{\mu-2}}{(a+b) + \frac{\mu}{\mu-2}} = (1-\mu)^{2k}$$

We deduce k as a function of μ:

$$k = \frac{Ln(\frac{(\mu-2)EMSE + \mu}{(\mu-2)(a+b) + \mu})}{Ln((1-\mu)^2)}$$

With $Ln(.)$ is the nuperian logarithm function.
The quantity k can be minimized and the smallest value is exactly the length of the training sequence n_f which can be determined as follows :

$$n_f = min_{\mu \in [0,2]}(k(\mu))$$

The step size μ_0 that gives the shortest training sequence is then :

$$\mu_0 = arg(min(k(\mu)))$$

Simulation result

We want to design a DFE which gives at the end of the training sequence $EMSE_0 = 0.1$, we suppose $\tilde{Q}(0) = [-10, 100]$. According to the last study, this is possible if we choose $\mu = 0.165$ and the length of the shortest training sequence is equal to 26. In figure (5), perfect agreement between simulation and theoretical results illustrates the exactness of the new approach.

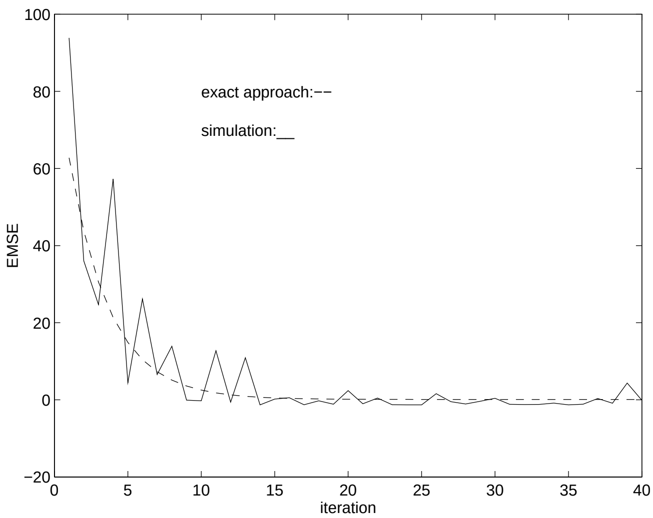

Fig. 5. $EMSE$ versus time

C. Initialization Strategy for DFE equalizer

1) Formulation of the problem: To avoid the ill-convergence (convergence to undesired minima) of the DFE, the DFE parameters must belong to regions (said polytopes $\mathcal{P}$) corresponding to correct decisions. The width of this polytope is not related to the adaptation. When the cardinality of the alphabet and the lengths of the equalizer and the channel increase, the polytope of correct decision becomes more narrow.
Note that for a good initialization, only a general a priori knowledge of the good polytope is needed.
Blind algorithms are developed with a general a priori knowledge of the input data structure. Due to the fact that in finite alphabet case we can analyze precisely the transient behavior of the DFE during the training phase, the main idea here is to propose the following initialization strategy with a general a priori knowledge of the channel allowing a good convergence to the global minima.

- First, after a short training sequence with imposed length n_f (some ten iterations) we determine an initialization domain which ensures that the equalizer parameter H_n enters in the mean sense, in the polytope corresponding to correct decisions. Note that the exact convergence to the minimum of the polytope is not needed, so it is possible to shorten the length of the training sequence. When n_f is small and the a priori knowledge on the channel is poor, this initialization domain is limited.
- Secondly, when the equalizer enters in the good polytope, the adaptation must verify that the equalizer remains in this polytope. This more restrictive condition is related to the a priori knowledge of the noise power on the channel. Higher is the noise power, more restrictive is the adaptation step domain. Exact analytical results can be deduced from equation (7). The size of the matrices increases drastically with the length of the equalizer and the cardinality of the data. However, numerical but exact conclusions can be deduced. We assign the following conditions:

$$\begin{aligned} &E(H_{n_f}) \in \mathcal{P} \\ &E(|H_{n_f} - E(H_{n_f})|^2) < \frac{1}{\alpha} distance(E(H_{n_f}), \mathcal{H}) \end{aligned} \quad (10)$$

where $\alpha > 1$ is a coefficient that limits the variations amplitude of H_n inside the polytope, $\mathcal{H}$ is the limiting hyperplanes of $\mathcal{P}$.
Note that in this analysis, we doesn't consider the trivial case where an adequate choice of the step size leads to a convergence in one iteration.

2) Practical case: In order to illustrate the idea, the case of an equalizer of length 1 and an alphabet $\{\pm 1\}$ with the transition matrix P such as $p_{ij} = 1/2$, is considered. We demonstrate in appendix B that the polytope of correct decision is then $I = [-1 + F, 1 + F]$. Consequently, H must be in $[-1+F, 1+F]$ in order to eliminate the ill-convergence problem and equation(10) becomes :

$$\begin{aligned} &-1 + F < E(H_{n_f}) < 1 + F \\ &E(|H_{n_f} - E(H_{n_f})|^2) < \\ &\frac{1}{\alpha} min(1 + F - E(H_{n_f}), 1 - F + E(H_{n_f})) \end{aligned} \quad (11)$$

If the a priori knowledge on the channel is $f_1 < F < f_2$, we prove in appendix C that the initial value of the equalizer parameter H_0 must lie in a domain D defined by :

$$\begin{cases} [f_2 - \frac{1}{|(1-\mu)|^{n_f}}, f_1 + \frac{1}{|(1-\mu)|^{n_f}}] \\ 1 - (\frac{2}{(f_2 - f_1)})^{1/n_f} < \mu < \mu' \end{cases}$$

μ' depends on equation (10), here $\mu' = 2$.
In order to enter in the good polytope in the mean sense. A choice of the initial value of the equalizer parameters in this domain eliminates the ill-convergence problem (in the mean sense).

Simulation results

For example, if we suppose that $-5 < F < 5$ and we impose $n_f = 16$, we must choose $0.1 < \mu < 2$ (see figure 6). If we

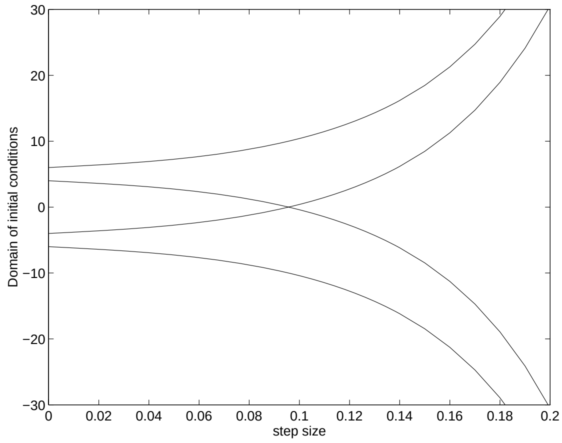

Fig. 6. The domain D of initial conditions allowing a correct convergence for all step size value

fixe $\mu = 0.2$ then we obtain $-30 < H_0 < 30$. We represent in figure 7, the evolution of the equalizer parameters for two cases : good initialization when H_0 belongs to D and bad initialization when H_0 is not in D.

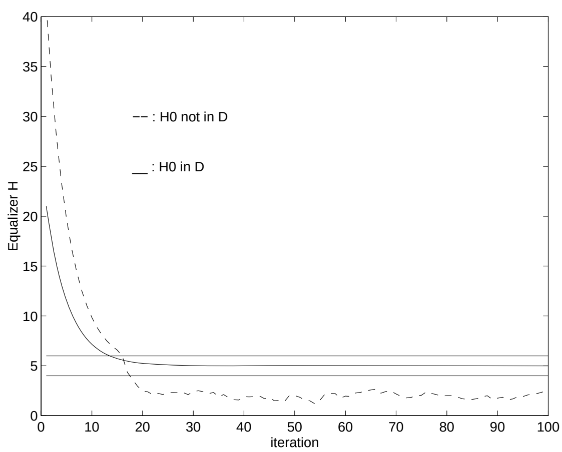

Fig. 7. evolution of the DFE Parameter versus time when it is goodly initialized (continuous line) and not goodly initialized (discontinuous line)

The equalizer with very short training sequence ($n_f = 16$) gives surely a correct decision (in the mean sense) in the blind mode if the equalizer initialization is in the domain D.

V. Conclusion

The design of semi-blind decision feedback equalizer with short training sequence is developed through an exact convergence analysis during the training phase. This study is possible because the input sequence belongs to a finite alphabet set.
We calculate the shortest training sequence that allows to reach a given MSE and we propose a convergence strategy for the blind decision directed (DD) algorithm that avoid ill-convergence of the DFE.

Appendix A

The goal is the calculus of MSE at the iteration k.
The error e_k is defined as :

$$e_k = F^T A_k + b_k - H_k^T A_k \tag{12}$$

So,

$$MSE(k) = E(| - V_k^T A_k + b_k|^2) \tag{13}$$

Since the noise b_k is assumed to be zero-mean, iid and independent of A_k, we have:

$$MSE(k) = \sigma_b^2 + E((V_k^T A_k)^2) \tag{14}$$

Since A_k belongs to a finite alphabet set, we have:

$$MSE(k) = \sigma_b^2 + \sum_{i=1}^{N} E(A_k^T V_k V_k^H A_k^* 1_{\theta(k)=i})$$

$$= \sigma_b^2 + \sum_{i=1}^{N} W_i^T Q_i(k) W_i^*$$

Using the tensor algebra properties [12] ($vec(A.B.C) = (C^T \otimes A)vec(B)$), we have :

$$MSE(k) = \sigma_b^2 + \sum_{i=1}^{N} W_i^H \otimes W_i^T vec(Q_i(k)) \tag{15}$$

Appendix B

The goal is the determination of the polytope I corresponding to the correct decision. This polytope must verify :

$$a(n) = \hat{a}(n)$$

Where $\hat{a}(n)$ is the received symbol at iteration n after decision ($\hat{a}(n) = sign(c_n)$)
Because $\hat{a}(n) = sign(a(n) + Fa(n-1) - H\hat{a}(n-1))$, we obtain:

$$a(n) = sign(a(n) + Fa(n-1) - H\hat{a}(n-1))$$

This is possible only when :

$$|Fa(n-1) - H\hat{a}(n-1)| < |a(n)|$$

Since $a_n \in \{\pm 1\}$, we have $|a(n)| = 1$, so :

$$-1 + F < H < 1 + F$$

Appendix C

The goal is to determine the initialization domain of the equalizer that eliminates the ill-convergence problem.
According to equation (8), we have :

$$\begin{aligned} \tilde{q}(n) &= \Lambda^n \tilde{q}(0) \\ &= \frac{1}{2}(1-\mu)^n \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tilde{q}(0) \end{aligned} \tag{16}$$

If we denote respectively c and d the components of vector $\tilde{q}(0)$, we have referring to equation (4) :

$$E(H_n) = (c+d)(1-\mu)^n + F \tag{17}$$

Since $E(H_0) = (c+d) + F$, we can deduce from equation (11), the domain D of initial conditions :

$$F - \frac{1}{|1-\mu|^{n_f}} < E(H_0) < F + \frac{1}{|1-\mu|^{n_f}} \tag{18}$$

If the a priori knowledge on the channel is $f_1 < F < f_2$, equalizer parameter H_0 must lie in a domain D defined by :

$$\begin{cases} [f_2 - \frac{1}{|(1-\mu)|^{n_f}}, f_1 + \frac{1}{|(1-\mu)|^{n_f}}] \\ 1 - (\frac{2}{(f_2 - f_1)})^{1/n_f} < \mu < \mu' \end{cases}$$

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