World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:3, No:3, 2009

PI Control for Second Order Delay System with Tuning Parameter Optimization

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Abstract—In this paper, we consider the control of time delay system by Proportional-Integral (PI) controller. By Using the Hermite-Biehler theorem, which is applicable to quasi-polynomials, we seek a stability region of the controller for first order delay systems. The essence of this work resides in the extension of this approach to second order delay system, in the determination of its stability region and the computation of the PI optimum parameters. We have used the genetic algorithms to lead the complexity of the optimization problem.

Keywords—Genetic algorithm, Hermit-Biehler theorem, optimization, PI controller, second order delay system, stability region.

I. Introduction

SYTEMS with delays represent a class within infinite size largely used for the modeling and the analysis of transport and propagation phenomena (matter, energy or information) [7, 15]. They naturally appear in the modeling of processes found in physics, mechanics, biology, physiology, economy, dynamics of the populations, chemistry, aeronautics and aerospace. In addition, even if the process itself does not contain delay, the sensors, the actuators and the computational time implied in the development of its control law can generate considerable delays [7]. The latter have a considerable influence on the behavior of closed-loop system and can generate oscillations and even instability [3].

PID controllers are of high interest thanks to their broad use in industrial circles [2]. Traditional methods of PID parameter tuning are usually used in the case of the systems without delays [5], [6]. A robust PI/PID design via numerical optimization for delay processes was proposed in [20]. Padma Sree and Sirinivas [8] propose PI/PID controllers design for first-order delay system by extracting the coefficients of the numerator and denominator of the closed-loop transfer function. Shafie and Shenton [9] proposed a graphical technique based on the D-partition method for PID controller tuning. Xiang et al [19] used the Nyquist stability criterion to solve the stabilization problem of second-order unstable delay process by PID controller. A tuning method using a first-order set-point filter was proposed for PI controller setting of unstable first-order delay system [18]. In [16], Roy and Iqbal have explored PID tuning of first order delay system using a first order Pad approximation and the Hermite-Biehler stabilization framework. An analytical approach was developed in [11, 12, 13] and allowed the characterization of the stability region of delayed systems controlled via PID. Indeed, by using the Hermit-Biehler theorem applicable to the quasi-polynomials [1, 13], the authors have developed an analytical characterization of all values of the stabilization gains (K_p, K_i, K_d) of the regulator for the case of first order delay system. In [17], Silva et al have determined all feedback gain values that stabilize a second order delay plant.

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In our work we extend the Silva et al approach [17] to a second order delay system using PI controller. Also, we look for optimum regulators under constraint of a global criterion which takes into account the ponderation actions of several conditions related to the error signal (i.e: speed, precision and/or control signal).

This paper is structured as follows: in section 2, we present the theorem of Hermit-Biehler applicable to the quasi-polynomials. Section 3 is devoted to the problem formulation for first order delay system controlled via PI controller. The extension of this approach is developed in section 4 where a solution for the second order delay system stabilization problem is detailed. In order to obtain optimal regulator in the zone of stability, a description of the genetic algorithms is presented in section 5. Section 6 is reserved for simulations' results.

II. Preliminary results for analyzing time delay system

Several problems in process control engineering are related to the presence of delays. These delays intervene in dynamic models whose characteristic equations are of the following form [10, 11]:

$$\delta(s) = d(s) + e^{-L_1 s} n_1(s) + e^{-L_2 s} n_2(s) \quad + ... + e^{-L_m s} n_m(s) \tag{1}$$

Where: $d(s)$ and $n(s)$ are polynomials with real coefficients and L_i represent time delays. These characteristic equations are recognized as quasi-polynomials. Under the following assumptions:

$$\begin{array}{l} (A_1)\ \ \deg(d(s)) = n \ and\ \deg(n_i(s)) < n \ for\, i = 1, 2, ..., m \\ (A_2)\ L_1 < L_2 < ... < L_m \end{array} \tag{2}$$

One can consider the quasi-polynomials $\delta^*(s)$ described by :

$$\begin{aligned} \delta^*(s) &= e^{sL_m}\delta(s) \\ \delta^*(s) &= e^{sL_m} d(s) + e^{s(L_m - L_1)} n_1(s) \\ &\quad + e^{s(L_m - L_2)} n_2(s) + ... + n_m(s) \end{aligned} \tag{3}$$

The zeros of $\delta^(s)$ are identical to those of $\delta^*(s)$ since e^{sL_m} does not have any finite zeros in the complex plan. However, the quasi-polynomial $\delta^*(s)$ has a principal term since the coefficient of the term containing the highest powers of s and e^s is nonzero. If $\delta^*(s)$ does not have a principal term, then it has an infinity roots with positive real parts [1, 13].

The stability of the system with characteristic equation (1) is equivalent to the condition that all the zeros of $\delta^*(s)$ must be in the open left half of the complex plan. We said that $\delta^*(s)$ is Hurwitz or is stable. The following theorem gives a necessary and sufficient condition for the stability of $\delta^*(s)$.

theorem 1[10, 11, 12, 13]
Let $\delta^*(s)$ be given by (3), and write:

$$\delta^*(j\omega) = \delta_r(\omega) + j\delta_i(\omega) \tag{4}$$

where $\delta_r(\omega)$ and $\delta_i(\omega)$ represent respectively the real and imaginary parts of $\delta^*(j\omega)$. Under conditions (A_1) and (A_2), $\delta^*(s)$ is stable if and only if:

1: $\delta_r(\omega)$ and $\delta_i(\omega)$ have only simple, real roots and these interlace,

2: $\delta_i^{'}(\omega_0)\delta_r(\omega_0) - \delta_i(\omega_0)\delta_r^{'}(\omega_0) > 0$ for some w_0 in $[-\infty, +\infty]$

where $\delta_r^{'}(\omega)$ and $\delta_i^{'}(\omega)$ denote the first derivative with respect to w of $\delta_r(\omega)$ and $\delta_i(\omega)$, respectively.

A crucial stage in the application of the precedent theorem is to verify that and have only real roots. Such a property can be checked while using the following theorem .

theorem 2[10, 11, 12, 13]
Let M and N designate the highest powers of s and e^s which appear in $\delta^*(s)$. Let η be an appropriate constant such that the coefficient of terms of highest degree in $\delta_r(\omega)$ and $\delta_i(\omega)$ do not vanish at $\omega = \eta$. Then a necessary and sufficient condition that $\delta_r(\omega)$ and $\delta_i(\omega)$ have only real roots is that in each of the intervals $-2l\pi + \eta < \omega < 2l\pi + \eta$, $l = l_0, l_0 + 1, l_0 + 2...$ $\delta_r(\omega)$ or $\delta_i(\omega)$ have exactly $4lN + M$ real roots for a sufficiently large l_0 .

III. PI CONTROL FOR FIRST ORDER DELAY SYSTEM

We consider the functional diagram of figure 1, in which the transfer function of delayed system is given by (5)

$$G(s) = \frac{K}{1 + Ts} e^{-Ls} \tag{5}$$

Where K , T and L represent respectively the state gain, the constant time and the time delay of the plant. These three parameters are supposed to be positive.

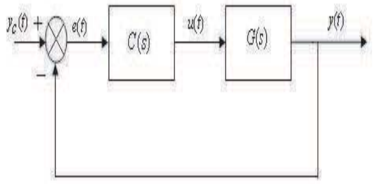

Fig. 1. Closed-loop control of a time delay system

The PI Controller is described by the following transfer function:

$$C(s) = K_p + \frac{K_i}{s} \tag{6}$$

Our objective is to analytically determine the region in the (K_p, K_i) parameter space for which the closed-loop system is stable.

theorem 3[10, 11, 12, 13]
The range of K_p value, for which a solution to PI stabilization problem for a given stable open-loop plant exists, is given by:

$$-\frac{1}{K} < K_p < \frac{T}{KL}\sqrt{\alpha_1^2 + \frac{L^2}{T^2}} \tag{7}$$

Where α_1 the solution of the equation $tan(\alpha) = -\frac{T}{L}\alpha$ in the interval $[\frac{\pi}{2}, \pi]$.

Proof. The closed loop characteristic equation of the system is given by:

$$\delta(s) = (KK_i + KK_p s)e^{-Ls} + (1 + Ts)s \tag{8}$$

we deduce the quasi-polynomials:

$$\delta^*(s) = e^{Ls}\delta(s) = (KK_i + KK_p s) + (1 + Ts)se^{Ls} \tag{9}$$

substituting $s = jw$, we have:
$\delta^*(j\omega) = \delta_r(\omega) + j\delta_i(\omega)$
where:

$$\begin{cases} \delta_r(\omega) = KK_i - \omega\sin(L\omega) - Tw^2\cos(L\omega) \\ \delta_i(\omega) = w\left[KK_p + \cos(L\omega) - Tw\sin(L\omega)\right] \end{cases} \tag{10}$$

Clearly, the parameter K_i only affects the real part of $\delta^*(j\omega)$ whereas the parameter K_p affects the imaginary part. According to the first condition of theorem 1, we should check that the roots of δ_i and δ_r are simple. By using the theorem 2, while choosing , $M = 2, N = l = 1$ and $\eta = \frac{\pi}{4}$, we observe that $\delta_i(\omega)$ has simple roots for any K_p checking (7)[1, 13].

the application of the second condition of theorem 2, led us to:
$E(\omega_0) = \delta_i^{'}(\omega_0)\delta_r(\omega_0) - \delta_i(\omega_0)\delta_r^{'}(\omega_0) > 0$
for $\omega_0 = 0$ (a value that annul $\delta_i(\omega)$) we obtain:
$E(\omega_0) = (\frac{KK_p+1}{L})KK_i > 0$ which implies $K_p > \frac{-1}{K}$ since $K > 0$ and $K_i > 0$.
This proofs the first inequality given by (7) in Theorem 3.
We consider that $z = L\omega$, we get:

$$\delta_r(z) = K\left[K_i - \frac{z}{KL}(\sin(z) + \frac{T}{L}z\cos(z))\right] \tag{11}$$

It results that $a(z) = \frac{z}{KL}(\sin(z) + \frac{T}{L}z\cos(z))$ then

$$\delta_r(z) = K\left[K_i - a(z)\right] \tag{12}$$

for $z_0 = 0$, we obtain :

$$\delta_r(z_0) = K(K_i - a(0)) = KK_i > 0 \tag{13}$$

for $z_j \neq z_0, j = 1, 2, 3...$, we obtain:

$$\delta_r(z_j) = K(K_i - a(z_j)) \tag{14}$$

Interlacing the roots of $\delta_r(z)$ and $\delta_i(z)$ is equivalent to $\delta_r(z_0) > 0$ (since $K_i > 0$), $\delta_r(z_1) < 0, \delta_r(z_2) > 0...$ We can use the interlacing property and the fact that $\delta_i(z)$ has only real roots to establish that $\delta_r(z)$ possess real roots too. From the previous equations we get the following inequalities:

$$\left\{\begin{array}{l} \delta_r(z_0) > 0 \\ \delta_r(z_1) < 0 \\ \delta_r(z_2) > 0 \\ \delta_r(z_3) < 0 \\ \delta_r(z_4) > 0 \\ \quad\vdots \end{array}\right. \Rightarrow \left\{\begin{array}{l} K_i > 0 \\ K_i < a_1 \\ K_i > a_2 \\ K_i < a_3 \\ K_i > a_4 \\ \quad\vdots \end{array}\right. \tag{15}$$

where the bounds a_j for $j = 1, 2, 3...$ are expressed by:

$$a_j = a(z_j) \tag{16}$$

Now, according to these inequalities, it is clear that we need only odd bounds (which to say a_1,a_3...). It has to be strictly positive to get a feasible range for the controller parameter K_i. From [10, 12], a_j is positive (for the odd values of j) for every K_p verifying (7). Hence, the conditions giving by (15) are reduced to:

$$0 < K_i < \min_{j=1,3,5...} \{a_j\} \tag{17}$$

IV. PI CONTROL FOR SECOND ORDER DELAY SYSTEM

A second order system with delay can be mathematically expressed by a transfer function having the following form:

$$G(s) = \frac{K}{s^2 + a_1 s + a_0} e^{-Ls} \tag{18}$$

Where K is the static gain of the plant, L is the time delay and a_0 and a_1 are the plant parameters. The parameters are always positive. The characteristic equation of the closed-loop system is given by:

$$\delta(s) = K(K_i + K_p s)e^{-Ls} + (s^2 + a_1 s + a_0)s \tag{19}$$

we deduce the quasi-polynomial $\delta^*(s)$

$$\delta^*(s) = e^{Ls}\delta(s) = K(K_i + K_p s) + s(s^2 + a_1 s + a_0)e^{Ls} \tag{20}$$

by replacing s by jw , we get:

$$\delta^*(j\omega) = \delta_r(\omega) + j\delta_i(\omega) \tag{21}$$

with:

$$\left\{\begin{array}{l} \delta_r(\omega) = KK_i + (\omega^3 - a_0\omega)\sin(L\omega) - a_1\omega^2\cos(L\omega) \\ \delta_i(\omega) = w\left[KK_p + (a_0 - \omega^2)\cos(L\omega) - a_1\omega\sin(L\omega)\right] \end{array}\right. \tag{22}$$

Clearly, the parameter K_i only affects the real part of $\delta^*(j\omega)$ whereas the parameter K_p affects the imaginary part. Let's put $z = Lw$, we get:

$$\left\{\begin{array}{l} \delta_r(z) = KK_i + \sin(z)(\frac{z^3}{L^3} - a_0\frac{z}{L}) - a_1\frac{z^2}{L^2}\cos(z) \\ \delta_i(z) = \frac{z}{L}(KK_p + \cos(z)(a_0 - \frac{z2}{L^2}) - a_1\frac{z}{L}\sin(z)) \end{array}\right. \tag{23}$$

Step1. The application of the second condition of theorem 2, led us to:
$E(z_0) = \delta_i^{'}(z_0)\delta_r(z_0) - \delta_i(z_0)\delta_r^{'}(z_0) > 0$
from (13) we get:
$\delta_i^{'}(z) = \frac{KK_p}{L} - \sin(z)(a_0 + \frac{2a_1 z}{L^2} - \frac{z^3}{L^3}) + \cos(z)(\frac{a_0}{L} - \frac{3z^2}{L^3} - a_1\frac{z^2}{L^2})$
for $z_0 = 0$ (a value that annul $\delta_i(z)$) we obtain:
$E(z_0) = \delta_i^{'}(z_0)\delta_r(z_0) = (\frac{KK_p + a_0}{L})KK_i > 0$
which implies $K_p > \frac{-a_0}{K}$ since $K > 0$ and $K_i > 0$.

Step2. We pass to the verification of the interlacing condition of $\delta_r(z)$ and $\delta_i(z)$ roots. For such purpose, we are going to determine the roots from the imaginary part, which is translated by the following relation:

$$\delta_i(z) = 0 \Rightarrow \left\{\begin{array}{l} z = 0 \\ or \\ KK_p + \cos(z)(a_0 - \frac{z^2}{L^2}) - a_1\frac{z}{L}\sin(z) = 0 \end{array}\right.$$

$$\Rightarrow \left\{\begin{array}{l} z = 0 \\ or \\ KK_p + \cos(z)(a_0 - \frac{z^2}{L^2}) = a_1\frac{z}{L}\sin(z) \end{array}\right.$$

$$\Rightarrow \left\{\begin{array}{l} z = 0 \\ or \\ f(z) = g(z) \end{array}\right.$$

We notice that $z_0 = 0$ is a trivial root of the imaginary part. The others are difficult to solve analytically. However, this can be made graphically. Two cases are presented. In each case the positive real roots of $\delta_i(z)$ will be denoted by z_j ,$j = 1, 2, 3$, arranged in increasing order of magnitude.

First case: $\frac{-a_0}{K} < K_p < K_u$

In this case, we graph the curves of$f(z)$, of $g(z)$ and the line h defined by $z = L\sqrt{a_0}$ which are presented in figure 2. Where K_u is defined in second case.

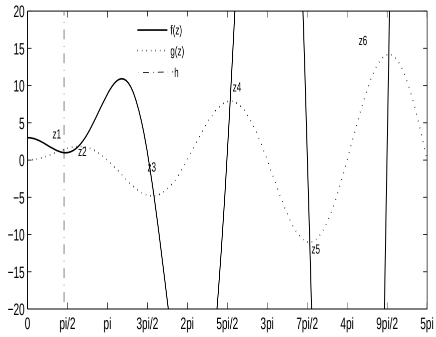

Fig. 2. Representation of the curves of $f(z)$, of $g(z)$ and of h (Case: $-\frac{a_0}{K} < K_p < K_u$)

We notice that for $-\frac{a_0}{K} < K_p$ the curve of $f(z)$ intersects the curve of $g(z)$ twice in the interval $[0, \pi]$. Also we can see the following properties:

$$\left\{\begin{array}{l} z_1 \in [0\,, \pi/2] \\ z_3 \in [3\,\pi/2\,, 2\,\pi] \\ z_5 \in [7\,\pi/2\,, 4\,\pi] \\ \quad\vdots \end{array}\right. \quad and \quad \left\{\begin{array}{l} z_2 \in [\pi/2, \pi] \\ z_4 \in [5\pi/2, 3\pi] \\ z_6 \in [9\pi/2, 5\pi] \\ \quad\vdots \end{array}\right.$$

i.e z_j verify $\left\{\begin{array}{l} z_1 \in [0\,, \pi/2] \\ and \\ z_j \in \left[(2j-3)\frac{\pi}{2}, (j-1)\pi\right] \quad for\, j \geq 2 \end{array}\right.$

and we have $\left\{\begin{array}{l} z_1 < z = L\sqrt{a_0} \\ and \\ z_j > z = L\sqrt{a_0},\ for\, j \geq 2 \end{array}\right.$

Second case: $K_p \geq K_u$

Figure 3 represents the case where $K_p = K_u$, and K_u is the maximal value of K_p such as the plots of $f(z)$ and $g(z)$ are tangent in the interval $[0, \pi]$.

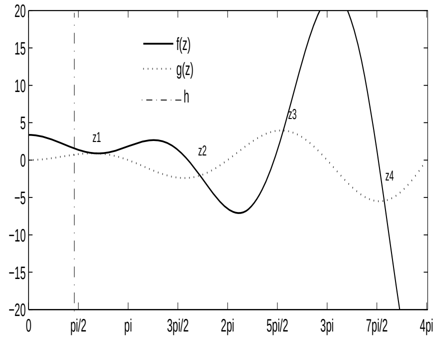

Fig. 3. Representation of the curves of $f(z)$, of $g(z)$ and of h (Case: $K_p = K_u$)

The plot in Figure 4 corresponds to the case where $K_p > K_u$ and the plot of $f(z)$ does not intersect $g(z)$ in the interval $[0, \pi]$.

To verify that $\delta_i(z)$ possess only simple roots we used

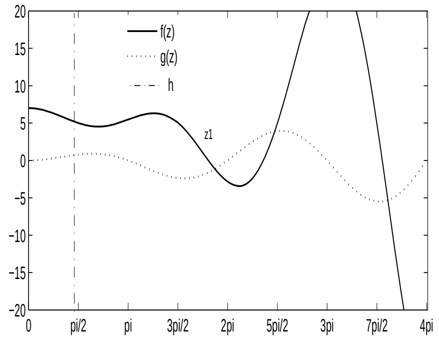

Fig. 4. Representation of the curves of $f(z)$, of $g(z)$ and of h (Case: $K_p > K_u$)

Theorem 3. Replacing Ls by s_1 in (10), we rewrite $\delta^*(j\omega)$ as follows:

$$\begin{aligned}\delta^*(s) &= e^{Ls}\delta(s) = e^{s_1}\delta(s_1)\\ &= e^{s_1}((\tfrac{s_1}{L})^3 + a_1(\tfrac{s_1}{L})^2 + a_0\tfrac{s_1}{L}) + K(K_p\tfrac{s_1}{L} + K_i)\end{aligned}$$

For this new quasi-polynomial, we see that $M = 3$ and $N = 1$ where M and N designate the highest powers of s and e^s which appear in $\delta^*(s)$. We choose $\eta = \frac{\pi}{4}$ that satisfies the condition giving by theorem 3 as $\delta_r(\eta) \neq 0$ and $\delta_i(\eta) \neq 0$. According to figure 2, we notice that for $-\frac{a_0}{K} < K_p < K_u$, $\delta_i(z)$ possess four roots in the interval $\left[0, 2\pi - \frac{\pi}{4}\right] = \left[0, \frac{7\pi}{4}\right]$ including the root at origin. As $\delta_i(z)$ is odd function of z so, it possesses seven roots in $\left[-2\pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}\right] = \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right]$. Hence, we can affirm that $\delta_i(z)$ has exactly $4N + M = 7$ in $\left[-2\pi + \frac{\pi}{4}, 2\pi + \frac{\pi}{4}\right] = \left[-\frac{7\pi}{4}, \frac{9\pi}{4}\right]$. In addition, it can be shown that $\delta_i(z)$ has tow real roots in each of the intervals $\left[2l\pi + \frac{\pi}{4}, 2(l+1)\pi + \frac{\pi}{4}\right]$ and $\left[-2(l+1)\pi + \frac{\pi}{4}, -2l\pi + \frac{\pi}{4}\right]$ for $l = 1, 2....$ It fallows that $\delta_i(z)$ has exactly $4lN + M$ real roots in $\left[-2l\pi + \frac{\pi}{4}, 2l\pi + \frac{\pi}{4}\right]$ for $-\frac{a_0}{K} < K_p < K_u$. At the end, according to theorem 3 $\delta_i(z)$ has only real roots for every K_p in $\left[-\frac{a_0}{K}, K_u\right]$. For $K_p \geq K_u$, corresponding to figure 3 and 4, the roots of $\delta_i(z)$ are not real. We pass to determine the superior value of K_p. According to the definition of K_u, if $K_p = K_u$ then the curves of $f(z)$ and $g(z)$ are tangent in the point α . Which is translated by:

$$\left\{\begin{array}{l} KK_u + \cos(\alpha)(a_0 - \frac{\alpha^2}{L^2}) = a_1\frac{\alpha}{L}\sin(\alpha) \\ and \\ \frac{d}{dz}\left[KK_u + \cos(z)(a_0 - \frac{z^2}{L^2})\right]_{z=\alpha} = \frac{d}{dz}\left[a_1\frac{z}{L}\sin(z)\right]_{z=\alpha} \end{array}\right.$$

$$\Rightarrow -2\alpha\cos(\alpha)(1 + a_1L) + \sin(\alpha)(\alpha^2 - a_0L^2 - a_1L) = 0$$

$$\Rightarrow \tan(\alpha) = \frac{\alpha(2 + a_1L)}{(\alpha^2 - a_0L^2 - a_1L)} \tag{24}$$

once α is determined, the parameter K_u is expressed by (25):

$$K_u = \frac{1}{K}(a_1\frac{\alpha}{L}\sin(\alpha) - \cos(\alpha)(a_0 - \frac{\alpha^2}{L^2})) \tag{25}$$

Now, we state theorem 4 determining the range of for the second order delay system stabilization controlled by PI regulator. This is equivalent to theorem 3 for a first delay system.

theorem 4

Under the above assumptions on K,L ,a_0 and a_1 , the range of (K_p values for which a solution exists to the PI stabilization problem of a open-loop stable plant with transfer function $G(s)$ is given by:

$$-\frac{a_0}{K} < K_p < \frac{1}{K}(a_1\frac{\alpha}{L}\sin(\alpha) - \cos(\alpha)(a_0 - \frac{\alpha^2}{L^2})) \tag{26}$$

Where α the solution of the equation $tan(\alpha) = \frac{\alpha(2+a_1L)}{(\alpha^2 - a_1L - a_0L^2)}$ in the interval $[0, \pi]$

After the determination of the roots of imaginary part $\delta_i(z)$, we pass to the evaluation of these roots by real part $\delta_r(z)$.

$$\begin{aligned}\delta_r(z) &= KK_i + \sin(z)(\tfrac{z^3}{L^3} - a_0\tfrac{z}{L}) - a_1\tfrac{z^2}{L^2}\cos(z)\\ &= K\left[K_i - a(z)\right]\end{aligned} \tag{27}$$

where $a(z) = \frac{z}{KL}\left[\sin(z)(a_0 - \frac{z^2}{L^2}) + a_1\frac{z}{L}\cos(z)\right]$. Let's put $z_j, j = 1, 2, 3...$ the roots of $\delta_r(z)$, for $z_0 = 0$, we have:

$$\delta_r(z_0) = K(K_i - a(0)) = KK_i > 0 \tag{28}$$

for $z_j \neq z_0$ where $j = 1, 2, 3...$, we get:

$$\begin{aligned}\delta_r(z_j) &= K(K_i - a(z_j))\\ &= K(K_i - a_j)\end{aligned} \tag{29}$$

with $a(z_j) = a_j$.

Interlacing the roots of $\delta_r(z)$ and $\delta_i(z)$ is equivalent to $\delta_r(z_0) > 0$ (since $K_i > 0$), $\delta_r(z_1) < 0, \delta_r(z_2) > 0...$ We can use the interlacing property and the fact that $\delta_i(z)$ has only real roots to establish that $\delta_r(z)$ possess real roots too. From the previous equations we get the following inequalities:

$$\left\{\begin{array}{l} \delta_r(z_0) > 0 \\ \delta_r(z_1) < 0 \\ \delta_r(z_2) > 0 \\ \delta_r(z_3) < 0 \\ \delta_r(z_4) > 0 \\ \vdots \end{array}\right. \Rightarrow \left\{\begin{array}{l} K_i > 0 \\ K_i < a_1 \\ K_i > a_2 \\ K_i < a_3 \\ K_i > a_4 \\ \vdots \end{array}\right. \tag{30}$$

From these inequalities, it is clear that the a_j odd bounds must be strictly positive; however the a_j even bounds are negative in order to find a feasible range of K_i . From which we have:

$$0 < K_i < \min_{j=1,3,5...} \{a_j\} \tag{31}$$

On the following, we are interesting to prove that the odd a_j are strictly positive and that the even a_j are negative in order to affirm the relation (31).
From figure 2, the roots z_j of $\delta_i(z)$ verify:

$$\begin{cases} z_1 \in [0\,, \pi/2] \\ and \\ z_j \in \left[(2j-3)\frac{\pi}{2}, (j-1)\pi\right] \quad for\, j \geq 2 \end{cases} \tag{32}$$

In addition we have: $\begin{cases} z_1 < L\sqrt{a_0} \\ and \\ z_j > L\sqrt{a_0},\ for\, j \geq 2 \end{cases} \Rightarrow$

$$\begin{cases} (a_0 - \frac{z_1^2}{L^2}) > 0 \\ and \\ (a_0 - \frac{z_j^2}{L^2}) < 0,\ for\, j \geq 2 \end{cases}$$

Proof:

- Let's prove that $a(z_j) > 0$ for $j = 1, 3, 5, 7...$
  Step 1: According to (32), for z_1 we have $\cos(z_1) > 0$ and $\tan(z_1) > 0$.
  We suppose that $a(z_1) < 0 \Rightarrow \left[\sin(z_1)(a_0 - \frac{z_1^2}{L^2}) + a_1\frac{z_1}{L}\cos(z_1)\right] < 0$
  $\Rightarrow \sin(z_1)(a_0 - \frac{z_1^2}{L^2}) < -a_1\frac{z_1}{L}\cos(z_1)$
  $\Rightarrow \tan(z_1)(a_0 - \frac{z_1^2}{L^2}) < -a_1\frac{z_1}{L}$
  $\Rightarrow \tan(z_1) < \frac{-a_1\frac{z_1}{L}}{(a_0 - \frac{z_1^2}{L^2})} < 0$
  Which is absurd, as a result: $\left[\sin(z_1)(a_0 - \frac{z_1^2}{L^2}) + a_1\frac{z_1}{L}\cos(z_1)\right] > 0 \Rightarrow a(z_1) > 0$.
  Step 2: According to (32), we have $\cos(z_j) > 0$ and $\tan(z_j) < 0$ for z_j where $j = 3, 5, 7, 9...$
  We suppose that $a(z_j) < 0 \Rightarrow \left[\sin(z_j)(a_0 - \frac{z_j^2}{L^2}) + a_1\frac{z_j}{L}\cos(z_j)\right] < 0$
  $\Rightarrow \sin(z_j)(a_0 - \frac{z_j^2}{L^2}) < -a_1\frac{z_j}{L}\cos(z_j)$
  $\Rightarrow \tan(z_j)(a_0 - \frac{z_j^2}{L^2}) < -a_1\frac{z_j}{L}$
  $\Rightarrow \tan(z_j) > \frac{-a_1\frac{z_j}{L}}{(a_0 - \frac{z_j^2}{L^2})} > 0$
  Which is absurd, so $\left[\sin(z_j)(a_0 - \frac{z_j^2}{L^2}) + a_1\frac{z_j}{L}\cos(z_j)\right] > 0 \Rightarrow a(z_j) > 0$ for $j = 3, 5, 7, 9...$

- Let's prove that $a(z_j) < 0$ for $j = 2, 4, 6, 8...$
  For $z_j, j = 2, 4, 6, 8....$ we have $\cos(z_j) < 0$ and $\tan(z_j) < 0$ according to (31),
  We suppose that $a(z_j) > 0 \Rightarrow \left[\sin(z_j)(a_0 - \frac{z_j^2}{L^2}) + a_1\frac{z_j}{L}\cos(z_j)\right] > 0$
  $\Rightarrow \sin(z_j)(a_0 - \frac{z_j^2}{L^2}) > -a_1\frac{z_j}{L}\cos(z_j)$
  $\Rightarrow \tan(z_j)(a_0 - \frac{z_j^2}{L^2}) < -a_1\frac{z_j}{L}$
  $\Rightarrow \tan(z_j) > \frac{-a_1\frac{z_j}{L}}{(a_0 - \frac{z_j^2}{L^2})} > 0$
  Which is absurd,so $\left[\sin(z_j)(a_0 - \frac{z_j^2}{L^2}) + a_1\frac{z_j}{L}\cos(z_j)\right] < 0 \Rightarrow a(z_j) < 0$ for $j = 2, 4, 6, 8...$

■

Recapitulation: For every K_p in the interval giving by theorem 4, the parameters $a(z_j)$ verify the following conditions:

$$\begin{cases} a(z_j) > 0,\ for\, j = 1, 3, 5, 7... \\ and \\ a(z_j) < 0,\ for\, j = 2, 4, 6, 8... \end{cases} \tag{33}$$

To determine the set of all stabilizing (K_p, K_i) values for a second order delay system, we propose a procedure summarized in the following algorithm.

Algorithm for determining PI parameters:

1) Choose K_P in the interval suggested by theorem 1 and initialize $j = 1$,
2) Find the roots z_j of $\delta_i(z)$,
3) Compute the parameter a_j associated with the z_j previously founded,
4) Determine the lower and the upper bounds for K_i as follows: $0 < K_i < \min_{j=1,3,5...} \{a_j\}$
5) Go to step 1.

Once the stability domain is determined, the question is what are the optimum parameters of the PI controller which guarantee the good performance of the closed-loop system? On the following, the genetic algorithm is proposed to answer this need.

V. Genetic Algorithms

The Genetic Algorithms (AGs) are iterative algorithms of global search of which the goal is to optimize a specific function called criterion of performance or cost function. In order to find the optimal solution of a problem by using AGs, we start by generating a population of individuals in a random way. The evolution from one generation to the following is based on the use of the three operators' selection, crossover and mutation which are applied to all the elements of the population . Couples of parents are selected according to their functions costs. The crossover operator is applied with a P_c probability and generates couples of children. The mutation operator is applied to the children with a P_m probability and generates mutant individuals who will be inserted in the new population. The reaching of a maximum number of generations is the criterion of stop for our algorithm. Figure 5 shows the basic flow chart of AGs. The principle of regulator parameters optimization by the genetic algorithms is shown by figure 6.

It is about the search of parameters K_p , K_i in the area of stability. This may be made by the selection according to the optimization criterion J is described by the following expression:

$$\begin{aligned} J &= \alpha ISE + \beta IAE + \delta ITAE + \gamma ITSE \\ &= \alpha \sum_0^{t_{\max}} e(t)^2 + \beta \sum_0^{t_{\max}} |e(t)| + \delta \sum_0^{t_{\max}} t\,|e(t)| + \gamma \sum_0^{t_{\max}} te(t)^2 \end{aligned} \tag{34}$$

Where α , β ,δ and γ are ponderation's factor ranging between 0 and 1 and whose their sum equalizes 1, can be

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:3, No:3, 2009

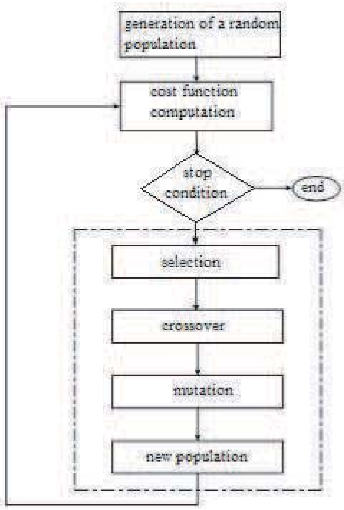

Fig. 5. Steps of the genetic algorithm evolution

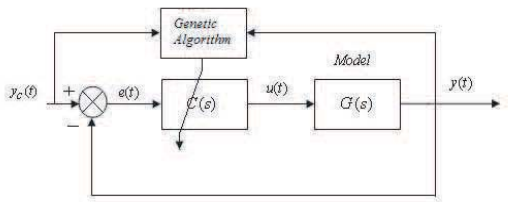

Fig. 6. The optimization principle by genetic algorithm

modified according to specifications' chart [14]. If we want to minimize the tuning energy, the $ITAE$ criteria and the IAE are considered. In the case where we privilege the rise time, we take the $ITSE$ criterion. In order to guarantee the tuning energetic cost, we choose the ISE criterion. The calculation steps of the control law are summarized by the following algorithm:

1) Introduction of the following parameters:
   - max_{pop}individuals number by population
   - initial population
   - gen_{max} generation number
2) Initialization of the generation counter ($gen = 1$)
3) Initialization of the individual counter ($j = 1$)
4) For $t = 1s$ to $t = t_{max}$ efficiency evaluation of j^{th} population individual $fitness(J) = \frac{1}{1+J}$
5) Individual counter incrementing ($j = j + 1$).
   - If $j < max_{pop}$, going back to step 4
   - If not: application of the genetic operators (selection, crossover, mutation) for the founding a new population
6) Generation counter incrementing ($gen = gen + 1$), If , going back to step 3.
7) taking K_popt and K_iopt which correspond to the best individual in the last population (individual making the best fitness).

On the following, the genetic algorithm characterized by generation number equal to 100, $P_c = 0.8$, $P_m = 0.08$ and individual number by population equivalent to 20.

VI. Simulation results

We consider a second order delay system described by the following transfer function [13, 17]:$G(s) = \frac{5e^{-3s}}{s^2+2s+5}$
In order to determine K_p values, we look for α in interval $[0, \pi]$ satisfying $\tan(\alpha) = \frac{8\alpha}{(\alpha^2-51)} \Rightarrow \alpha = 2.685$. K_p range is given by: $-1 < K_p < 0.9116$.
The system stability region, obtained in K_p, K_i plane is presented in figure 7. From figure 7, our K_p and K_i population's

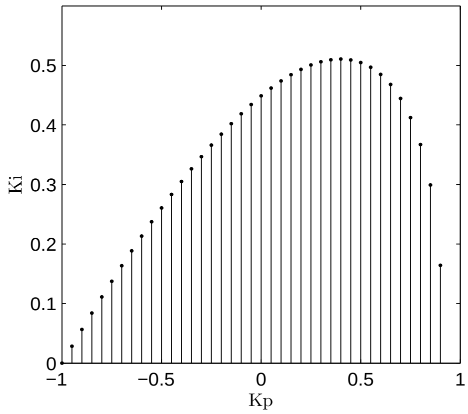

Fig. 7. PI controller stability domain

individuals are choosing between $[-1, 0.911]$ and $[0, 0.5]$. PI controller optimum parameters supplied by genetic algorithm which are $K_popt = 0.3371$ and $K_iopt = 0.2203$. α, β, δ and γ are choosing successively equivalent to 0.3, 0.2 , 0.2 and 0.3. Figure 8 presents the evolution of the set point, the output and the control law.

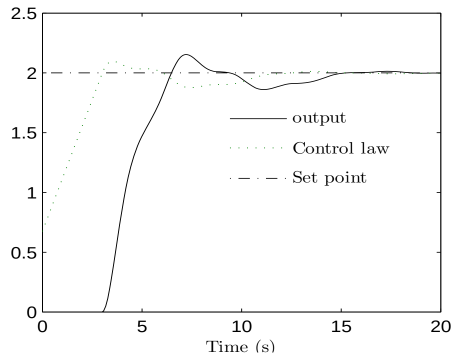

Fig. 8. The optimization principle by genetic algorithm

VII. Conclusions

In this work, we proposed an extension of Hermit-Biehler theorem to compute the region stability for second order delay system controlled by PI regulator. The procedure is based first on determining the range of proportional gain value K_p for which a solution to PI stabilization exists. Then, it is shown that for a fixed K_p inside this range, the integral gain value K_i

is computing. Lastly, we were interested in search of optimal PI for a given performance criteria, inside the stability region. The criterion used to find the PI parameters is based on a linear combination of classical performance criteria: the ISE, the IAE, the $ITSE$ and the $ITAE$. In regard to the complexity of the optimization problem, we used the genetic algorithms. The validation of these results has been done on a delayed second order plant.

REFERENCES

[1] S. P. Bhattacharyya, H. Chapellat, and L. H. Keel, *Robust Control:The Parametric Approach*, Upper Saddle River, NJ: Prentice-Hall, 1995.
[2] L. Cedric, Pontryagin revisit: implication sur la stabilit des PID retards, Confrence Internationale Francophone d'Automatique CIFA'02
[3] M. Dambrine, *Contribution l'tude de la stabilit des systmes retards*, Thse de doctorat, Universit de Lille, 1994.
[4] D.E. Godelberg, *Genetic Algorithms in search, optimization and machine learning*, Addison-Wesley, Massachusetts, 1991.
[5] O.Lequin, M. Gevers, M. Mossberg, E. Bosmans, L. Triest, *Iterative feedback tuning of PID parameters: comparison with classical tuning rules*, Control Engineering Practice, N11, pp. 1023-1033, 2003.
[6] G.P. Liu, S.Daley, Optimal-tuning PID control for industrial systems, Control Engineering Practice, N 9, pp. 1185-1194, 2001.
[7] S.-I. Niculescu, *Delay effects on stability*, Springer, London, 2001.
[8] R.Padma Sree, M.N. Sirinivas, M. Chidambaram, A simple method of tuning PID controllers for stable and unstable FOPTD systems, Comp. Chem. Eng, pp. 2201-2218, 2004.
[9] Z. Shafiei, A.T Shenton, tuning of PID-type controllers for stable and unstable delay systems with time delay Automatica, pp. 1609-1615, 1994.
[10] G. J. Silva, A. Datta and S. P. Bhattacharyya, Stabilization of Time Delay Systems, Proceedings of the American Control Conference, pp. 963-970, 2000.
[11] G. J. Silva, A. Datta and S. P. Bhattacharyya, Stabilization of First-order Systems with Time Delay using the PID controller, Proceedings of the American Control Conference, pp. 4650-4655, 2001.
[12] G. J. Silva, A. Datta and S. P. Bhattacharyya, New synthesis of PID controller, IEEE transactions on automatic control, vol.47,No.2, 2002.
[13] G.J. Silva, A. Datta, S.P. Bhattacharyya, *PID controllers for time delay systems*, Springer, London, 2005.
[14] M. Villain, *Systmes asservis linaires*, ellipses/ dition marketing S.A., 1996.
[15] Q.C. Zhong, *Robust control of time delay system*, Springer, London, 2006.
[16] A. Roy, K. Iqbal, PID controller tuning for the first-order-plus-dead-time process model via Hermite-Biehler theorem, ISA Transaction, pp. 362-378, 2005.
[17] G. J. Silva, A. Datta and S. P. Bhattacharyya, Determination of Stabilizing gains for Second-order Systems with Time Delay, Proceedings of the American Control Conference, pp. 25-27, 2001.
[18] C. S. Jung, H. K. Song, J. C. Hyun, A direct synthesis tuning method of unstable first-order-plus-time-delay processes, Journal of process control, pp. 265-269, 1999.
[19] C. Xiang, Q.G. Wang; X. Lu, L.A. Nguyen, T.H. Lee, Stabilization of second-order unstable delay system, Journal of process control, pp. 675-682, 2007.
[20] R. Toscano, A simple robust PI/PID controller design via numerical optimization approach, Journal of process control, pp. 81-88, 2005.

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