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A Dynamic Model for a Drill in the Drilling Process

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***Abstract*—**The dynamic model of a drill in drilling process was proposed and investigated in this study. To assure a good drilling quality, the vibration variation on the drill tips during high speed drilling is needed to be investigated. A pre-twisted beam is used to simulate the drill. The moving Winkler-Type elastic foundation is used to characterize the tip boundary variation in drilling. Due to the variation of the drill depth, a time dependent dynamic model for the drill is proposed. Results simulated from this proposed model indicate that an abrupt natural frequencies drop are experienced as the drill tip tough the workpiece, and a severe vibration is induced. The effects of parameters, e.g. drilling speed, depth, drill size and thrust force on the drill tip responses studied.

***Keywords*—**Drilling, vibration of drill, twisted beam.

I. INTRODUCTION

IN manufacturing, drilling has been used widely to drill holes. Due to the severe drill tip vibration induced in the drilling, the high accuracy and quality drilling still be a difficult task. The reliability in drilling process may affect the production capacity and the yield rate significantly, especial in the electronic board manufacturing. To improve drilling quality and capabilities, it is necessary to understand the dynamic drilling process characteristics. In this study, a time dependent dynamic model of the drill is proposed to explore the effect of drill size, drill speed and tip thrust force on the dynamic responses of a drill during the drilling process.

Only a few studies have been conducted on the time varying dynamic properties of a drill during the drilling process Traditional drilling analysis has always focused on the drill itself, such as Rosard [1], Jarrett and Warner [2], Tekinalp and Ulsoy [3] and [4], …., etc. The interaction between the drill stiffness and the drill depth are ignored. The periodically pretwisted beams with different cross section shapes have been proposed to approximate the drills. e.g. Rosard [1], Jarrett & Warner [2], Liao & Dang [5] and Liao & Huang [6], etc. The effects of the pre-twisted angle and spinning speed on vibrations in the drill were also presented. The buckling load and natural frequency of a drill bit were investigated in a number of published papers [7]-[9].

In previous published papers, different vibration was introduced to model the complex drill bits to estimate the distribution of its natural frequencies. The effects of drill geometry shape and the cutting chip on the drilling and dynamic drill bit properties were examined. Results indicated that even a small variation in the geometry or symmetry can introduce a very strong influence on the drilling stability and dynamic responses of a drill [10]. These proposed drill models have provided very useful information for drill bit design. Some investigators extended their research to the instability analysis of a drill under different drilling conditions [6], [11]-[12]. Results indicated that the spinning speed, pre-twisted angle and axial force may change the distribution of instability zooms.

Recently, a number of structural models with moving masses, forces and boundaries have been developed [13]-[17]. It should be noted that the dynamic characteristics of these time dependent systems are quite different from the time invariant problems. Most important of all, the traditional time invariant natural modes and frequencies become meaningless because the system and its corresponding boundary conditions are time dependent. In this article, a dynamic model of the drill with time varying drilling depth and thrust force is proposed to simulate the variation of dynamic response during the drilling process.

In a mass production drilling process, e.g. the PC board drilling manufacture, the drill is fractured frequently at the moment as the drill tip touch the work piece. Hence, to explore the dynamic response of a drill in the high speed drilling process is important for improving the drilling reliability and capacity. Initially, the axial thrust force applied on the drill tip will dominate the dynamic response of a drill. For convenience, a time dependent axial force $P u_s\left(t-t^*\right)$, is used to simulate the applied drilling force for a drill. An unit step function $u_s\left(t-t^*\right)$ is employed to describe the axial load during the drilling process. The time varied boundary, i.e. drilling depth, is used in the system formulation. Two moving Winkler-Type elastic foundations are used to simulate the time dependent boundary. As above, the dynamic model of a drill in drilling process can be formulated by using a pretwisted beam with the time varied thrust force, drilling depth and boundary stiffness.

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II. Theory and Formulations

The drill approximated by a cantilever pre-twisted beam with a spinning speed Ω is shown in Fig. 1(a). The length of drill is L, t and b denote the thickness and breadth of the drill cross section. The deflection components $v(r,t)$ and $u(r,t)$ denote the two transverse deflections of the drill. Consider the drill to be pre-twisted with a uniform twist angle β .

By applying Hamilton's principle, the equation of motion of a spinning drill can be derived as follows, where the effects of coriolis and rotary inertia have also been included.

$$\frac{\partial^2}{\partial r^2}\left(EI_{yy}\frac{\partial^2 u}{\partial r^2}+EI_{xy}\frac{\partial^2 v}{\partial r^2}\right)-\frac{\partial}{\partial r}\left(\bar{I}_{yy}\frac{\partial^3 u}{\partial t^2\partial r}+\bar{I}_{xy}\frac{\partial^3 v}{\partial t^2\partial r}\right)+\mu\frac{\partial^2 u}{\partial t^2}-2\mu\Omega\frac{\partial v}{\partial t}-\mu\Omega^2 u=0 \quad (1)$$

$$\frac{\partial^2}{\partial r^2}\left(EI_{xx}\frac{\partial^2 v}{\partial r^2}+EI_{xy}\frac{\partial^2 u}{\partial r^2}\right)-\frac{\partial}{\partial r}\left(\bar{I}_{xx}\frac{\partial^3 v}{\partial t^2\partial r}+\bar{I}_{xy}\frac{\partial^3 u}{\partial t^2\partial r}\right)+\mu\frac{\partial^2 v}{\partial t^2}+2\mu\Omega\frac{\partial u}{\partial t}-\mu\Omega^2 v=0 \quad (2)$$

Where E and μ are the Young's modulus and mass per unit length respectively. I_{xx} , I_{yy} and I_{xy} are the moments of area.

$$I_{xx}=I_{XX}\cos^2\left(\frac{r}{L}\beta\right)+I_{YY}\sin^2\left(\frac{r}{L}\beta\right) \quad (3)$$

$$I_{yy}=I_{XX}\sin^2\left(\frac{r}{L}\beta\right)+I_{YY}\cos^2\left(\frac{r}{L}\beta\right) \quad (4)$$

$$I_{xy}=\left(I_{YY}-I_{XX}\right)\sin\left(\frac{r}{L}\beta\right)\cos\left(\frac{r}{L}\beta\right) \quad (5)$$

with

$$I_{XX}=\frac{b\,t^3}{12} \quad (6)$$

$$I_{YY}=\frac{b^3 t}{12} \quad (7)$$

The boundaries are

$$u=v=u'=v'=0, \quad \text{at} \quad r=0 \quad (8)$$

$$u''=v''=u'''=v'''=0, \quad \text{at} \quad r=L \quad (9)$$

where a symbol prime $(')$ denotes a partial derivative with respect to r. $\bar{I}_{xx}$, $\bar{I}_{xy}$ and $\bar{I}_{yy}$ are the principal mass moments of inertia.

Dynamic Responses during the Drilling through Process

In order to investigate the effects of time dependent axial thrust force and drilling depth on the dynamic response of a drill during the drilling process. The moving boundary, i.e. two moving Winkler-Type elastic foundations [18], are used to simulate the time dependent drilling depth, as shown in Fig. 1(b).=The uniform distribution stiffness k_{xx} and k_{yy} are assumed for the elastic foundations. To explore the severe vibration experienced when the drill tip touch the work piece, axial load, $P\left[u_s\left(t-t^*\right)-u_s\left(t-t^{**}\right)\right]$, is applied at the drill tip during the drilling process. The work introduced from this thrust force can be derived as

$$W_p(t)=\int_0^L \frac{1}{2}Pu_s\left(t-t^*\right)\left[\left(\frac{\partial u\ (r,t)}{\partial r}\right)^2+\left(\frac{\partial v\ (r,t)}{\partial r}\right)^2\right]dr-\int_0^L \frac{1}{2}Pu_s\left(t-t^{**}\right)\left[\left(\frac{\partial u\ (r,t)}{\partial r}\right)^2+\left(\frac{\partial v\ (r,t)}{\partial r}\right)^2\right]dr \quad (10)$$

where $u_s\left(t-t^*\right)$ is the unit step function. The drilling process is initiated at $t=t^*$, i.e. the drill tip will touch the workpiece at $t=t^*$. The drill will drill through the work piece at $t=t^{**}$. The same unit step functions are employed on the corresponding boundary constraints. With considering the drilling time and moving boundary effects, then the strain energy introduced in the Winkler-Type elastic foundations can be derived as

$$U_k(t)=2\int_0^L\left[\frac{1}{2}k_{xx}u_s\left(r-r^*\right)u_s\left(t-t^*\right)\ u^2(r,t)+\frac{1}{2}k_{yy}u_s\left(r-r^*\right)u_s\left(t-t^*\right)\ v^2(r,t)\right]dr-2\int_0^L\left[\frac{1}{2}k_{xx}u_s\left(r-r^{**}\right)u_s\left(t-t^{**}\right)\ u^2(r,t)+\frac{1}{2}k_{yy}u_s\left(r-r^{**}\right)u_s\left(t-t^{**}\right)\ v^2(r,t)\right]dr \quad (11)$$

where

$$r^*(t)=L-f\times\left(t-t^*\right)u_s\left(t-t^*\right) \quad (12)$$

$$r^{**}(t)=L-f\times\left(t-t^{**}\right)u_s\left(t-t^{**}\right) \quad (13)$$

f is the feed velocity of the drill.

For=convenience, a nondimensional depth $\xi(t)$ is used to denote the relative drilling displacement. This non-dimensionless drilling displacement is defined as follows,

$$\xi(t)=\frac{f\times\left(t-t^*\right)u_s\left(t-t^*\right)}{L} \quad (14)$$

When the drill drills throughout a work piece, then the above displacement can be rewritten as

$$\xi(t)=\frac{f\times\left(t-t^*\right)u_s\left(t-t^*\right)}{L}-\frac{f\times\left(t-t^{**}\right)u_s\left(t-t^{**}\right)}{L} \quad (15)$$

Hence, the equation of motion for the drill, as it drills throughout the work piece, can be rewritten as follows

$$\frac{\partial^2}{\partial r^2}\left(EI_{yy}\frac{\partial^2 u}{\partial r^2}+EI_{xy}\frac{\partial^2 v}{\partial r^2}\right)-\frac{\partial}{\partial r}\left(\overline{I}_{yy}\frac{\partial^3 u}{\partial t^2\partial r}+\overline{I}_{xy}\frac{\partial^3 v}{\partial t^2\partial r}\right)$$
$$+Pu_s\left(t-t^*\right)\frac{\partial^2 u}{\partial r^2}-Pu_s\left(t-t^{**}\right)\frac{\partial^2 u}{\partial r^2}$$
$$+2k_{xx}u_s\left(r-r^*(t)\right)u_s\left(t-t^*\right)u\ -2k_{xx}u_s\left(r-r^{**}(t)\right)u_s\left(t-t^{**}\right)u$$
$$+\mu\ \frac{\partial^2 u}{\partial t^2}-2\mu\ \Omega\frac{\partial v}{\partial t}-\mu\ \Omega^2 u\ =0 \qquad (16)$$

$$\frac{\partial^2}{\partial r^2}\left(EI_{xx}\frac{\partial^2 v}{\partial r^2}+EI_{xy}\frac{\partial^2 u}{\partial r^2}\right)-\frac{\partial}{\partial r}\left(\overline{I}_{xx}\frac{\partial^3 v}{\partial t^2\partial r}+\overline{I}_{xy}\frac{\partial^3 u}{\partial t^2\partial r}\right)$$
$$+Pu_s\left(t-t^*\right)\frac{\partial^2 v}{\partial r^2}-Pu_s\left(t-t^{**}\right)\frac{\partial^2 v}{\partial r^2}$$
$$+2k_{yy}u_s\left(r-r^*(t)\right)u_s\left(t-t^*\right)v\ -2k_{yy}u_s\left(r-r^{**}(t)\right)u_s\left(t-t^{**}\right)v$$
$$+\mu\ \frac{\partial^2 v}{\partial t^2}+2\mu\ \Omega\frac{\partial u}{\partial t}-\mu\ \Omega^2 v\ =0 \qquad (17)$$

Consider the dynamic responses of the drill during the drilling process be expressed as

$$u\ (r,t)=\sum_{i=1}^{m}p_i\ (t)\,\phi_i\ (r) \qquad (18)$$

$$v\ (r,t)=\sum_{i=1}^{m}q_i\ (t)\,\phi_i\ (r) \qquad (19)$$

where $\phi_i\ (r)$ are the comparison functions for Eqs. (17) and (18), and $p_i\ (t), q_i\ (t)$ are the corresponding weighting coefficients, those are to be determined. By applying Galerkin's method, the equations of motion for the drill can be derived in matrix form as

$$[M]\begin{Bmatrix}\ddot{p}\\ \ddot{q}\end{Bmatrix}+2\Omega[G]\begin{Bmatrix}\dot{p}\\ \dot{q}\end{Bmatrix}+\Big\{[K]_a+\left[P\ u_s\left(t-t^*\right)-P\ u_s\left(t-t^{**}\right)\right][K]_b$$
$$+\Omega^2\,[K]_c+\left[u_s\left(r-r^{**}(t)\right)u_s\left(t-t^{**}\right)\right.$$
$$\left.-u_s\left(r-r^{**}(t)\right)u_s\left(t-t^{**}\right)\right][K]_d\Big\}\begin{Bmatrix}p\\ q\end{Bmatrix}=0 \qquad (20)$$

where

$$[M]=\begin{bmatrix}[M]^1+[M]^2 & [M]^4\\ [M]^4 & [M]^1+[M]^3\end{bmatrix} \qquad (21)$$

$$[G]=\begin{bmatrix}[0] & -[M]^1\\ [M]^1 & [0]\end{bmatrix} \qquad (22)$$

$$[K]_a=\begin{bmatrix}[K]^1 & [K]^3\\ [K]^3 & [K]^2\end{bmatrix} \qquad (23)$$

$$[K]_b=\begin{bmatrix}-[K]^4 & [0]\\ [0] & -[K]^4\end{bmatrix} \qquad (24)$$

$$[K]_c=\begin{bmatrix}-[M]^1 & [0]\\ [0] & -[M]^1\end{bmatrix} \qquad (25)$$

$$[K]_d=\begin{bmatrix}2k_{xx}[K]^5 & [0]\\ [0] & 2k_{yy}[K]^5\end{bmatrix} \qquad (26)$$

These matrices are

$$M_{ij}=\int_0^1\mu\ \phi_i\left(\overline{r}\right)\phi_j\left(\overline{r}\right)d\overline{r} \qquad (27)$$

$$M_{ij}^2=\frac{\overline{I}_{yy}}{L^2}\int_0^1\frac{d\phi_i\left(\overline{r}\right)}{d\overline{r}}\frac{d\phi_j\left(\overline{r}\right)}{d\overline{r}}d\overline{r} \qquad (28)$$

$$M_{ij}^3=\frac{\overline{I}_{xx}}{L^2}\int_0^1\frac{d\phi_i\left(\overline{r}\right)}{d\overline{r}}\frac{d\phi_j\left(\overline{r}\right)}{d\overline{r}}d\overline{r} \qquad (29)$$

$$M_{ij}^4=\frac{\overline{I}_{xy}}{L^2}\int_0^1\frac{d\phi_i\left(\overline{r}\right)}{d\overline{r}}\frac{d\phi_j\left(\overline{r}\right)}{d\overline{r}}d\overline{r} \qquad (30)$$

$$K_{ij}^1=\frac{E}{L^4}\int_0^1 I_{yy}\frac{d^2\phi_i\left(\overline{r}\right)}{d\overline{r}^2}\frac{d^2\phi_j\left(\overline{r}\right)}{d\overline{r}^2}d\overline{r} \qquad (31)$$

$$K_{ij}^2=\frac{E}{L^4}\int_0^1 I_{xx}\frac{d^2\phi_i\left(\overline{r}\right)}{d\overline{r}^2}\frac{d^2\phi_j\left(\overline{r}\right)}{d\overline{r}^2}d\overline{r} \qquad (32)$$

$$K_{ij}^3=\frac{EI_{xy}}{L^4}\int_0^1\frac{d^2\phi_i\left(\overline{r}\right)}{d\overline{r}^2}\frac{d^2\phi_j\left(\overline{r}\right)}{d\overline{r}^2}d\overline{r} \qquad (33)$$

$$K_{ij}^4=\frac{1}{L^2}\int_0^1\frac{d\ \phi_i\left(\overline{r}\right)}{d\overline{r}}\frac{d\ \phi_j\left(\overline{r}\right)}{d\overline{r}}d\overline{r} \qquad (34)$$

$$K_{ij}^5=\int_0^1\phi_i\left(\overline{r}\right)\ \phi_j\left(\overline{r}\right)d\overline{r} \qquad (35)$$

and

$$\overline{r}=\frac{r}{L} \qquad (36)$$

$$\overline{r}^*(t)=\frac{r^*(t)}{L} \qquad (37)$$

$$\overline{r}^{**}(t)=\frac{r^{**}(t)}{L} \qquad (38)$$

III. RESULTS AND DISCUSSIONS

For comparison with published results in previous study [19], the same geometrical parameters of the drill are consider as: $L = 0.1m$, $t_0 = 0.005m$, $b = 0.01m$ and $\beta = 7.854\,rad/m$. During the drilling process, the axial thrust load $P=3000N$, feeding speed $f = 0.002m/s$ and $k_{xx} = k_{yy} = 1\times 10^8\ N/m^2$ are considered. A dimensionless natural frequency was employed in this analysis, it is defined as follows,

$$\bar{\omega} = \omega \Big/ \sqrt{EI_{XX} / \rho A L^4} \tag{39}$$

where ρ and A are the density and cross area of a drill, respectively. The corresponding dimensionless spinning speed is displayed as

$$\bar{\Omega} = \Omega \Big/ \sqrt{EI_{XX} / \rho A L^4} \tag{40}$$

For checking the accuracy of this proposed model, a comparison between the natural frequencies of the drill with different twist angles solved from the proposed model and the results published in previous study [5]. Table I lists the deference in the lowest natural frequencies of these drills solved in this study and the reference [5]. It was observed that the difference in solved natural frequencies between two different models are so close, the largest difference is even lower than 0.5%. However, the time varying effect on the variation of its natural frequencies of the drill as it drills into a work piece has not discussed in these studies.

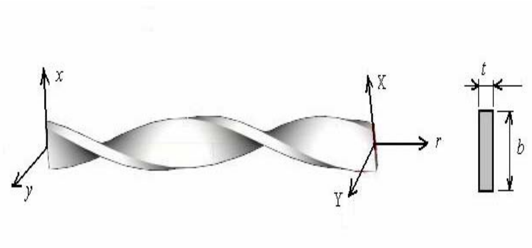
The variation of the lowest natural frequency of the drill during a drilling process is plotted in Fig. 2. In this drilling process, the drill tip touches and drills into the work piece at time $t = t^* = 0.15\,\text{sec}$. The results indicate that the lowest natural frequency of the drill is remained the same before it touches the work piece. Then an abrupt frequency drop is observed as the drill tip touches the work piece, i.e. at $t = t^* = 0.15\,\text{sec}$. The corresponding drilling depth is $\xi(t) = 0^+$. The abrupt dynamic characteristics change is introduced from the axial loading as the drill tip touches the work piece. In the drilling process, a severe vibration is introduced from the interactions of the drill periphery with the work piece. This severe vibration of drill tip may offsets the hole from the axis of the drill. After the drill penetrates into the work piece, the drilling depth and the natural frequencies of the drill increase with the drilling time. During the drilling process the flexible stiffness of the drill also is increased with the drilling time. As shown in Fig. 2, it is assumed that the drill goes throughout the work piece at the time of $t = t^{**} = 0.45\,\text{sec}$. The simulated results show an abrupt increase of the the lowest natural frequency of the drill at the instant that the drill goes through the work piece. $t = t^{**} = 0.45\,\text{sec}$ The lowest natural frequency of drill remains almost constant after the time $t > t^{**}$.

The dynamic time response of drill tip during a drilling process is simulated and illustrated in Fig. 3. A non-dimensional vibration amplitude is employed to display the variation of the amplitude of the drill tip in a drilling process. A severe vibration occurs as the drill tip touches the work piece, i.e. at $t = t^* = 0.15\,\text{sec}$. The drilling depth is $\xi(t) = 0^+$ as displayed in Fig. 3. After the drill drills in the work piece, the amplitude response of the drill tip is depressed significantly. The simulated results also show that the amplitude remains depressed even after the drill drilling throughout the work piece. Fig. 4 displays the variation of the lowest natural frequency of the drill with different thickness in the drill cross section. Before the drill touches the work piece, the lowest natural frequencies of different drills are almost same. However, the larger thickness drill has the smaller value of the lowest natural frequency as the drill drills into a work piece. Results indicate that the boundary between the drill and work piece has affected the lowest natural frequency of the smaller thickness drill much than the thicker drill does. The effect of width on the lowest natural frequency of the drill during a drilling process has also been studied in this work. Fig. 5 displays the variations in the lowest natural frequencies of drill with different width in the drill cross section. The similar effect, as shown in Fig. 5, has been observed for the drill width.

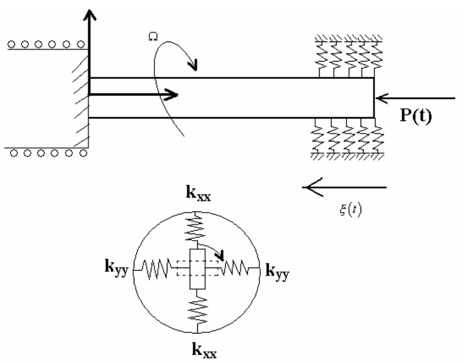
Fig. 6 shows the effect of axial load on the variation of lowest natural frequency during the drilling process. Generally, a smaller lowest natural frequency has been calculated for the drill with the larger axial load on the drill tip. The axial load may affect the lowest natural frequency significantly in the initial period as the drill tip goes into the work piece. The axial drilling load effect will reduce as the drill goes deeper into the work piece. In other words, the boundary effect will dominate the dynamic behavior of the drill. Fig. 7 illustrates the variation in the lowest natural frequency with the rotation speed. The simulated results indicate that a smaller lowest natural frequency is calculated for the drill with higher drilling speed.

TABLE I
THE VARIATION IN THE LOWEST NATURAL FREQUENCY BETWEEN THE STUDY OF LIAO [5] AND THIS INVESTIGATION, $L = 20cm$, $t = 0.5cm$, $b = 2cm$

Pre-twisted angle	Liao [5]	This study	Difference
0^0	3.516	3.516	0%
90^0	3.606	3.606	0%
180^0	3.843	3.844	0.02%
360^0	7.188	7.230	0.5%



(a) The schematic diagram of a drill



(b) The time dependent drilling process model

Fig. 1 A sketch of a drill in the drilling process

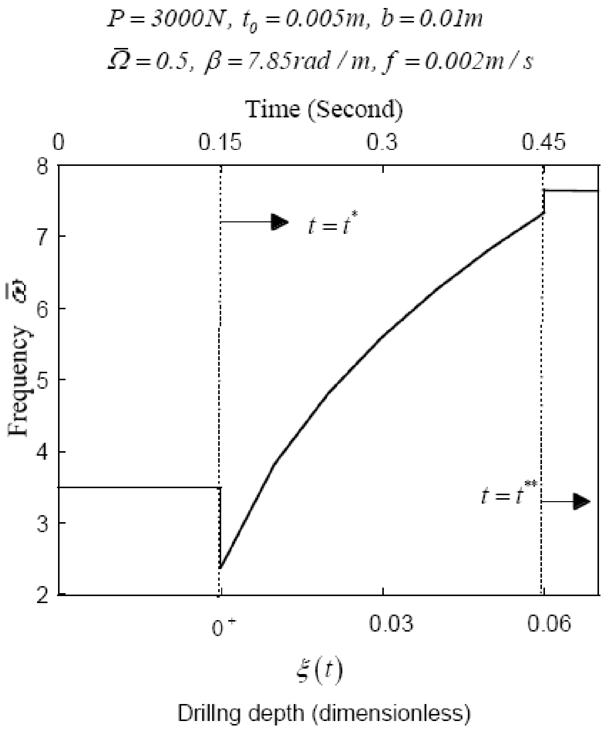


Fig. 2 The variation of the lowest natural frequency of a drill during a drilling process

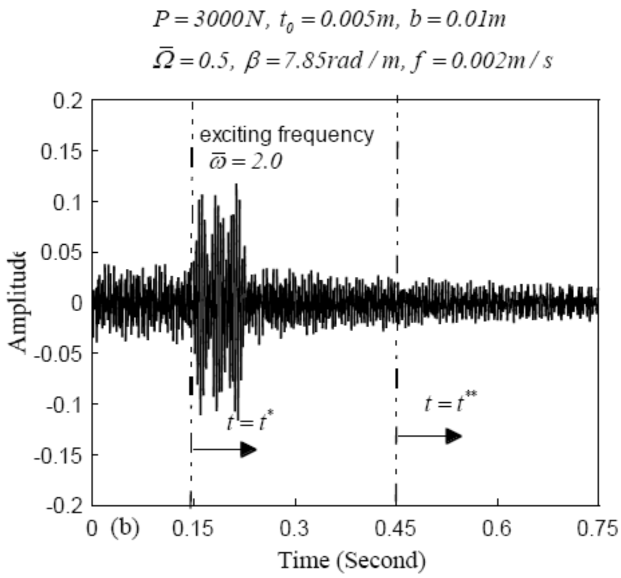


Fig. 3 The vibration response of a drill tip during the drilling

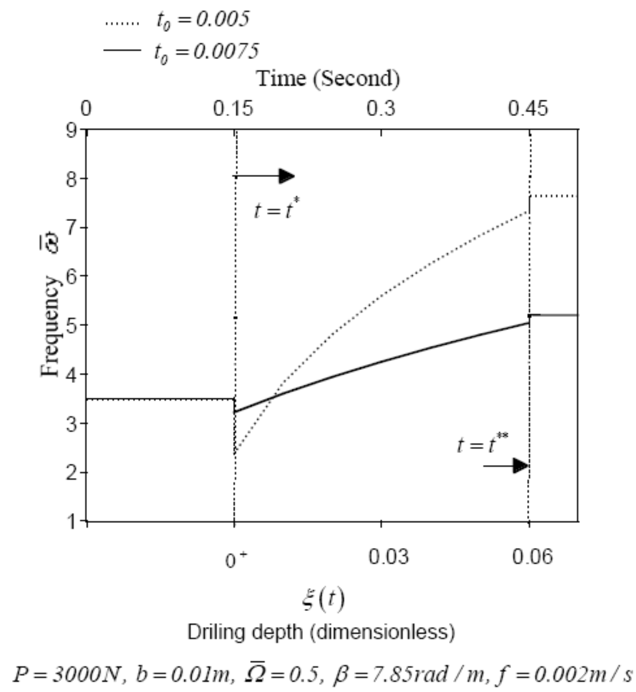
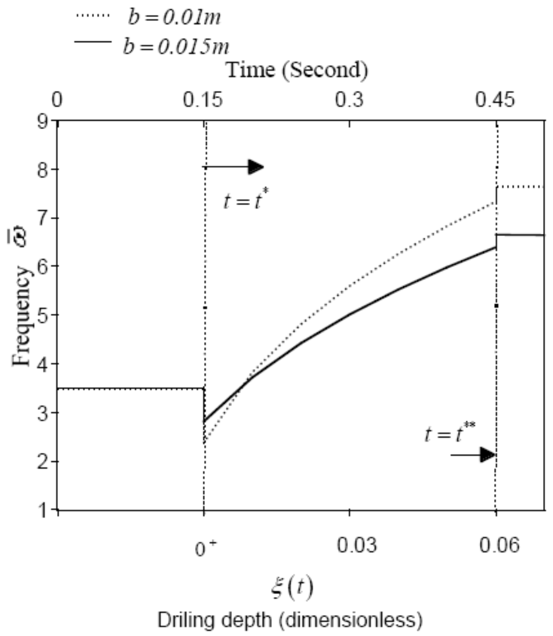
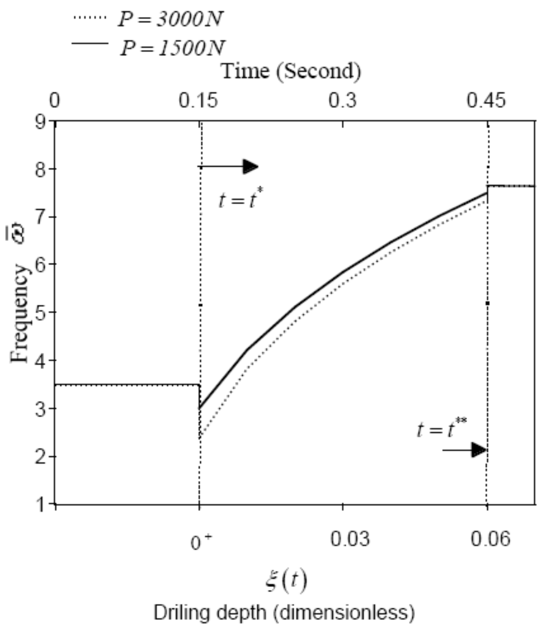


Fig. 4 The variation of lowest natural frequency of a drill in drilling process with different thickness in drill cross section



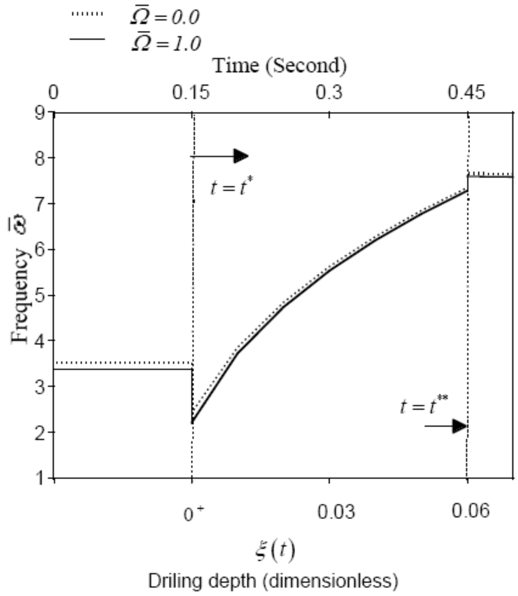
$P = 3000N,\ t_0 = 0.005,\ \bar{\Omega} = 0.5,\ \beta = 7.85rad/m,\ f = 0.002m/s$

Fig. 5 The variation of lowest natural frequency of drill in drilling process with different width in drill cross section



$t_0 = 0.005m,\ b = 0.01m,\ \bar{\Omega} = 0.5,\ \beta = 7.85rad/m,\ f = 0.002m/s$

Fig. 6 The variation of lowest natural frequency of a drill in drilling process with different axial drilling loads



$P = 3000N,\ b = 0.01m,\ t_0 = 0.005m,\ \beta = 7.85rad/m,\ f = 0.002m/s$

Fig. 7 The variation of lowest natural frequencies of a drill in drilling process with different rotational speeds

IV. Conclusion

The dynamic model of high-speed drills during a drilling process is proposed. The effect of time dependent drilling depth on the dynamic responses of drill has also been studied in this work. The major conclusions drawn from the simulated results can be summarized as follows:

(1) The simulated results indicate that the dynamic behavior of drill is varied in the drilling process. In other words, the drill system has varied values of the lowest natural frequency in different periods, i.e. before and after the drill tip touch the work piece. An abrupt drop of the lowest frequency is observed at the moment as the drill tip touches the work piece. A corresponding serve vibration response on the drill tip has also be found.

(2) The results indicate that the time dependent drilling depth is the key effect to dominate the dynamic response of drill during the drilling process.

(3) The axial thrust force, spinning speed, geometrical parameters in drill cross section corresponding dynamic response of drill during the drilling process.

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