

# GSM Position Tracking using a Kalman Filter

Jean-Pierre Dubois, Jihad S. Daba, M. Nader, C. El Ferkh

**Abstract**—GSM has undoubtedly become the most widespread cellular technology and has established itself as one of the most promising technology in wireless communication. The next generation of mobile telephones had also become more powerful and innovative in a way that new services related to the user's location will arise. Other than the 911 requirements for emergency location initiated by the Federal Communication Commission (FCC) of the United States, GSM positioning can be highly integrated in cellular communication technology for commercial use. However, GSM positioning is facing many challenges. Issues like accuracy, availability, reliability and suitable cost render the development and implementation of GSM positioning a challenging task. In this paper, we investigate the optimal mobile position tracking means. We employ an innovative scheme by integrating the Kalman filter in the localization process especially that it has great tracking characteristics. When tracking in two dimensions, Kalman filter is very powerful due to its reliable performance as it supports estimation of past, present, and future states, even when performing in unknown environments. We show that enhanced position tracking results is achieved when implementing the Kalman filter for GSM tracking.

**Keywords**—Cellular communication, estimation, GSM, Kalman filter, positioning

## I. INTRODUCTION

GLOBAL POSITIONING SYSTEM (GPS) is considered the keystone in the general field of localization. High accuracy, worldwide capabilities, and low cost renders GPS the most popular positioning technique in navigation. However, GPS equipment is highly expensive and requires open sky to receive the GPS signals. For these reasons, it is desirable to search for an alternative positioning technology that is lower in cost, more resistant to obstacles, and performs efficiently.

This is where GSM technology [1 – 4] becomes useful because it has the potential to provide localization information. It relies on existing hardware (base stations and mobile telephone) instead of non-network based methods using GPS for instance, thus reducing the cost of implementing position services. In addition, the mobile telephone is always related to the user's location.

The transmitter system between the base station (BTS) and the user mobile can be processed in order to determine the location of the user. Many techniques are involved in this scenario, some require one base station, some require at least two base stations, and others require more than three base stations.

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When the signals propagate in the atmosphere, they are jeopardized by many interference sources. This will affect the accuracy of the positioning results. It would be better to use highly advanced techniques, processors, algorithms and filters to estimate the location of the user through the location information exchanged between the handset and the base station. For this purpose, we employ an innovative scheme where we integrate the Kalman filter [5 – 14] in the localization process, especially that it has great tracking characteristics. The Kalman filter can recognize that the measurements are noisy and should be ignored sometimes or have only small effects on the estimated state. By integrating more information from reliable data more than unreliable data, the Kalman filter can smooth out the effects of noise in the estimated state variables. In addition, the Kalman filter algorithm makes it very easy to combine measurements from different sources (locomotion data) and different times. The main objective of this paper is to investigate optimal GSM positioning techniques and to evaluate the gain behind using Kalman filter in the localizations process of mobiles. The paper is organized as follows: In section II, we conduct a literature survey for GSM positioning services and applications. In section III, we introduce different system classifications, measurement techniques (CI, TOA, TDOA, AOA), heterogeneous and homogenous systems, in addition to GSM positioning architecture. In section IV, we present the different aspects of positioning performance consideration by first introducing the GSM positioning system and wave-based system. Then, we present a study of the signal ambiguity. At the end of the section, we consider other contributing sources of errors. In section V, we implement the novel Kalman filter tracking scheme. In section VI, we present our results and discuss them.

## II. POSITIONING SERVICES

### A. Overview of Positioning Services

Since it was developed and launched, it was clear that GSM has the potential of providing localization information. But this idea has only recently started to be realized with the contribution of many organizations that are investigating GSM positioning and interested in GSM equipment and operators of GSM-based networks. Other localization methods such as GPS require open sky to receive the weak satellite signals. And instead of relying on non-network based methods using GPS for instance [15, 16], it will be more practical and helpful to use the existing mobile network infrastructure in the localization process. The advantage of using existing hardware to deploy the localization is that the equipment (e.g. mobile telephones and base stations) in use today can be used for the localization, thus reducing the

investment cost. In order to examine the ability of deriving position information from GSM signals, we need to inspect the potential applications for mobile telephone positioning. Also, we need to study the generic position measurement techniques and positioning system classification and architecture (as described in section III). It is necessary for this work to investigate what services GSM offers and to describe what techniques it uses and with what accuracy it can estimate a mobile's position. We describe below important characteristics of positioning systems.

### 1. Terminal Based Location

One terminal based solution is SnapTrack (developed by Snap Track Incorporated). In this case, the wireless network sends an estimation of the location of the mobile to a server. Then the server informs the mobile which GPS satellites are in its area, and the mobile takes a "snapshot" of the GPS signals, calculates its distance from all satellites in view and sends this information back to the server. The server software performs error correction and calculates the caller's precise latitude, longitude and altitude. In case of emergency calls, the location is determined. Despite of its high accuracy and world wide ability and low cost, this technique based on GPS positioning suffers from many problems because it is very sensitive to obstruction of the line-of-sight between the mobile and the GPS satellites. Therefore, complication may occur when the mobile user is inside a building, or if he or she is driving on multi-levels highways, tunnels or underground parking for instance [15]. In these cases, satellite coverage may be poor because the GPS signals are corrupted by multi-path and fading. Alternatively, a position can always be calculated using GSM positioning.

### 2. Network Based Location

The more appropriate solutions for this research are the network based techniques. One example of such a solution is the systems that are based on time of arrival (TOA), time difference of arrival (TDOA), and angle of arrival (AOA) measurements. In certain cases, TDOA techniques are augmented with AOA capability to improve coverage and accuracy [3]. One example of this is the coverage of a rural highway where the base station arrangement is often in a line along the highway.

### 3. Accuracy of Systems

Cambridge Positioning Systems use a technique called Enhanced Observed Time Difference (E-OTD) in their system Cursor. E-OTD means that the time differences of arrival are calculated in the mobile and in the network. According to the Cambridge Positioning Systems web page, this gives an accuracy in the positioning of 50 meters. In the minutes from a FCC E911 meeting on July 6, 2000, Cell-Loc indicated that with AMPS (an analog signal cellular telephone service) they have achieved test results of 31-92 meters 67% of the time and 21-148 meters 95% of the time. It is unclear under what circumstances these results have been obtained. Cell-Loc uses TDOA/voice positioning technology and the accuracy improves with longer conversations, which may indicate that tracking is used. During mobile telephone conversations more

measurements are available and it is not certain that Cell-Loc uses tracking [17]. TruePosition has developed a network-based TDOA solution for E911. Because TruePosition's TDOA system uses existing antennas, it is less expensive than a system using AOA, which usually requires independent antennas with a time-consuming and expensive installation. On July 6, 2000, TruePosition's solution was tested in field tests, but only met the accuracy requirements the majority of the time.

### B. Applications of Cellular Positioning

There are many reasons why it is useful to locate the position of a mobile telephone, as described below.

#### 1. Increased Subscriber Safety

There is a continuous increase in the number of the emergency calls that are being made from mobile phones, where in many cases the caller can't provide accurate information about their location. This necessitates an automatic provision of position information [3], similar to that already available for calls made from fixed phones, to ensure that the emergency help is directed to the right place. Thus, faster response time and more efficient use of emergency resources are provided. For example, The Federal Communications Commission (FCC) in the USA intends to improve the reliability of wireless 911 services, by requiring wireless carriers to provide to emergency call dispatchers information on the location from which a wireless call is being made. These requirements are called enhanced-911. After December 31, 2002, 100 percent of all new digital handsets are activated to be Automatic Location Identification-capable, meaning that they are capable of delivering specific latitude and longitude location information. The FCC has set the following standards for location accuracy and reliability: For network-based solutions: 100 meters for 67% of calls and 300 m for 95% of calls; For terminal-based solutions: 50 m for 67% of calls and 150 m for 95% of calls.

#### 2. Location Services

Subscribers can get information about their location by sending a SMS in case they lost their way. In addition, they can get information on a narrow selection of services (gas stations, restaurants, pharmacies, hospitals...) and get direction to street addresses from the position that the request is made [13]. In addition, the location service is very helpful for taxi dispatch [4]. When a customer has called the taxi dispatch center for a taxi, the customer's location is broadcasted over the taxi radio network so that the nearest unoccupied taxi will go directly to the customer. This technology is very popular nowadays in modern cities such as New York, London, and Paris.

#### 3. Location-Sensitive Billing

Depending on the location of the mobile, network operators can provide differential tariffs. This allows network operators, without a copper-based public switched telephone network (PSTN), to offer competitive rates for calls from home or office.

#### 4. Internal Location Service

Positioning is used for internal operations in telephony networks (eg. demographic surveys where the subscriber identity is not revealed).

#### 5. Lawful claimed Location Service

It is a legal claimed positioning, such as detection of mobiles entering a security area. In this case, the identity of the user is revealed.

#### 6. Intelligent Transport Systems (ITS)

Many services envisaged under the ITS initiative will require position information, often in conjunction with a communications channel, to be effective. The ability to position a mobile telephone could enable services such as providing information to travelers, more effective dispatch of vehicles in fleets, and detecting traffic incidents and congestion.

#### 7. Enhanced Network Performance

At the microscopic level, the cellular communications network makes better decision when handing over from one cell to another, while monitoring the displacement of mobile telephone. Macroscopically, long-term monitoring of mobile positions [3] provides useful input to the cellular network planning.

### III. POSITIONING TECHNIQUES

#### A. Position System Classification

To classify the positioning system, we need to consider where the position measurements are made and the position information is used. Two broad classifications are made: self-positioning and remote positioning.

##### 1. Self-Positioning

In a self-positioning system the positioning receiver should make the suitable signal measurements from geographically distributed transmitters and then it should use these data to determine its position. The most known self-positioning system is GPS. A self-positioning receiver, thus knows where it is, and applications collocated with it can use this information to make position-based decisions required for navigation.

##### 2. Remote Positioning

In a remote positioning system, the receivers should measure a signal originating from, or reflected off, the object to be positioned. Then to be able to estimate the location of the object, the collected data are sent to a central site where they will be analyzed. This position information can then be used at the central site or be sent to another system running an application such as a computer-aided dispatch (CAD) system. A good example of remote-positioning systems are remote sensing radars.

#### 3. Indirect Positioning

Using a data link, it is possible to send position measurements from the self-positioning receiver to a remote site or vice-versa. A self-positioning system that sends position data to a remote location is referred to as indirect remote positioning, and a remote positioning system transmitting an object's position to the object is referred to as indirect self-positioning.

#### B. Position System Classification

In order to locate the position of a signal, many measurements should be taken into consideration. The most important measurements are: Cell Identification (CI), time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), and carrier phase. Each measurement defines a locus on which the mobile phone must lie.

The intersection of multiple loci from multiple measurements intersects defines the position of the mobile phone. When more measurements can be made than are required to uniquely define the position, a least squares approach can be used to combine all the measurements into a more accurate position estimate [8]. If a few measurements are available, the loci will intersect at more than one point, resulting in ambiguous position estimates. It is important to note that all the techniques that are discussed can be used in either a self- or remote-positioning mode, depending on whether the measurements are made at the mobile phone or at one or more base stations.

##### 1. Cell Identification

CI, illustrated in Fig. 1, is considered the most basic positioning technique. It uses the base station to identify the user in cell area. The measurement only puts the user in a particular cell's circle of coverage. The position's ambiguity can be reduced if the user is within reach of more than 1 cell. CI is popular among the operators because it does not require any modifications in the handset or the network and because it is cheap to deploy.

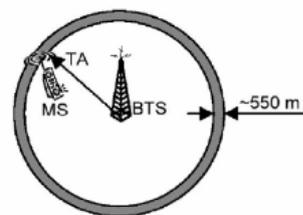


Fig. 1 Cell identification method

##### 2. Time of Arrival

It involves measuring the time it takes for a signal to travel between a base station and a mobile phone (self-positioning) or vice-versa (remote-positioning). This approach might measure the round-trip time of a signal transmitted from the transmitter to the receiver and then echoed back to the transmitter. This will generate the result twice than that of the one-way measurement. In this scenario, the transmitter requires that the receiving mobile phone know the exact time at which the transmitting base station will transmit, and that the receiver have an accurate clock. The receiver does not rely

on such synchronization between the mobile and base station, and is the more common means of measuring propagation time.

Either measurement reduces position of the mobile phone to a circular locus around the base station. If another propagation time measurement is made with respect to a second base station, a second circular locus is produced. The two circular loci will produce an ambiguity because they intersect in two points. This ambiguity can be resolved by using a priori information concerning the trajectory of the mobile phone or by making a propagation time measurement to a third base station. In the case where a third base station is included, the three transmitters must be synchronized. In an ideal terrain-less environment, only three synchronized transmitters are needed in order to locate exactly a fixed receiver. If the receiver is moving, its position can only be approximated due to the distance traveled while TDOA measurements are taken [3]. Hence, the faster the mobile is moving, the greater the error will be. Thus, the use of more than three transmitters is better when pinpointing moving objects. Positioning systems are classified by listing the loci used to determine the position, so a system that makes propagation time measurements can be referred to as a circular-circular-circular system.

### 3. Time Difference of Arrival

A mobile phone can track a series of base stations and measure the time difference between each pair of arrivals. Each TDOA measurement defines a hyperbolic locus on which the mobile telephone must lie. The intersection of the two hyperbolic loci will define the position of the mobile telephone. In the case where two hyperbolas intersect in two points, an ambiguity will result. This problem is solved in a manner similar to that of the propagation time measurements: a third TDOA measurement from a third base station will resolve the problem or by approximating a priori information [3]. It is important to mention that in the TDOA systems the base station needs to be synchronized according to the operating mode of the system: (1) For self-positioning mode, the base stations are considered as transmitters and the transmitted signal must leave each base station at the same time, otherwise the TDOA measurements will generate a bias error in the resulting hyperbolic locus; (2) For remote positioning mode, the base stations are considered as receivers. The transmission originating from the mobile phone is detected at many base stations, so there must be a known time relationship between the receiver clocks at these base stations, otherwise a bias errors will result.

### 4. Angle of Arrival

The angle of arrival is measured from the base station to the mobile telephone or by measuring the angle of arrival of a signal from a mobile phone to the base station. In either cases, this technique calculates the angles (or directions) at which a signal arrives at two base stations from a handset using triangulation to find the location [14]. The measurement of a signal will produce a straight line locus from the base station to the mobile telephone.

In the same way, another AOA measurement will yield a 2nd straight line, the intersection of the 2 lines giving a

position fix for this angle-angle system. In this case, there is no ambiguity, because 2 straight lines can only intersect at 1 point [3].

This method requires a complex antenna array at each cell site, and these antennas together should be synchronized in order to determine the direction. AOA performs well in areas with low multipath reflections.

### 5. Carrier Phase

The phase of a carrier is used to provide position estimates with an error considerably less than the carrier wavelength. There are many problems using this approach. First, we have a large number of ambiguities that take place in the positioning solution [3]. The phase of the received signal can be measured by the positioning receiver but it cannot measure the integer number of cycles (wavelengths) between the transmitter and receiver. Second, in the carrier phase measurement process, a continuous lock on the carrier signal must be maintained.

### C. Heterogeneous and Homogeneous Systems

Homogenous systems only use one type of measurements (e.g., circular-circular), whereas heterogeneous system use different types of measurements. Concerning the mixing of two types of measurements, the radar uses propagation time measurement with an AOA measurement to yield a position fix. Regarding the performance consideration of positioning systems, the RMS is a way to measure the accuracy.

### D. GSM Positioning Architectures

Different physical architectures can be implemented to position the GSM mobile phones. These architectures have significant differences affecting infrastructure costs, coverage area, total number of users that can be supported, and the number of users that can be simultaneously positioned [3]. Thus according to the needs of the positioning application, each architecture can be implemented. Two major categories arise: Mobile-based and network-based positioning. In addition the combination of these two architectures is called Hybrid positioning.

The first category is the mobile-based positioning architecture where the positioning is based on the mobile handset. The mobile handset uses the signals from the BTS to calculate its position. One way of calculating this position is to use the TDOA requiring two fundamental changes to the GSM equipment. The first is to modify the handset so that it is able to make highly accurate TDOA measurements. The second is to provide information to the handset on the synchronicity of the network [17]. This is easily done by sending an sms or a paging service to the receivers. Another way of mobile positioning is the implementation of sophisticated software, which will combine the information from a variety of sources such as information located at the BTS, signal strength indications, sector information etc...

The second category is the network-based positioning architecture. It is called network-based because the mobile network, in conjunction with network-based position determination equipment, is used to position the mobile device. Many methods can be used such as AOA. This method is used to capture information, to make calculation, and to

determine an estimate of the mobile device position. Another method is TOA, which is used to capture TDOA information, to make calculations, and to determine an estimate of the mobile device position. Another method is Radio Propagation which uses a previously determined mapping of the RF to estimate the mobile device position.

A combination of both categories is the hybrid-based positioning architecture. To calculate the location, a given mobile will measure the TOA of bursts from various transceiver stations. These are then sent to the location service center which generates TDOA measurements and computes a position estimate for the given mobile.

#### *E. Location Problems Due to Channel Interference*

There are two major location problems caused by channel interference: Multipath and non-line of sight propagations

Multipath propagation is especially disturbing when signals that have been reflected on their way to the base station arrive within a chip period of the first arriving signal. For AOA-based systems, multipath propagation can give an angular spread which varies from 1 to 10 degrees even in a Line-of-sight (LOS) situation where one signal is significantly stronger than the others [14]. The solution at hand is to perform some spatial averaging with the drawback of reducing the resolution of the signal.

Non-line of sight (NLOS) is the dominating problem in TOA measurements, characterized by multipath Rayleigh fading. The LOS measurement is needed to calculate the position. To counteract NLOS effects, analysis should be performed on the incoming signal as to avoid positioning based on signals that have traveled a longer path than the direct one or whose direction is greatly incorrect. There are different strategies to avoid NLOS, most of which exclude unreasonable measurements by comparing all incoming signals and keeping those that are likely to be LOS.

### IV. POSITIONING PERFORMANCE CONSIDERATIONS

#### *A. Positioning System Overview*

Positioning systems measures the location of one or more objects. Examples include GPS, radars, sonars and urban vehicle tracking systems. Most important systems fall into the category of wave-based positioning systems because they use the propagation properties of the waves to measure position.

A wave-based positioning system consists of one or more reference sites. The positions of remote objects are measured relative to these sites. Each reference site may have a transmitter or/and a receiver. On the other hand, each remote object may have a transmitter, a receiver, reflective properties or some combination of these. For example, in GSM positioning, each base station is a reference site and the mobile has a receiver which picks up the signals from the base station.

Obviously in the case of positioning systems, performance is considered as one of the major parameters. However, there are many important factors that also should be taken into consideration. Ultimately, normal system engineering concerns also apply to GSM positioning systems [4]. These include reliability, availability, maintainability, resilience,

robustness, usability, functionality, integrity, cost, accuracy, quality and timescale consideration must apply.

In this way, one difference between normal communications system and positioning system is that in a typical communications system, small deterministic time delays in the delivery of the information are not important. Contrary, the situation is different in a positioning system where a few seconds delay can make a position measurement useless. For example, in a fighter plane radar, a few seconds delay can cause an error of a kilometer or maybe more. There are cases where the time delay will not be important, for example, the data from surveillance tracking system may be used many years later in a court case. Thus, the amount of time delay is an important factor for most positioning systems [4].

#### *B. Wave-based Systems*

Wave-based systems include sonars, radars, and Omega, where the position information is coded directly onto the wave due to the physical properties of the wave propagation. Sonars and radars use similar techniques that rely on time delay or direction of wave-front arrival. Omega measures the phase shifts. In these systems, the user relates the objects positions to an absolute, real world Cartesian measurement frame. Some systems may not use Cartesian coordinates (some use latitude, longitude and height), but this can still be related to some absolute Cartesian frame.

#### *1. System Model*

In this model, the source is the moving object. The waveform coding is the process of the moving object emitting a signal modulated with suitable waveform. Unlike the conventional communications model, this is done without reference to the message being sent by the source, although the chosen waveform can affect system performance by causing trade-offs between various parameters, e.g. between range and range-rate. The signal is then transmitted over the physical medium, during which time physical coding occurs. It is during the process of physical coding that the position information is transformed from the target to the communications frame. This coding is due to the physical nature of the wave propagation and the overall configuration of the transmitters and receivers. At the receiver, the signal must be demodulated, in the process is called wave-form decoding. This provides positional information expressed in the communication frame. Physical decoding is simply transforming this information to the target frame. Finally, a series of measurements can be combined, using estimation to form the final estimates. This process would normally use knowledge of the moving object's kinematics. A good example is Kalman filtering, which will be treated in the next section. Note that in some systems, estimation can occur concurrently with waveform decoding, or after waveform decoding, but surely prior to physical decoding.

Thus, the process of waveform decoding requires a receiver to estimate various signal parameters. But first we need to discuss some important factors that can highly affect the estimation process: Signal ambiguity and the performance of coding schemes.

### C. Signal Ambiguity

Signal ambiguity occurs when a system cannot distinguish between a signal and a translated version of that signal. For example, a repetitive waveform will have autocorrelation peaks occurring at the repetition rate of the waveform. A positioning system using such a waveform for ranging measurements will be unable to determine which repetition of the waveform produced a particular echo. Another form of ambiguity can occur if two or more objects are being measured simultaneously. For instance, if two objects are close together, then it may not be possible to distinguish between them. The ability to distinguish simultaneously between two or more objects will be called the close resolution. For more practical waveforms, this autocorrelation function (ACF) can be divided into 2 parts: the main peak and the side lobes. The width of the main peak and the height of the side lobes are important for 3 reasons: accuracy, resolution, and performance.

#### 1. Accuracy

The system accuracy depends inversely on the root-mean-square (rms) widths of the power spectrum for high signal-to-noise ratios. Since the Fourier transform of the power spectrum is the auto-correlation function, the accuracy will depend directly on the rms widths of the auto-correlation function. The narrower the main peak, the greater the system accuracy. However, if the side lobes are high, the noise ambiguity will be large and the overall system accuracy cannot be determined from an examination of the width of the peak alone.

#### 2. Resolution

The size of the main peak will be an important factor in determining the close resolution performance of the system, which is a measure of the ability to differentiate closely spaced objects. The most common definition of resolution is the two-point resolution which is based on the ability of a system to resolve two closely spaced points. Accordingly, the width of the main peak can be used as a guide to close resolution performance. The size of the side lobes will also have an effect on resolution, but in this case the resolution targets that are far apart. If one target has a much higher SNR than another, then the side lobes of the stronger target can obscure the main peak of the weaker target. This is often called the near-far problem because the differences in signal strength result from the stronger target being much closer. As for the case of noise ambiguity, the optimal auto-correlation function will be the one that has the smallest side lobes.

#### 3. Performance

The width of the peak affects is the overall system performance because the number of independent messages that can be sent depends on this width. To a first order, the smaller the size of the main peak, the greater the system performance is. In a finite bandwidth system, it is not possible to adjust the size of the main peak without affecting the height of the side lobes, which in turn will increase the noise

ambiguity and thus reduce the performance. This tradeoff is the key to waveform coding.

From the above discussion we can now visualize the ACF of the optimum waveform for a positioning system. It should have zero side lobes and an infinitely narrow main peak. Such a waveform would have infinite bandwidth.

### D. Performance of Coding Schemes

If the source statistics and measurement constraint are known, then it is possible to calculate the performance of the system for a particular coding scheme, but such an answer would be specific to the source statistics and measurement constraints. The normal assumptions for communication systems are that each source symbol is independent, each letter of the source alphabet is sent independent and only one symbol is sent at a time. This allows the publication of a system bit rate which can be readily used to compare different systems and coding schemes. The aim here is to provide reasonable assumptions concerning the measurement strategy and source statistics that will allow a meaningful comparative performance figure for different coding schemes.

#### 1. Source Statistics

There are two necessary assumptions concerning the source statistics. The first is the correlation between individual measurements. The performance will be best when each measurement is independent, so it will be assumed that each measurement is independent. The second aspect is the nature of the a priori distribution of the objects. This constraint could be in terms of specifying a known area that the objects must lie within, or specifying a known positional pdf. This requires less knowledge on the part of systems engineer than the assumption of a particular pdf. The objects are considered to be uniformly distributed over a particular domain. It is assumed uniform over the communication frame and not on the target frame simply because the calculations concerning coding will be carried out in the communications frame.

#### 2. Measurement Strategy

There are two characteristics of the measurement strategy that need to be respected: the measurement order and the measurement accuracy. It will be assumed that only one measurement will be taken at a time. The assumption concerning independence of the measurements makes the selection of the measurement order simple: the order of measurement makes no difference. Concerning the measurement accuracy, each measurement should allow sufficient integration time to achieve this accuracy. This corresponds to a requirement in a communications system to achieve a particular error rate.

#### 3. Other Factors

In addition to the above mentioned factors, there are other performance factors that also affect the performance of the system. One source of signal ambiguity is known as the physical ambiguity which represents the nature of the transformation between the coordinate frame defined by the loci and a Cartesian frame. The sources of error that are common to all cellular systems include multipath fading, non-

line-of-sight signal propagation and multiple-access interference.

Another performance parameter is the source information rate, which can be used to determine the ultimate limits on the compression of position data. In this way, just as for a normal communication system, a position measurement conveys a certain amount of information which can be quantified in terms of bits. Accordingly, it is possible to define Shannon theoretic measures of performance.

Furthermore, the aspect of any positioning system is the area over which successful position measurements can be made. In general, this is expressed as a percentage of the area of interest that is provided with an acceptable level of service. For a cellular communications system, the principal area of interest is the area in which an acceptable level of voice and/or data service is available. We anticipate, however, that for GSM positioning, it will be possible to determine position with lower SNRs than are required to maintain an adequate level of voice service.

## V. KALMAN ESTIMATOR

### A. Kalman Filter Characterization

Kalman filter is an optimal recursive data processing algorithm. One aspect of this optimality [13] is that the Kalman filter integrates all information that can be provided to it. In other way, the Kalman filter processes all available measurements, regardless of their precision, to estimate a current value of the studied object. Concerning recursiveness, the Kalman filter does not need all previous data to be stored and reprocessed every time a new measurement is taken [5]. This is important for the filter's implementation, which is in fact a data processing algorithm. This filter naturally integrates discrete-time measurements instead of continuous time inputs. The Kalman filter merges all the available measurement data with a prior knowledge about the system and measuring devices, to generate an estimation of the desired variables in such a way the error is minimized statistically. A typical Kalman filter application is depicted in Fig. 2, where a system is driven by some known controls and measuring devices to provide the value of certain quantities. What is available for estimation purposes is the knowledge of the system's inputs and outputs [6], thus the need for a filter to estimate the system's state. Furthermore, this filter should be aware of the impact of noises affecting the system and occurring during the measurement phases.

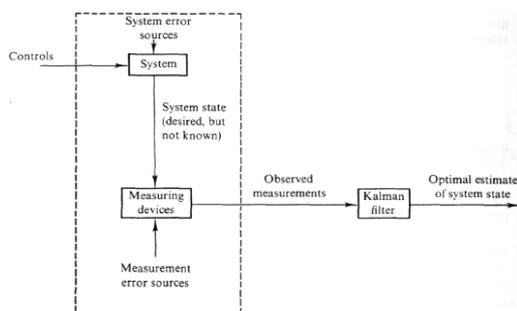


Fig. 2 Kalman filter block diagram

### B. Basic Assumptions

Kalman filter was built on the basic assumptions that the system modal can be linear and the white Gaussian noise is applicable [6]. When nonlinear systems exist in real life, the engineers' strategy is to linearize these systems in order to get more practical means. Linear systems are more desirable because they are easier and more practical from a mathematical perspective [11]. As a matter of fact, there are many ways to extend the Kalman filter approach to be applicable to nonlinear applications, but this alternative is considered only when linear models are insufficient.

The white noise assumption means that the value of the noise is not associated in time and the noise has equal power at all frequencies. But actually this does not exist in real life. However, the white noise modal is still applicable because the bandpass frequency, for every system, looks identical to the real wideband noise [6]. This will be an advantage from a mathematical point of view.

The Gaussian assumption is relevant to the noise's amplitude. At any time, the probability density of a Gaussian noise amplitude has a bell shape curve. Physically, this can be validated because the measurement noise is caused by a large number of small sources [6] and the central limit theorem guarantees the Gaussian statistics. Moreover, Gaussian densities make the mathematics more tractable since it offers the first and second order statistics of a noise process in terms of mean and variance.

### C. The Discrete Kalman Filter Algorithm

The Kalman filter's strategy in estimation is based on feedback control in the sense that the process state is estimated, then the filter obtains feedback in the form of noisy measurements [12]. The Kalman filter's equations are divided into two groups: (1) Time update equations that predict forward the actual state and error covariance estimates to obtain the a priori estimates for the next time step; (2) Measurements update equations which include a new measurement into the a priori estimate to get an improved a posteriori estimate. These equations interact in a way that the update equations can function as predictor equations and the measurement update equations can be thought as corrector equations [10]. Hence, the algorithm resembles a predictor-corrector processor.

The Kalman filter addresses the general problem of trying to estimate the state  $x$  of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}, \quad (1)$$

with a measurement  $z$  represented by

$$z_k = Hx_k + v_k, \quad (2)$$

where  $w_{k-1}$  and  $v_k$  are the process and the measurement noise, respectively. The update equations are given by

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}, \quad (3)$$

$$P_k^- = AP_{k-1}A^T + Q. \quad (4)$$

In this phase, the state and covariance estimates are projected forward from discrete time  $k-1$  to  $k$ . The measurements update equations are given by

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}, \quad (5)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H\hat{x}_k^-), \quad (6)$$

$$P_k = (I - K_k H) P_k^-. \quad (7)$$

The first step in this phase is to compute the Kalman gain  $K_k$ , then to obtain  $z_k$  by actually measuring the process. The next step is to generate an a posteriori estimate. The final step is to compute the a posteriori error covariance estimate.

**Filter parameters and tuning:** In the actual implementation of the filter  $R$ , the measurement covariance error is usually measured prior to the operation of the filter. In order to determine the variance of the measurement noise, we need to take some off-line sample measurements. However, it is more difficult to get  $Q$ , the process noise covariance, since we can't observe directly the process we are estimating. In this case, we assume that we have access to "reliable" process measurements.

Noting here that under the condition that  $Q$  and  $R$  are constant [5], both estimation error covariance  $P_k$  and the Kalman gain  $K_k$  will stabilize quickly and then remain constant. Thus, these two parameters can be pre-computed by running the filter offline.

#### D. Numerical Application

After measuring the position through the previously discussed methods, these measured positions are then entered in a Kalman Filter as shown in Fig. 3.



Fig. 3 Block diagram of position estimation using Kalman filter.

The Kalman filter addresses the general problem of trying to estimate the state  $z$  of a discrete-time controlled process that is governed by the linear stochastic different equation

$$z_{k+1} = Az_k + \Gamma \omega_k, \quad (8)$$

where  $z = \left[ x_1, \frac{dx_1}{dt}, x_2, \frac{dx_2}{dt} \right]^T$ ,  $x_1, x_2$  are the Cartesian coordinates of the object,

$$A = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (9)$$

$T$  is a parameter representing the "duration" in kilometers,

$\Gamma = \begin{bmatrix} T/2 & 1 & 0 & 0 \\ 0 & 0 & T/2 & 1 \end{bmatrix}^T$ , and  $\omega$  is zero-mean wgn.

The measurement model is

$$y_k = H_0 z_k + v_k, \quad (10)$$

where  $H_0 = [1 \ 1 \ 0 \ 0]$ ,  $v$  is a zero mean white Gaussian noise (wgn), with the initial parameters set as  $\hat{x}_0 = [0 \ 0 \ 0 \ 0]^T$ , and

$$V_x(0) = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

## VI. RESULTS AND DISCUSSIONS

### A. Simulations

The implementation of a Kalman Filter using Matlab for different number of runs and various values for the parameters  $T$  and  $dt$  is illustrated in Figs. 4 – 8.

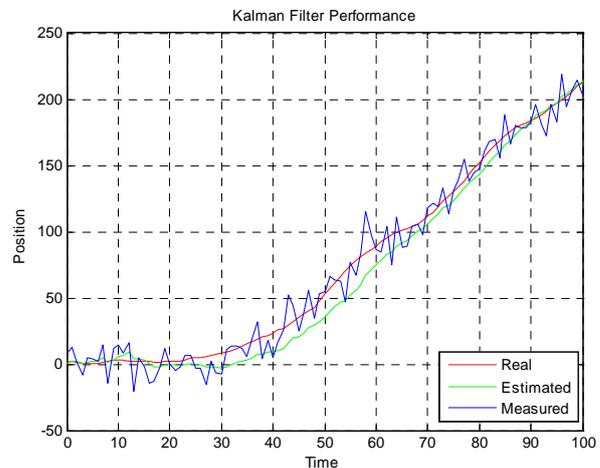


Fig. 4 Position estimated after 1 run for  $T = 100$  and  $dt = 1$ .

It is evident that the estimated position (green line) differs from the measured position (blue line), indicating that the Kalman filter is not providing accurate position estimation. Simulation shows large values of 176.3406 and 6.1678 for both mean bias and variance, respectively.

Comparing the measured position with the estimated one, position by position, we obtain the figure below.

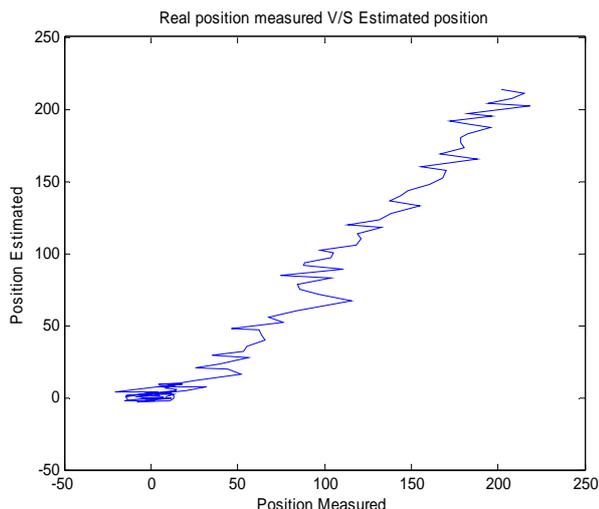


Fig. 5 Estimated versus measured positions.

Since the gradient of this curve is not unity, the estimated position is not consistent with the measured one.

In Fig. 6, the simulation is conducted for 200 runs. It is clear that the estimated positions are consistent with the real ones, indicating high accurate positions estimation. Both the mean bias and variance are very close to 0, with values of  $4.4806 \times 10^{-5}$  and  $9.9161 \times 10^{-8}$ , respectively.

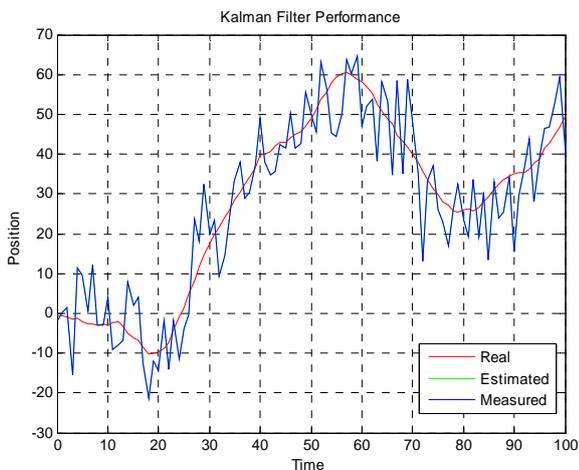


Fig. 6 Estimated position after 200 runs for  $T = 100$  and  $dt = 1$ .

In Fig. 7, the real measured position is compared with the estimated one, position by position.

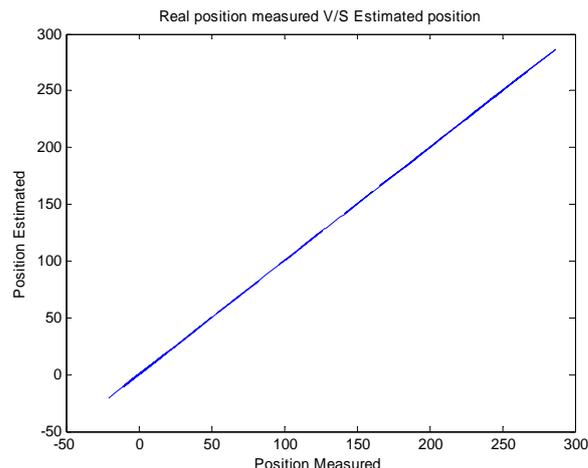


Fig. 7 Real position versus estimated position using 200 runs

The unit gradient of the straight line shows that for each measured position, a highly accurate estimated position is calculated. Thus, the more the number of runs, the more accurate the estimator is, at the cost of computational time.

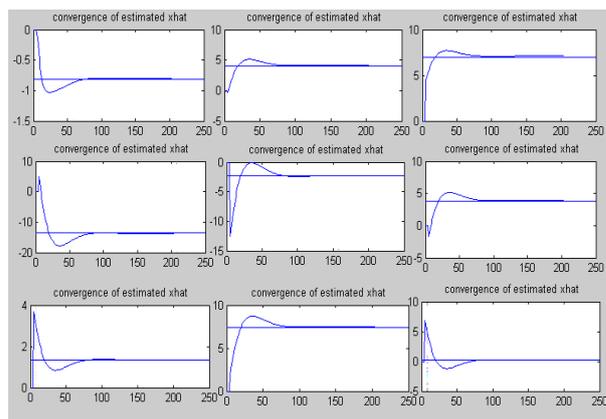


Fig. 8 Position convergence using multiple runs.

Figure 8 shows the convergence of each position. The results indicate that convergence is achieved after nearly 90 iterations. Estimating the position using more than 90 runs is not needed and will simply waste computation and time.

### B. Discussions

Comparing the error (bias) and the variance for the estimated positions using different number of runs (1, 200 and 90), we can first conclude that a single run is not sufficient and yields an estimated position that is not trustworthy. Thus, multiple runs are needed to get a good estimated position. In this way, the mean bias and the variance for 200 runs ( $4.4806 \times 10^{-5}$  and  $9.9161 \times 10^{-8}$ , respectively) are very reliable and at this stage, 200 runs are recommended. However, after plotting the position convergence, we found that after 90 runs the position will converge. When further investigating the mean bias and the variance (0.0633 and 0.0049, respectively) for 90 runs, we noticed that they are at an acceptable low practical level, although higher than the values obtained with 200 runs.

From an engineering perspective we can consider that when estimating position for many users, where each user is continuously changing position, adopting 90 runs will be more efficient, more practical and less time consuming for data processing. Furthermore, in a real life environment it is better to adopt the initial state of a user to be the previous measured position. The enhanced system will provide faster and reliable results.

Thus, as a tradeoff between estimation accuracy and processing speed, we conclude that 90 runs are needed for the implementation of the proposed Kalman filter scheme.

## VII. CONCLUSIONS AND FUTURE WORK

In this paper, we examined different GSM positioning techniques used in GSM technology. The positioning accuracy, performance and the practical challenges in realizing reliable GSM position tracking were investigated. A powerful tracking filter, the Kalman filter, was used in the simulation part in order to reach suitable results. The Kalman filter's algorithm yielded impressive tracking results after a relatively short number of runs.

Initially, our results showed that after 200 runs, the Kalman filter was able to reach extremely accurate tracking results. However, from an engineering perspective, and trading off estimation accuracy with processing speed while maintaining practically low estimation errors in terms of mean bias and variance, we recommended 90 runs as being sufficient for the Kalman filter algorithm processing.

In addition, when adopting the initial state to be the previously measured position and not a zero state position, the performance of the filter further improved. In this way, the Kalman filter can have a promising future when combining GSM technology with tracking services.

As future work, and after researching the Kalman filter and proving its high efficiency in tracking using simulation, it will be useful to know how it will perform in real life applications of GSM positioning and to compare its performance to other well established estimators.

In particular, one can refer to a number of research papers treating estimation theory [18 – 23].

Another aspect of future work lies in investigating how to integrate GSM positioning methods with global positioning system (GPS) techniques. This combination can be represented by employing very sensitive GPS receivers that can improve the performance of localization.

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