Effective Class of Discreet Programing Problems

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Abstract—We consider herein a concise view of discreet programming models and methods. There has been conducted the models and methods analysis. On the basis of discreet programming models there has been elaborated and offered a new class of problems, i.e. block-symmetry models and methods of applied tasks statements and solutions.

Keywords—Discreet programming, block-symmetry, analysis methods, information systems development.

I. INTRODUCTION

AT present, various classes and designation information systems are used in all spheres of human beings activities. In the process of information systems creation there have been used advanced instrumental programming means, database control systems, works elaboration and design automation systems, domain-specific intelligence, modern technical base as different levels of computing networks.

Alongside with that, the dynamic conditions and requirements to the information systems development and operation, necessity in adaptation to enterprises and organizations demands, quick re-profiling their activities in market conditions determine the necessity in continuous solution of acute information systems creation. Therefore tasks of information systems analysis, design, operation, modernization, reliability are quite acute.

Majority of above mentioned applied problems, as a rule, resolves itself by means of discreet programming tasks the statement and solution of which, in its turn, cause sufficient difficulties. In the first instance it is a computation complexity (NP-complete problems), dimensionality of solvable applied problems, accuracy and efficiency of developed algorithms for practical application.

In the process of setting and solving the new applied problems there appears the necessity in the elaboration of more effective discreet programming new classes comparing to the existing ones.

But prior to transferring them let conduct a concise view of discreet programming classical problems.

Let define discreet programming problem in the following way [1].

Let name the discreet programming problem as the task of scalar function extreme problem finding (max, min), prescribed at discreet (unconnected) multiplicity, i.e. such

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mathematical programming tasks which apply the discreet requirements to all or a part of variables, determining the area of permissible solutions. Let write down mathematical programming solution in the form of:

$$extr\{f(x); x \in \Omega\} \quad (1)$$

where $x = (x_1,...,x_j,...,x_n)$ - n-dimensional vector; $f(x)$ - scalar function; Ω -a multiplicity in Rn, $\Omega \subset R_n$.

In case Ω_k is a finite (or countable) multiplicity or Cartesian product of finite multiplicity by multiplicity of power of continuum, then there will appear the discreet programming problem. Then the condition of x belonging to multiplicity can be written in the form of:

$$x_j \in \Omega, V_j = \overline{1, n_1}$$
$$x \in \Omega, V_j = \overline{1, n}, \Omega \subset R_n; n_1 < n \quad (2)$$

At $n_1 < n$ there appears the problem of partially discreet programming.

If Ω_k is the multiplicity of all integral-valued vectors then at n1=n there takes place the problem of integer programming. And at $n_1 < n$ there takes place a partial integer programming task.

The problems of linear integer programming solution have been studied to the fullest extent

$$extrm\{(c, x) : Ax < b, x \in \Omega \quad (3)$$

Here Ω_1 is the multiplicity of all nonnegative integers, special case of linear integer programming x_j of Boolean variables problems where in (3):

$$x \in (0,1), V_j = \overline{1, n_1}$$

In a number of cases integer programming requirement to integrality is superposed on an objective function as well.

Upon setting and solving discreet programming tasks there can be differentiated the following classes of them: problems with nondivisibility, extremum combinatorial problems, tasks with specified diffused target function, problems on nonclassic areas, multiextremal problems, discreet problems connected with extremums finding at finite multiplicities.

The above classes of applied problems, in their turn, can

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have various mathematical statements and methods of their implementation. Therefore discreet programming development is fulfilled according to the following scheme: applied problem statement, discreet programming mathematical elaboration model, development of problem solution method (algorithm).

As a rule, effective solution of the problem is closely linked with mathematical model of the task, the model structure and its peculiarities.

Let's consider some discreet programming mathematical models and methods of their solving.

II. Discreet Programming Problems Models

Classical example of the mentioned class models is the one of linear integer programming the variables of which is a nondivisible amount. That class models, in their turn, generated various versions of applied problems statements and defined as models with nondivisibility.

In the development process of discreet programming theory there is differentiated a class of combinatorial models [1].

In those models it is necessary to define the integral function point of extremum specified in elements finite multiplicity, either in the elements of that finite multiplicities delivering extremum to the target function.

One of standard combinatorial model samples is the problem on a voyageur [2].

In the problem herein there is the shortest close path crossing once through all cities provided there are **n**-cities and the distance matrix between them is specified.

In combinatorial statement there is necessary to find the restatement which is minimizing the target function value.

Various combinatorial problems statement can be frequently formulated in the form of models with Boolean variables with merely two values 0 or 1.

Lots of applied problems are considered as Boolean models which witness that class models prospects. [3].

In a number of applied problems tasks there are some features concerning the target function or restriction areas. For instance, it is necessary to define specified discontinuous function extreme point at convex polyhedron type

$$\sum_{j=1}^{n} C_j \left(x_j \right)$$

where

$$C_j \left(x_j \right) = \begin{cases} 0, & if \ x_j = 0 \\ C_j x_j + d_j, & if \ \ x_j \succ 0 \end{cases}$$

The models create the class of ones with non-homogeneous disconnected target function.

Models of extreme points finding on the area are prescribed not only with linear inequalities (limitations) but also with logical conditions. Such areas are non-convex or unconnected. These problems form models on non-classical areas [2].

Researchers pay particular interest to multi-extremum models in which optimal values of more than one target function upon availability (or non-availability) of restriction systems shall be defined. As a rule that class models have computation complexity. Alongside with that, a number of applied problems statement comes to the models of the class herein. Denoted problems solution is actual [1].

One of the initial models surely is the model of transport problem with which a lot of researches in discreet programming are connected. Those researches have brought to the model flows in the networks and other modifications of the models denoted above.

It should be noted that models elaboration is closely linked with their implementation method and, vice versa, new methods development, in its turn, leads to new models appearance for applied problems statement.

III. Solution Method of Discreet Programming Problems

Discreet programming problems solution methods are often linked with their mathematical settlement and features. There are a lot of methods for solving them and it is expedient to differentiate the following of them: i.e., precision and approximated ones. Among precision methods the most popular are combinatorial and truncation methods.

A typical combinatorial method is the method of branch-and-bound principle [4]. The method concept is in the channel enumeration of admissible decisions based on evaluation calculation. Key stages of the approach are as follows:

1) Initial solutions multiplicity G is broken down into subsets of g_i (branching process);

2) For every g_i subset there is calculated evaluation values (lower and upper boundaries);

3) Based on the selected evaluation values there calculated the permissible solutions;

4) Iteration branching process upon the prescribed rule and evaluation calculation continued until the optimal solution is gained.

Truncation method idea lies in the following: The initial problem is being solved. If the received solution satisfies integrality condition then it is considered that the problem has been solved. Otherwise a new linear delimitation is added to initial problem restrictions. Afterwards the problem with additionally introduced restriction is solved. Iteration process is repeated until the integral solution is received.

A specimen of successful truncation method implementation is Gomori algorithm.

Alongside with that it should be noted that precision methods are restrictedly used in high dimensionality precision methods. In spite of large memory powerful computing systems application the problem of perfection and development of mathematical apparatus, «discreet curse», is still actual.

Therefore in respect of applied tasks effective solution and precision methods computational complexity overcoming there is occurred the necessity in developing approximation and heuristic methods closely linked with the structure and

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features of those problems statement.

As distinguished from precision methods the rough ones allowed to solve high dimensionality problems and to receive solutions satisfying practice demands. At that in a number of cases there is appeared a chance to assess deviation from optimal solution or to define near neighborhood from optimal solution.

All that allowed using rough methods as effective apparatus for practical problems solution.

In proportion to discreet programming models and methods development, new problems and other attachments statement there is occurring the necessity in elaboration of new approaches and methods for problems solutions.

One of those approaches is block-symmetry models and methods [5].

A number of applied tasks: designing modular software and information system database set of lattice points, program modules distribution and database set of lattice points per computing network units, project selection under restricted resources can be formulated as a new class of discreet programming block-symmetry models problems. As distinct from traditional ones the models of the class herein allow to formulate the problems with several types of various nature variables, perform decomposition of intricate problem into blocks with a common target function and elaborate effective algorithms having polynomial computation complexity.

Let consider general statement and solution of discreet programming block-symmetry problems [6].

IV. STATEMENT OF THE PROBLEM

Let us assume prescribed objects multiplicity $A = \{a_i; i = 1, I\}$ and objects multiplicity $B = \{b_j; j = 1, J\}$ with different type elements as well links between those multiplicities components which are defined by means of matrix

$$W = \|\omega_{is}\|, i = \overline{1, I}, j = \overline{1, J}$$

elements of which are integral-valued or Boolean. It is necessary to enter multiplicity A elements into disjoint subsets A_n, $n = \overline{1, N}$, and multiplicity B components into disjoint subsets B_m, $m = \overline{1, M}$, in order to add a target function extremum $F(A_n, B_m)$.

Let introduce following variables for formalized problem statement. Let $X = \|x_{in}\|$, $i = \overline{1, I}$, $n = \overline{1, N}$ is a Boolean matrix where $x_{in} = 1$, if i element is distributed into the n group, otherwise $x_{in} = 0$. Similarly $Y = \|y_{in}\|$, $j = \overline{1, J}$, $m = \overline{1, M}$, where $y_{jm} = 1$, if j element is distributed into m group and otherwise $y_{jm} = 0$. In general matrix case variables X and Y can be integral-valued.

Let define by means of $A \times B$ the function $F(X, Y)$ depending on A and B elements distribution per subsets A_n

and B_m. Respectively on multiplicity of A - function $\varphi_k(X)$, $k = \overline{1, K}$ and on multiplicity of B - function $\psi_s(Y)$, $S = \overline{1, S}$, defining delimitations at A and B multiplicities correspondingly.

Discreet programming block-symmetry problem is formulated as follows:

$$F(X, Y) \rightarrow extr \quad (4)$$

upon delimitations

$$\varphi_k(X) \leq \varphi_{k0}, \ k = \overline{1, K} \quad (5)$$

$$\psi_s(Y) \leq \psi_{so}, s = \overline{1, S} \quad (6)$$

In restriction multitude (5) and (6) dependent on problem statement non-equality signs can be reversed.

In general, double index matrix is X and Y variables and the specified W matrix can be integral-valued.

Let assume the problem under condition when variables are X, Y and W - Boolean matrixes. Function $F(Z)$ is frequently used as functions $F(X, Y)$, where

$$Z = XWY \quad (7)$$

Let's consider expression (7) representing product of X and Y variables matrix and prescribed matrix Z on which a target function has been defined. In distinction from traditional discreet programming problems statement the statement herein has two types of variables X and Y, variables X and Y are symmetric as respect to the specified matrix W.

In the problem (4)-(6) we can differentiate constraint set as (5) which depends on X variable and constraint set as (6) depending on variable Y.

Functional of $F(X, Y)$ type can be represented as follows:

$$F(p(X), g(Y)) \rightarrow extr \quad (7^*)$$
$$p(X) \rightarrow extr \quad (8)$$
$$\varphi_k(X) \leq \varphi_{k0}, k = \overline{1, K} \quad (9)$$
$$g(Y) \rightarrow extr \quad (10)$$
$$\psi_s(Y) \leq \psi_{s0}, s = \overline{1, S} \quad (11)$$

Let distinguish functions block (8), (9) in problem statement (7^*)-(11) depending solely on variable X, and functions block (10), (11) depending only on variable Y united with a single functional of type (7). Let note that in a number of problem statement there can be a set of restrictions as

$$f_r(X, Y) \leq f_{r0}, r = \overline{1, R} \quad (12)$$

depending on variables X and Y.

In this case we can differentiate a target functional block of (7), (12) type.

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Thus, let name the problem (7)-(11) as discreet programming block-symmetry problem.

Let's consider (7). It confirms that the variables X and Y are symmetric in respect to a prescribed matrix W and function (7) can be defined both from left to right and vice versa, i.e.

$$Z = XWY = YWX \qquad (13)$$

Based on the general statement let define the main properties of formulated problems class distinguishing it from traditional discreet programming problems statement

Property 1. Availability of two variable types X and Y represented in the form of Boolean matrixes defined at prescribed matrix W.

Property 2. Problem modularity lies in distinguishing separate function blocks as (7), (13); (8), (9) and (10), (11) in the statement;

Property 3. Problem symmetry is the in possibility of calculation (13) both in direct and reverse directions.

In a number of problems statements a function (4) can be represented as a vector of F function. In that case there formulated multicriteria discreet programming problem.

Problem solution. Formulated problem peculiarities and properties analysis allow offering effective algorithms of the given class problems solution. Let consider the solution of block-symmetry discreet programming problems provided that X, Y and W are Boolean matrixes. It is easy to prove following statement.

V. CONCLUSION

Distribution of A multiplicity elements per disjoint subsets A_n corresponds to Boolean addition of matrix lines $\|\omega_{ij}\|$ and distribution of B multiplicity elements per disjoint subsets B_m correspond to Boolean addition of Matrix columns $\|\omega_{ij}\|$. Results of conclusion herein allow calculating the evaluations and guidelines for solutions search on effective algorithm development.

Let introduce the notion of the problem solution basis. The basis is understood as a preliminary prescribed elements composition of A_n and B_m subsets.

In matrix W the basis is represented as a sub-matrix Z with defined elements. The given matrix by means of derangement of matrix W lines and columns numbers and their renumbering always can be defined in the left upper corner. Such representation simplifies the procedure of evaluating and defining the solution search direction.

To solve a discreet programming block-symmetry problem under the condition when X, Y and W are Boolean matrixes there elaborated and offered an effective scheme of problem solution. Solutions search scheme consists of following key stages:

1) Let differentiate sub-matrix $Z = \|Z_{nm}\|$, $n = \overline{1,N}$, $m = \overline{1,M}$ in Boolean matrix W and define it as the problem solution basis.

2) Let define matrix $D = \|d_{i^1 n}\|$, $i^1 = \overline{n+1,I}$, $n = \overline{1,N}$ of the solution search direction X by means of Boolean addition of non-basis W matrix lines with the lines of the basis and calculate assessment values merely per basis positions.

3) In compliance with the received evaluation let fulfill A multiplicity elements distribution per subsets A_n. In the result we shall fix the solution X and intermediate matrix $P = \|\pi_{nj}\|$, $n = \overline{1,I}$, $j = \overline{1,J}$.

4) Let define matrix $D = \|d_{j^1 m}\|$, $j^1 = \overline{m+1,I}$, $m = \overline{1,M}$ as the direction of Y solution search by means of Boolean addition to non-basis columns of intermediate matrix $P = \|\pi_{nj}\|$ with basis columns and calculate assessment values merely per P basis positions.

5) In compliance with the received evaluation of P matrix let distribute B multiplicity elements per subsets B_m. In the result let's fix solution Y and target matrix Z on which $F(Z)$ target function value has been defined.

It should be noted that the problem solution search can be fulfilled both in straight direction according to the scheme $DX\ \widetilde{D}Y$ and in reverse direction according to the scheme $\widetilde{D}Y\ DX$.

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