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Numerical Solution of Infinite Boundary Integral Equation by using Galerkin Method with Laguerre Polynomials

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Abstract**—**In this paper the exact solution of infinite boundary integral equation (IBIE) of the second kind with degenerate kernel is presented. Moreover Galerkin method with Laguerre polynomial is applied to get the approximate solution of IBIE. Numerical examples are given to show the validity of the method presented.

Keywords**—**Approximation, Galerkin method, Integral equations, Laguerre polynomial.

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I. Introduction

PROJECTION method has been applying for a long time, and its general abstract treatment goes back to the fundamental theory of Kantorovich (see [6]). In the paper Kantorovich gave a general schema for defining and analyzing the projection method to solve the linear operator equations. The detail of the method is given in [7, part II]. Elliott [4], collocation method based on the Chebyshev polynomials and Chebyshev expansions is applied to solve the numerical solution of Fredholm integral equation (FIE), and this often leads to the linear system of algebraic equations.
To solve approximately the integral equation

$$g(s) = f(s) + \lambda \int_D k(s,t) g(t)dt, \ \ s \in D, \tag{1}$$

we usually choose a finite dimensional family of function that is believed to contain a function $g_n(s)$ close to true solution $g(s)$. The desired approximate solution $g_n(s)$ is selected by forcing it to satisfy the equation (1). There are various means in which $g_n(s)$ can be said to satisfy equation (1) approximately, and this leads to different type of methods. The most popular and powerful tools are collocation and Galerkin method (see [2]).

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Many problems of electromagnetics, scattering problems, boundary integral equations (see [12-14] leads to infinite boundary integral equation of the second kind

$$g(s) = f(s) + \lambda \int_0^\infty k(s,t) g(t)dt, \tag{2}$$

where $f(s)$ is continuous function and the kernel $k(s,t)$ might has singularity in the region $D = \{(s,t): \ 0 \le s,t < \infty\}$ and $g(s)$ is to be determined.

The theory of singular integral equations in which the integration contour of (2) is smooth, closed or open curve of finite length and the kernel has strong singularity, have been comprehensively developed by Gakhov and Muskhelishvili [5,10]. Many researchers have developed the approximate method to solve integral equation (2) when the limit of integration is finite (see [1], [3], [8-9], [11]) and literature cited therein. But for IBIE, few works have been done (see [13-14]).

In this paper we develop Galerkin method with Laguerre polynomials to solve IBIE (2). Since Laguerre polynomials are orthogonal with weight function *w(x)=exp(-x)* on the interval [0,∞] it good fits the density function *g(s)*. The details of the method is given in section 2. The exact solution for degenerate kernel *k(s,t)* is outlined in section 3. Finally, some numerical examples for different kernel *k(s,t)* and *f(t)* are presented in section 4.

II. Galerkin Method

Consider Laguerre base functions as

$$\{L_0(s), L_1(s), ..., L_n(s)\}$$

where

$$L_0(s) = 1, \ \ L_n(s) = \sum_{m=0}^{n} \frac{(-1)^m}{m!} \binom{n}{m} s^m$$

with the following properties

$$\left(L_m(s), L_n(s)\right) = \int_0^\infty e^{-s} L_m(s) L_n(s) ds = 0, \ \ m \ne n$$

and

$$\|L_m(s)\| = 1, \ \ m = 0,1,2,... \ .$$

By taking the linear combination of Laguerre polynomials

$$g_n(s)=\sum_{j=0}^{n} c_j L_j(s). \tag{3}$$

and substituting into (2), yields

$$\sum_{j=0}^{n} c_j L_j(s) = f(s)+\lambda\int_0^\infty k(s,t)\left(\sum_{j=0}^{n} c_j L_j(t)\right)dt \tag{4}$$

Let $h_j(t)=\int_0^\infty k(s,t)L_j(t)dt$, then equation (4) can be written as

$$\sum_{j=0}^{n} c_j\left(L_j(s)-\lambda h_j(s)\right)= f(s). \tag{5}$$

Multiplying (5) by $L_i(s)$, we obtain

$$\sum_{j=0}^{n} c_j\left(L_j(s)-\lambda h_j(s), L_i(s)\right)=\left(f(s),L_i(s)\right) \tag{6}$$

where (a,b) is the inner product of a and b.

Using orthogonolity condition the equation (6) can be written as

$$c_i-\lambda\sum_{j=0}^{n} c_j\left(h_j(s),l_i(s)\right)=\left(f(s),L_i(s)\right),\quad i=0,\ldots,n \tag{7}$$

where

$$D(\lambda)=\begin{vmatrix} 1-\lambda\left(h_0(s),L_0(s)\right) & -\lambda\left(h_1(s),L_0(s)\right) & . \quad . & -\lambda\left(h_n(s),L_0(s)\right) \\ -\lambda\left(h_0(s),L_1(s)\right) & 1-\lambda\left(h_1(s),L_1(s)\right) & & -\lambda\left(h_n(s),L_1(s)\right) \\ . & . & & . \\ . & . & & . \\ -\lambda\left(h_0(s),L_n(s)\right) & -\lambda\left(h_1(s),L_n(s)\right) & . \quad . & 1-\lambda\left(h_n(s),L_n(s)\right) \end{vmatrix}$$

The system of equation (7) has unique solution if λ is not eigenvalues.

III. Exact Solution for the Degenerate Kernel

Let $k(s,t) = p_1(s)p_2(t)$ then the equation (2) becomes

$$g(s) = f(s)+\lambda p_1(s)\int_0^\infty p_2(t)g(t)dt, \tag{8}$$

Denoting the integral on the right side of (8) by c

$$c=\int_0^\infty p_2(t)g(t)dt \tag{9}$$

we get

$$g(s) = f(s)+\lambda c p_1(s). \tag{10}$$

Substitution (10) into (9) gives

$$c=\frac{\int_0^\infty p_2(t)f(t)dt}{1-\lambda\int_0^\infty p_1(t)p_2(t)dt}, \tag{11}$$

From (11) and (10) we obtain

$$g(s) = f(s)+\frac{p_1(s)\int_0^\infty p_2(t)f(t)dt}{1-\lambda\int_0^\infty p_1(t)p_2(t)dt}, \tag{12}$$

where $\int_0^\infty p_1(t)p_2(t)dt \neq \frac{1}{\lambda}$.

IV. Numerical Example

Examples 1: Let λ=1 and

$$k(s,t)=e^{-t^2-s},\ f(s)=s^5-e^{-s}.$$

Due to (12) the exact solution of (2) is $g(s)=s^5$. For fixed λ=1, the system of equation (7) has unique solution and numerical results are shown in Fig. 1 and 2 for *n=6*.

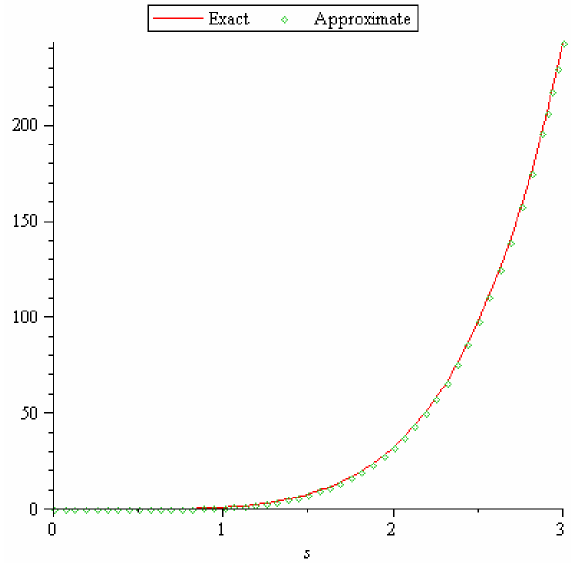

Fig. 1 For n=.6

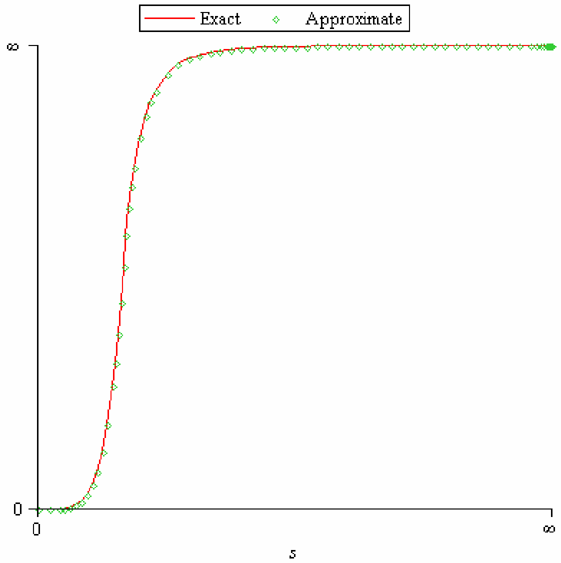

Fig. 2 For n=.6

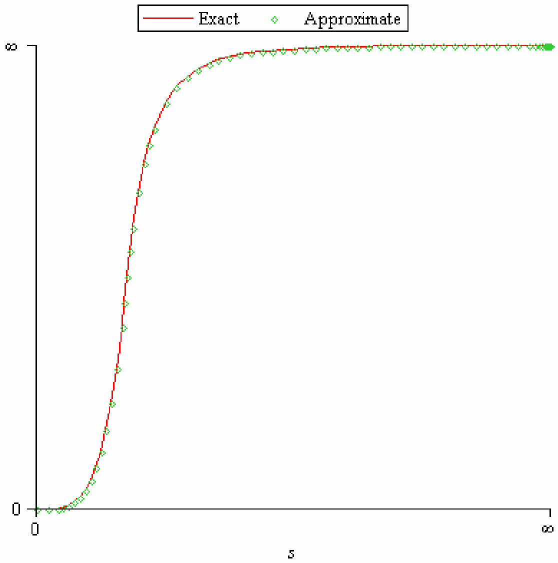

Fig. 4 For *n*= *6*.

Example 2: Let λ=1 and

$$k(s,t)=e^{-t^2-s^2}, \qquad f(s)=s^4-\frac{3\sqrt{\pi}}{8}e^{-s^2}.$$

Since the kernel *k(s,t)* is a degenerate kernel from (12) it follows that $g(s)=s^4$. For n=6, the results are shown in Figs. 3 and 4.

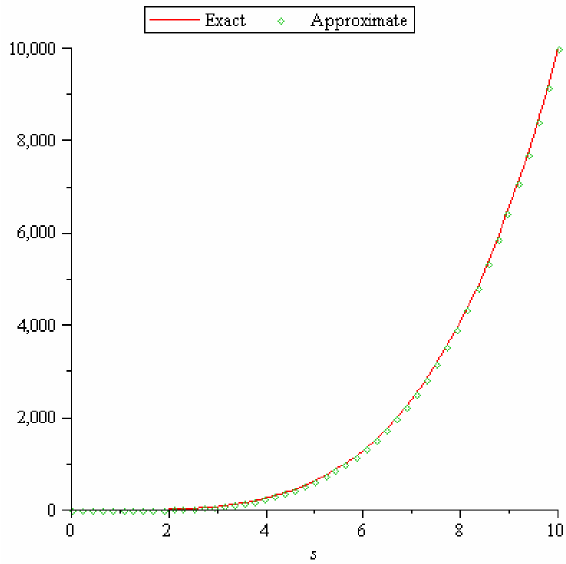

Fig. 3 For *n*= *6*.

V. Conclusion

Due to the orthogonality property on the interval $[0,\infty)$ with exponential weight function, we have used the Laguerre polynomials as an approximate solution. Galerkin method is applied to find the unknown coefficients. Mapple software is used to obtain the approximate solution. Figure 1 and 3 show good convergence for finite interval for small n, while Figure 2 and 4 demonstrate excellent convergence for infinite interval [0,∞].

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