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Ranking DMUs by Ideal PPS in Data Envelopment Analysis

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Abstract**—**An original DEA model is to evaluate each DMU optimistically, but the interval DEA Model proposed in this paper has been formulated to obtain an efficiency interval consisting of Evaluations from both the optimistic and the pessimistic view points. DMUs are improved so that their lower bounds become so large as to attain the maximum Value one. The points obtained by this method are called ideal points. Ideal PPS is calculated by ideal of efficiency DMUs. The purpose of this paper is to rank DMUs by this ideal PPS. Finally we extend the efficiency interval of a DMU under variable RTS technology.

Keywords**—** Data envelopment analysis (DEA), Decision making unit (DMU), Interval DEA, Ideal points, Ideal PPS, Return to scale (RTS).

I. Introduction

DEA is a non-parametric technique for measuring the efficiency of DMUs with common input And output terms [1,2]. The DEA models may be generally classified into radial and non-radial Models. Russell [7] discussed four conditions that are desirable in measuring "technical efficiency". Fare and Lovell [6] proposed an analytical model that aggregates both output and input efficiencies in the framework of a radial measure, the efficiency measure of the model is called the "Russell measure(RM)". RM has a major difficulty in efficiency measurement, because its objective function is formulated as a nonlinear programming problem. Cooper and Pastor [14] have, therefore, considered this problem and have proposed an adjustment to the Russell measure. Since DEA is a model for evaluation from the optimistic viewpoint, Entani and Tanaka [11] have already proposed the interval DEA model to obtain the efficiency interval. The efficiency interval is represented by its upper and lower bounds. The upper and lower bounds of the efficiency interval denote the evaluations from the optimistic and pessimistic viewpoints, respectively. The problem that obtains the upper bound of the efficiency interval is formulated as follows:

$$\Theta_o^u = \max_{U,V} \frac{\dfrac{U^T Y_0}{V^T X_0}}{\max\limits_j \dfrac{U^T Y_j}{V^T X_j}} \qquad (1)$$

$$s.t\, U \geq 0, V \leq 0.$$

Where Xj and Yj are the input and output vectors of DMUj, respectively, whose elements are all positive and the decision variables are the weight vectors U and V.

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Also, the lower bound of the Efficiency interval for DMUo can be determined as follows:

$$\Theta_o^L = \max_{U,V} \frac{\dfrac{U^T Y_0}{V^T X_0}}{\max\limits_{U,V} \dfrac{U^T Y_J}{V^T X_j}} \qquad (2)$$

$$s.t\, U \geq 0, V \geq 0.$$

According to [11], Model (2) can be changed to the following problem:

$$\theta_o^L = \max_{r,i} \frac{\dfrac{y_{ro}}{x_{io}}}{\max\limits_j \dfrac{y_{rj}}{x_{ij}}} \qquad (3)$$

Where the ith element of the input weight vector V and the rth element of the output vector U are One and the other elements are all zero. Theorem 1. The optimal value of (2) and (3) are equal. Proof: The proof of the theorem is provided in [11]. DMUs are improved so that their lower bounds become so large as to attain the maximum value One. The points obtained by this method are called ideal points. The ith input element and the Rth output elements of the ideal point for DMUo are denoted as follows:

$$\overline{x}_{io} = \min_r \left\{ \frac{y_{ro}}{\max\limits_j \dfrac{y_{rj}}{x_{ij}}} \right\} \qquad i = 1,\ldots,m. \qquad (4)$$

$$\overline{y}_{ro} = \max_i \left\{ \max_j \frac{y_{rj}}{x_{ij}} x_{io} \right\} \qquad r = 1,\ldots,s. \qquad (5)$$

When DEA models are used to calculate the efficiency of DMUs, a number of them may have an equal efficiency score of one. Many methods have been proposed in order to rank best performers; Andersen and Petersen (AP) [8] and Mehrabian et al. [9] (MAJ) presented two most popular of these methods. These methods would fail if data have certain structures. There are some methods based on norms. Jahanshahloo et al. [4] introduced an l1−norm approach that removes some deficiencies arising from AP and MAJ, but that cannot rank non-extreme DMUs.

The method we propose in this paper, ranks such DMUs, and does not have the above-mentioned problems.

The current paper is organized as follows. The next section addresses proposed model. In section 3 we give a proposed ranking and compare it with other models and, we consider the interval DEA with variable return to scale. End the paper concludes in section 4.

II. PROPOSED MODEL

We are dealing with n DMUs with the input and output matrices X = (x_{ij}) $\in$ $R^{m\times n}$ and Y= (y_{rj}) $\in$ $R^{s\times n}$, respectively. The data set is positive, i.e. X > 0 and Y > 0. The production possibility set (PPS) of n DMUs is as follows:

$T_c = \{(X, Y) \mid X \geq X\lambda, Y \leq Y\lambda, \lambda \geq 0\}$

First we obtain efficiency DMUs by adjusting Russell measure DMUs become divided into two categories:

1) Efficiency (E)
2) Inefficiency (F)

Inefficiency DMUs that have higher $\Theta^*_{\text{Russell}}$ than they will have better Rank.

We know all DMUs that belong into E set:

$\forall j \in E\ \Theta^*_{\text{Russell}}=1$

For ranking efficiency DMUs, we do on aspect following:

1) We calculate the ideal points of efficiency DMUs (E) by models (4), (5).
2) We calculate ideal PPS by Ideal of efficiency DMUs, and already DMUs.

$$\text{Ideal } PPS = \left\{ \left(x_1,\dots,x_m,y_1,\dots,y_s\right) \middle| \ x_i \geq \sum_{j=1}^{n}\lambda_j x_{ij} + \sum_{j\in E}\lambda'_j \bar{x}_{ij},\ y_r \leq \sum_{j=1}^{n}\lambda_j y_{rj} + \sum_{j\in E}\lambda'_j \bar{y}_{rj,i} = 1,\dots,m, r=1,\dots,s \right\}$$

Theorem 2. Ideal of strong efficiency DMUs are not dominated with any DMUs. ($(\overline{\text{DMU}_o}\ , o\,\varepsilon \{1,\dots,n\})$)

Proof: To prove the theorem, it is sufficient to show that the following inequalities are true.

$$\bar{x}_{io} \leq x_{ij},\ i=1,\dots,m,\ j=1,\dots,n. \tag{6}$$

$$\bar{y}_{ro} \geq y_{ij},\ r=1,\dots,s,\ j=1,\dots,n. \tag{7}$$

Recall that, in the beginning of this paper, we assumed that all of elements of the input and output vectors of DMU_j (j = 1, · · · , n) are positive. Furthermore, we assume that $\min_r \left\{ \frac{y_{ro}}{\max_j \frac{y_{rj}}{x_{ij}}} \right\}$ will happen in index r', that is:

$$\max_r \left\{ \frac{y_{ro}}{\max_j \frac{y_{rj}}{x_{ij}}} \right\} = \frac{y_{r'o}}{\max_j \frac{y_{r'j}}{x_{ij}}} \tag{8}$$

Also, we assume that $\max_j \frac{y_{r'j}}{x_{ij}}$ will happen in index k:

$$\max_j \frac{y_{r'j}}{x_{ij}} = \frac{y_{r'k}}{x_{ik}} \quad asaresult\ \forall j\ \frac{y_{r'j}}{x_{ij}} \leq \frac{y_{r'k}}{x_{ik}} \tag{9}$$

DMU_o is strong efficiency so we have:

$\forall i\ \forall_j\ x_{io} \leq x_{ij}$ Therefore $\frac{y_{r'j}}{x_{ij}} \leq \frac{y_{r'o}}{x_{io}}$

If we set j = 0 in (9) then we will have: $\frac{y_{r'o}}{x_{io}} \leq \frac{y_{r'k}}{x_{ik}}$

So we have $\frac{y_{r'o}}{x_{ij}} \leq \frac{y_{r'k}}{x_{ik}}$ as a result $\frac{x_{ik}.y_{r'o}}{y_{r'k}} \leq x_{ij}$

According to (8): $\min_r \frac{x_{ik}.y_{ro}}{y_{rk}} \leq x_{ij}$ as a result

$\max_r \frac{y_{ro}}{y_{rk}/x_{ik}} \leq x_{ij}$

According to (9): $\min_r \left\{ \frac{y_{ro}}{\max_j \frac{y_{rj}}{x_{ij}}} \right\} \leq x_{ij}$

According to (4): $\bar{x}_{io} \leq x_{ij}$

Inequality (6) is thus proved to be true.

For proving inequality (7), we assume that $\{(\max_i \frac{y_{rj}}{x_{ij}} x_{io}\}$ will happen in index i', that is:

$$\max_i \left\{ \left(\max_j \frac{y_{rj}}{x_{ij}}\right) x_{io} \right\} = \left(\max_j \frac{y_{rj}}{x_{i'j}}\right) x_{i'o} \tag{10}$$

Also, we assume that $\max_j \frac{y_{rj}}{x_{i'j}}$ will happen in index l:

$$\max_j \frac{y_{rj}}{x_{i'j}} = \frac{y_{rl}}{x_{i'l}} \quad as\ a\ result\ \forall_j\ \frac{y_{rj}}{x_{i'j}} \leq \frac{y_{rl}}{x_{i'l}} \tag{11}$$

DMU_o is strong efficiency so we have:

$\forall_r \forall_j \quad y_{rj} \le y_{ro}$ Therefore $\dfrac{y_{rj}}{x_{i'j}} \le \dfrac{y_{ro}}{x_{i'o}}$

If we set j = 0 in (11) then we will have: $\dfrac{y_{ro}}{x_{i'o}} \le \dfrac{y_{rl}}{x_{i'l}}$

So we have $\dfrac{y_{rj}}{x_{i'o}} \le \dfrac{y_{rl}}{x_{i'l}}$ as a result $y_{rj} \le \dfrac{y_{rl}}{x_{i'l}} . x_{i'o}$

By (10): $y_{rj} \le \max\limits_{i} \dfrac{y_{rl}}{x_{i'j}} . x_{io}$

By (11): $y_{rj} \le \max\limits_{i} \left[\max\limits_{j} \dfrac{y_{rj}}{x_{ij}} \right] . x_{io}$

By (5): $y_{rj} \le \bar{y}_{ro.}$

Inequality (7) is hence proved true.
For ranking efficiency DMUs, we calculate efficiency their ideal DMUs in ideal PPS with the following problem:

$$\Theta^*(\bar{X}_o, \bar{Y}_O) = \min \ \left(\frac{1}{m}\sum_{i=1}^{m}\theta_i\right) / \left(\frac{1}{s}\sum_{r=1}^{s}\theta_r\right)$$

$$\text{s.t.} \quad \sum_{j=1}^{n}\lambda_j x_{ij} + \sum_{j\in E} \lambda'_j \bar{x}_{ij} \le \theta_i \bar{x}_{io} \quad i=1,\dots,m.$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} + \sum_{j\in E} \lambda'_j \bar{y}_{rj} \ge \phi_r \bar{y}_{ro} \qquad r = 1,\dots,s,$$

$$\lambda_j \ge 0 \qquad , j = 1,\dots,n$$

$$0 \le \theta_i \le 1, \qquad , i = 1,\dots,m$$

$$\phi_r \ge 1, \qquad , r = 1,\dots.s.$$

The performance of a DMU will be better if its Θ^* has lower than others, because $\Theta^*(\bar{x}_{o,} \bar{Y}_o)$ is efficiency of $\overline{DMU_O} = (\bar{x}_{o,} \bar{Y}_o)$ in ideal PPS.The Performance of a DMU_O will be better if it's Θ^* ideal point has a smaller Because, if DMUo has further output then DMU_o will have more input and if DMUo has less input then DMUo will have less output i.e. if DMUo have better performance then its ideal point is closer to own DMU,as a result DMUo is far from frontier Ideal PPS.To elaborate, we apply our proposed model in the following example.Example1: We consider 7 DMUs with two inputs and two outputs. The data and the adjusting Russell measure Θ^* of the DMUs are shown in Table 1.

TABLE I INPUTS , OUTPUTS AND Θ^* OF DMUS IN EXAMPLE 1

DMU	*A*	*B*	*C*	*D*	*E*	*F*	**G**
x_1	2	3	2	2	3	2	3
x_2	3	4	2	3	4	2	4
y_1	4	5	4	5	4	4	5
y_2	5	6	4	4	5	6	4
θ^*	0.3358	0.6481	0.8000	1	0.5303	1	0.5072

According to the results of the adjusting Russell measure model, DMUD, and DMUF are evaluated as efficient. In order to rank the two DMUs by the proposed method, first we obtain their ideal point and ideal PPS then we calculate the distance of ideal point to Ideal PPS . According to the method, if DMU has a shorter distance, then it has better rank. The ideal points, and the ranking of DMUs are shown in Table 2.

TABLE II IDEAL POINTS AND RANKING OF EACH DMU

Ideal DMU	*A*	*B*	*C*	*D*	*E*	*F*	**G**
Input_1	-	-	-	1.33	-	1.6	-
Input_2	-	-	-	1.33	-	2	-
Output_1	-	-	-	6	-	5	-
Output_2	-	-	-	9	-	6	-
$\text{d}_j(\bar{x}_j, \bar{y}_j)$	-	-	-	4.73684	-	2.64205	-
RANK	4	5	3	2	6	1	7

The important property of this method is its ability to rank extreme and non-extreme DMUs. We show this property with the following example.

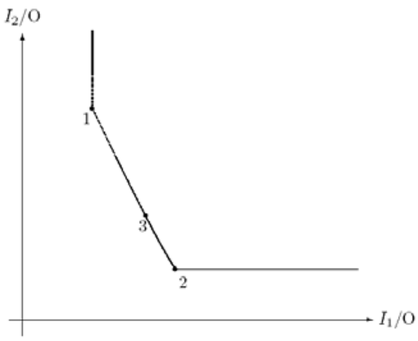

Fig. 1: Farell frontier for three DMUS.

Example2: We consider the three DMUs with two inputs and one output. The Farell frontier for These DMUs is shown in Fig 1. As can be seen in Fig 1, DMU1 and DMU2 are extreme efficient DMUs and DMU3 is non-extreme. The data, ideal points and ranking of DMUs, as well as their Efficiency results by model (12) are shown in Table 3. It can be seen that this method ranks all of DMUs.

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TABLE III DATA AND IDEAL POINTS AND RANKING

DMU_s	DMU_1	DMU_2	DMU_3	$\overline{\mathrm{DMU1}}$	$\overline{\mathrm{DMU2}}$	$\overline{\mathrm{DMU3}}$
Input_1	1	3	2	1	1	1
Input_2	4	1	2.5	1	1	1
Output_1	1	1	1	4	3	2.5
$d_j(\bar{x}_j, \bar{y}_j)$	-	-	-	14	10.50	8.75
Bank	3	2	1	-	-	-

III. EFFICIENCY INTERVAL UNDER VARIABLE RTS TECHNOLOGY

Entani et al. [11] improved the efficiency intervals of a DMU by adjusting its given inputs and outputs under constant RTS technology. We want to develop their model under variable RTS technology.

Lemma 1. For

$a,b,c,d \triangleright a',b',c',d' > 0, t_1,t_2,t_2' \ \geq 0, (t_1+t_1' \neq 0), (t_2+t_2' \neq 0)$ $, if$

$$\frac{d}{b} \leq \frac{d'}{b'} \, and \leq \frac{a'}{c'}$$

Then:

$$\frac{ad}{bc} \leq \frac{(t_1 a + t_1' a')(t_2 d + t_2' d')}{(t_2 b + t_2' b')(t_1 c + t_1' c')}$$

Proof. The proof of the theorem is provided in [10].
The upper and lower limits of interval efficiency for DMUo under variable RTS technology
be defined as follows:

$$\Theta_O^{E^*} = \max_{u,v} \frac{\dfrac{u^t y_0 + u_o}{u^t x_o}}{\max\limits_j \dfrac{u^t y_j + u_o}{u^t xj}} \quad s.t \ \ u\geq 0, \ v\geq 0. \tag{13}$$

$$\Theta_{O^*}^{E} = \min_{u,v} \frac{\dfrac{u^t y_0 + u_o}{v^t x_o}}{\max\limits_j \dfrac{u^t y_j + u_o}{v^t x_j}} \quad s.t \ \ u\geq 0, v\geq 0. \tag{14}$$

We can obtain the lower limit of efficiency directly by

$$\Theta_{0^*}^{E} = \min_{p,r} \frac{\dfrac{y_{op} + u_0}{x_{or}}}{\max\limits_j \dfrac{y_{jp} + u_0}{x_{jr}}} \tag{15}$$

Which is proven in what follows
The optimal value in (15) can be said that the optimal weight vectors U and V in (14) have the entry 1, respectively, and all other entries are 0. This fact is proven by theorem 3.First, we assume to have following inequality:

$$(A_1): \frac{x_{j2_1}}{x_{o\,1}} \leq \frac{x_{j2\ 2}}{x_{o\,2}}$$

$$(B_1): \frac{y_{0\ 1} + u_0}{y_{j2\ 1} + u_0} \leq \frac{y_{0\ 2} + u_0}{y_{j2\ 2} + u_0}$$

According to lemma 1, we must have following condition:

$$x_{j2}1, y_{j2}1 + u_0, x_0 1, y_0 1 + u_0 > 0$$

$$x_{j2}2, y_{j2}2 + u_0, x_0 2, y_0 2 + u_0 > 0$$

So. We must have $u_0 > -\min\limits_{r,j}\{y_{rj}\} or \, u_0 > \max\limits_{r,j}\{-y_{rj}\}$.

$If \ u_0 > \max\limits_{r,j}\{-y_{rj}\}$ Then we will have:

Therefore according to lemma1, we will have

$$\frac{(x_{j2}1)(y_0 1 + u_0)}{(y_{j2}1 + u_0)(x_0 1)} \leq \frac{(t_1 x_{j2}1 + t_1' x_{j2}2)(t_2(y_0 1 + u_0) + t_2'(y_0 2 + u_0))}{(t_2(y_{j2}1 + u_0) + t_2'(y_{j2}2 + u_0))(t_1 x_{01}1 + t_1' x_0 2)}$$

$$\frac{\dfrac{y_0 1 + u_0}{x_0 1}}{\dfrac{y_{j2}1 + u_0}{x_{j2}1}} \leq \frac{t_2(y_0 1 + u_0) + t_2'(y_0 2 + u_0)/t_1 x_{01}1 + t_1' x_0 2}{t_2(y_{j2}1 + u_0) + t_2'(y_{j2}2 + u_0)/t_1 x_{j2}1 + t_1' x_{j2}2}$$

Theorem 3: If $u_0 = \dfrac{u_0^*}{u_1^* + u_2^*}$ $and \ u_0 > \max\limits_{r,j}\{-y_{rj}\}$ then the optimal value of (14) and (15) are equal (where u_0^*, u_1^*, u_2^* are optimal answer of model 14).

Proof. In order to simplify the notation, we consider the case of two-dimensional input and two-dimensional output data.

$$(14): \Theta_{0^*}^{E} = \min_{u,v} \min_j \frac{(u^t y_0 + u_0)/v^t x_0}{(u^t y_j + u_0)/v^t x_j} = \frac{(u^{*t} y_0 + u_0^*)/v^{*t} x_0}{(u^{*t} y_{j2} + u_0^*)/v^{*t} x_{j2}}$$

$$(15): \Theta_{0^*}^{E} = \min_{p,r} \min_{j} \frac{\left(y_{0p} + u_0\right)/x_{0r}}{\left(y_{jp} + u_0\right)/x_{jr}} = \frac{\left(y_{0p1} + u_0\right)/x_{0r1}}{\left(y_{j1p1} + u_0\right)/x_{j1r1}}$$

Where j2 and j1 denote the optimal values of j, respectively. (15) is the special case of (14) where v^* has r_1 th entry is 1, u^* has p_1 th entry is 1 and all other entries are 0. Thus the following holds.

The variable space of (14) $\supset$ that of (15).Then, we have the following relation:

The optimal of (14) = $\dfrac{\left(y_{0p1} + u_0\right)/x_{0r1}}{\left(y_{j1p1} + u_0\right)/x_{j1r1}} \leq$ the optimal of

$$(15) = \frac{\left(u^{*t} y_0 + u_0^*\right)/v^{*t} x_0}{\left(u^{*t} y_{j2} + u_0^*\right)/v^{*t} x_{j2}} \qquad (16)$$

Here we have the following two cases with respect to inputs and outputs:

$$(A_1): \frac{x_{j21}}{x_{01}} \leq \frac{x_{j22}}{x_{02}}$$

$$(A_2): \frac{x_{j22}}{x_{02}} \leq \frac{x_{j21}}{x_{01}}$$

$$(B_1): \frac{y_{01} + u_0}{y_{j21} + u_0} \leq \frac{y_{02} + u_0}{y_{j22} + u_0}$$

$$(B_2): \frac{y_{02} + u_0}{y_{j22} + u_0} \leq \frac{y_{01} + u_0}{y_{j21} + u_0}$$

Thus, we consider the following four cases:
(A1, B1), (A1, B2), (A2, B1), (A2, B2)

We have $u_0 = \dfrac{u_0^*}{u_1^* + u_2^*}$ and $u_0 > \max_{r,j}\{-y_{rj}\}$ so, one of them holds by lemma 1,

$$(14) = \frac{\left(u^{*t} y_0 + u_0^*\right)/v^{*t} x_0}{\left(u^{*t} y_{j2} + u_0^*\right)/v^{*t} x_{j2}}$$

$$(14) = \frac{\left(u_1^{*t} y_{01} + u_0^{*t} y_{02} + u_0^*\right)/v_1^{*t} x_{01} + v_2^{*t} x_{02}}{\left(u_1^{*t} y_{j2^1} + u_2^{*t} y_{j2^2} + u_0^*\right)/v_1^{*t} x_{j2^1} + v_0^{*t} x_{j2^2}} \geq \begin{cases} \dfrac{\left(y_{01} + u_0\right)/x_{01}}{\left(y_{j2^1} + u_0\right)/x_{j2^1}} & \text{for } (A_1, B_1) \\ \dfrac{\left(y_{02} + u_0\right)/x_{01}}{\left(y_{j2^2} + u_0\right)/x_{j2^1}} & \text{for } (A_1, B_2) \\ \dfrac{\left(y_{01} + u_0\right)/x_{02}}{\left(y_{j2^1} + u_0\right)/x_{j2^2}} & \text{for } (A_2, B_1) \\ \dfrac{\left(y_{01} + u_0\right)/x_{02}}{\left(y_{j2^2} + u_0\right)/x_{j2^2}} & \text{for } (A_2, B_2) \end{cases} \qquad (17)$$

$$= \min_{p,r} \frac{\left(y_{0p} + u_0\right)/x_{0r}}{\left(y_{j2p} + u_0\right)/x_{j2r}} \geq \min_{p,r} \min_{j} \frac{\left(y_{0p} + u_0\right)/x_{0r}}{\left(y_{jp} + u_0\right)/x_{jr}} = \frac{\left(y_{0p1} + u_0\right)/x_{0r1}}{\left(y_{j1p1} + u_0\right)/x_{j1r1}} = (15)$$

From (16), (17), the following equation holds:

$$\frac{\left(u^{*t} y_0 + u_0^*\right)/v^{*t} x_0}{\left(u^{*t} y_{j2} + u_0^*\right)/v^{*t} x_{j2}} = \frac{\left(y_{0p1} + u_0\right)/x_{0r1}}{\left(y_{j1p1} + u_0\right)/x_{j1r1}} = (15)$$

Employing inductive inference, Theorem 3 for general case can be proven.

Therefore,by assumption $u_0 = u_0^* / \sum_{r=1}^{s} u_r^*, u_0 > \max_{r,j}\{-y_{r,j}\}$, DMUs are improved in variable RTS technology so that their lower bounds become so large as to attain the maximum value one. The ith input element and the rth output element of the ideal point for DMUo by variable RTS technology are denoted as follows:

$$\overline{x}_{io} = \min_{r} \left\{ \frac{y_{ro} + u_0}{\max_{j} \dfrac{y_{rj} + u_0}{x_{ij}}} \right\} \quad i = 1, \ldots, m. \qquad (18)$$

$$\overline{y}_{ro} = \max_{i} \left\{ \max_{j} \frac{y_{rj} + u_0}{x_{ij}} x_{io} \right\} \quad r = 1, \ldots, s. \qquad (19)$$

Therefore, for obtaining ideal DMUs in PPS with variable RTS technology, we should do following process:

1) First, we calculate efficiency DMUs by adjusting Russell measure in the PPS with variable RTS technology, then we obtain strong efficiency DMUs (E).
2) If DMUo is strong efficient then we calculate following LP:

$$\begin{aligned} \min \quad & u_0 \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i}^{m} v_i x_{ij} + u_0 \leq 0, \quad j = 1, \ldots, n, \\ & \sum_{r=1}^{s} u_r y_{ro} - \sum_{i}^{m} v_i x_{io} + u_0 = 0, \\ & v_i \geq \varepsilon, \quad i = 1, \ldots, m, \\ & u_r \geq \varepsilon, \quad r = 1, \ldots, s. \end{aligned} \qquad (20)$$

3) We calculate $u_0 = u_0^* / \sum_{r=1}^{s} u_r^*$ that $\left(u_0^*, U^*, V^*\right)$ is optimal answer of model (20). If $u_0 > \max_{r,j}\{-y_{rj}\}$ then we can obtain ideal DMUo by (18), (19).

Note: In the PPS with non-decreasing RTS (BCC − CCR) we can obtain ideal DMUo by (18),and (19),because uo is always non-negative.

IV. Conclusion

Our aim in this paper was to obtain a method for ranking DMUs. For ranking efficiency DMUs, We calculate the ideal points of efficiency DMUs and ideal PPS by Ideal of efficiency DMUs, and already DMUs. The Performance of a DMU will be better if it's ideal point has a smaller θ^* in model (12). Because, if DMUo have better performance then it's ideal point is closer to own DMU, as a result DMUo is far from frontier Ideal PPS. For DMUs that $\theta^*=1$ we calculate the distance of ideal point from frontier Ideal PPS, According to the method, if DMU has a shorter distance, then it has better rank.

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