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Recursive Filter for Coastal Displacement Estimation

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Abstract—All climate models agree that the temperature in Greece will increase in the range of 1° to 2°C by the year 2030 and mean sea level in Mediterranean is expected to rise at the rate of 5 cm/decade. The aim of the present paper is the estimation of the coastline displacement driven by the climate change and sea level rise. In order to achieve that, all known statistical and non-statistical computational methods are employed on some Greek coastal areas. Furthermore, Kalman filtering techniques are for the first time introduced, formulated and tested. Based on all the above, shoreline change signals and noises are computed and an inter-comparison between the different methods can be deduced to help evaluating which method is most promising as far as the retrieve of shoreline change rate is concerned.

***Keywords*—**Climate Change, Coastal Displacement, Kalman Filter

I. Introduction

IT is known that coastal zones are directly affected by the occurring climate change and that they are particularly vulnerable to extreme weather events, such as storm surges, intense erosion and landslides [1].

It must be firstly mentioned that the model created herein, is not capable of incorporating on its prediction procedure the possible action of extreme weather events or unexpected catastrophes. The actual capability of the model is to produce a reliable estimation of the future position of the shoreline based on the observed and combined action of a large number of factors. Using the available geo-information, it is possible to test the reliability of the mathematical model. If the predicted shoreline positions coincide with the actual positions in pre-defined epochs, then it is proved that the model, that is the prediction algorithm, can determine with accuracy the future evolution of the shoreline. The amount of computations and the complexity of the designed model depend directly upon the volume of the available data and the desired accuracy of the future projections. The calculation burden is depending on the number of historical shorelines available in a coastal area. Nevertheless, more data means more complete description of the past behavior of the particular coastal area. The combination of large amount of geoinformation and data, the computation of shoreline change rates using different statistical (and non-statistical) methods and the adjusted Kalman filter, were carried out using mathematical tools and programs such as the MATLAB [2]. Through that program, the shoreline change rates were computed for some coastal areas of Greece that were selected in the current study, using 10 statistical and non-statistical methods and the adjusted Kalman filter. These computed shoreline change rates and their prediction errors provide the primary information and tool for any further study concerning the coastal evolution.

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II. Materials and Methods

A. Coastal Dynamic Models

Sea level rise can activate two important mechanisms that result in the loss of land, namely, erosion and inundation. Erosion represents the physical removal of sediment by wave and current action, while inundation is the permanent submergence of low-lying land. Sea level rise contributes to the erosion of erodible cliffs, sandy and muddy coasts by promoting the offshore transport of sedimentary material. Land loss resulting from inundation is simply a matter of slope [3]. Therefore, the coastal areas with lower slope, will suffer the greater land losses. The ability to identify areas vulnerable to future changes in local sea level, as a result of local vertical movements (i.e. subsidence) and sea level rise is necessary if a timely response is to be made to the rising sea.

As far as concerning beach evolution, there are several factors that influence this complicated process, such as [4]:

i. Waves
ii. Currents and sediment transport along the beaches
iii. Coastal morphology
iv. Sea level rise
v. Vegetation
vi. Storms
vii. Tsunamis
viii. Seasonal nature of beach erosion and
ix. Man-induced changes in sand supply

In Greece, where the largest part of the population and the vast majority of all the land uses are accumulated in its coastal areas, the implementation of a model to predict shoreline evolution, is important. Such o model would become a useful tool and offer the opportunity to coastal managers to plan with great efficiency new rules of future development, according to the predicted, future position of the shoreline. Also, the knowledge of the shoreline transgression would allow the government to imply rules and protection measures for all the coastal areas that are already developed and populated and which will face future hazards and land losses because of the shoreline recession. Actions of this kind and protection strategies of light intervention with minimum effects for the environment (in opposition to "hard" protection measures such as sea walls, dikes or other structures) can be proved successful and viable only if the predictions of the future positions of the shoreline produced by the model are relatively accurate.

The assessment of the shoreline movement is a complex, parametrical problem and many attempts were made worldwide during the last years in order to describe it in the best possible way and find realistic solutions. The consequences of the rapid climate change have been intensively studied and also how human intervention (development in coastal areas) effect upon the evolution of each coastline [5]. Furthermore, a large number of climate models have been developed in order to predict the magnitude and periodic frequency of future possible extreme weather events and also mathematical models were built that are capable of predicting future positions of the shoreline, by using parameters and coefficients. Based on these experimental predictions, protection strategies were proposed, for certain coastal areas, especially in U.S.A and the Mediterranean.

Simple models that are capable of projecting the future position of a shoreline have been tasted in certain cases, with their predictions followed though by significant uncertainties and questionable effectiveness. Besides, any attempt to describe a complicated phenomenon such as the time wise movement of a shoreline using simplified mathematical models and formulas, will be futile and will produce inaccurate results. One of the most important factors concerning the future movement of a shoreline is the reliability of the model which is used to describe the evolution of the study coastal area. All natural parameters that effect upon the shoreline movement change according to the location each coast and the climate conditions of each region. The main challenge in the building process of a prediction model is to create models with sophisticated mathematical analysis (space - time), which will be able to produce reliable predictions concerning the coastal system dynamics. With the use of the G.I.S., these models are now in position to give results with greater efficiency and accuracy, handle bigger data bases and offer more capabilities of analysis to the users.

In conclusion, the accuracy and reliability of the predictions of a coastal dynamic model depends on the following three factors:

i. The quality of the original data and the accuracy (in close connection to the available scale) of the aerial photographs [6]. In the present study, one of the main difficulties during the building process and the application of the model was the high magnitude of the uncertainties of the available original information (the high errors in the digitized positions of the historical shorelines). These significant uncertainties were attributed either to the rather small scale of the available aerial photographs, or generally because of the poor quality (and limited size and variety) of the acquired geo-information.

ii. The shape and complexion of the polynomial. During the final building process of the prediction model which is presented here, certain assumptions had to be made. These assumptions concerned the computational procedure of the final filter and the primary values of the main coefficients (A, B and H) that were used.

iii. The precision of the predictions produced by the current model. One of the main advantages of the use of the Kalman filter compared to the 10 known statistical (and non-statistical) methods of computing shoreline change is that the final model incorporates the high uncertainties of the original data and produces estimations for the future shoreline position with integrated prediction errors, closely approximating reality.

In the present study, a Greek coastal area was chosen in order to model its coastal dynamics by calculating shoreline change rates (accretion or recession). Furthermore, the rates of change of the shoreline positions has been calculated using the following procedure: at first, using aerial photographs taken at different time periods and topographic maps, the shoreline of the study area was digitized at these different time periods resulting to a group of historical shorelines [7]. Then, cross sections along the shoreline with the greatest scale of all the available shorelines were drawn intersecting the rest of the historical shorelines at certain points. Measuring the distances between these points along the cross sections, the shoreline change rate can be computed using a certain method.

The 10 well-known statistical (and non-statistical) methods that are used worldwide to calculate the rates of shoreline change are presented briefly below and have been analysed extensively [8], [9], [10] and [11]:

i. End Point Rate (EPR)
ii. Average of rates (AOR) [
iii. Minimum Description Length (MDL)
iv. Jackknifing (JK)
v. Ordinary Least Squares (OLS)
vi. Reweighted Least Squares (RLS)
vii. Weighted Least Squares (WLS)
viii. Reweighted Weighted Least Squares (RWLS)
ix. Least Absolute Deviation (LAD)
x. Weighted Least Absolute Deviation (WLAD)

The above 10 methods were tested in the present study using original data taken from the considered Greek coastal area and their results were compared to each other (and with the results of the Kalman filter applications) offering useful conclusions about possible advantages or disadvantages of each method.

B. The Kalman Filter

The known Kalman filter is theoretically an estimation of the linear problem of the least squares, where the instant state of a linear dynamic system is estimated (using the "state vector"), which is disrupted by "white" noise [12], [13] and

[14]. The measurements are related in a linear way to the dynamic state of the system but there are "infected" by "white" noise. The estimator is statistically the best by any integrand of the square error of the estimation (f (ε^2) = min).

Practically, the Kalman filter is a unique tool for the control of complicated procedures such as the flow of a river or the movement of boats, airplanes and satellites [14]. The most important aspect though is that the Kalman filter provides predictions about the future, dynamic state of a system, incorporating the effect of all the previous observations. It must be mentioned that the modification and the application of a mathematical "feedback" filter such as the Kalman filter, in cases of low-dynamic systems like coastal areas, using as original data the measured distances along cross-sections, is presented for the first time in the present study.

The "escalating" equations of the measurements incorporate every time the new measurement in the a priori estimation, in order to produce a new and improved a posteriori estimation of the parameters. In this way, the more available data (positions of the historical shorelines in our case), the more complete and accurate will be the produced estimation vector.

C. Equations

The main equations that are used for the proposed modified Kalman filter are presented below [14] and [15]:

The a priori estimation of the state vector:

$$\begin{aligned} \hat{x}_k^- &= A \times \hat{x}_{k-1}^- + B \times u_{k-1} \\ \sum \hat{x}_k^- &= A \times \sum \hat{x}_{k-1}^- \times A^{\mathrm{T}} + Q \end{aligned} \tag{1}$$

The a posteriori estimation of the state vector:

$$\hat{x}_k = \hat{x}_k^- + K_k \times (z_k - H \times \hat{x}_k^-) \tag{2}$$

The gain matrix:

$$K_k = \sum \hat{x}_k^- \times H^T \times (H \times \sum \hat{x}_k^- \times H^T + R)^{-1} \tag{3}$$

The variability matrix:

$$\sum \hat{x}_k = (I - K_k \times H) \times \sum \hat{x}_k^- \tag{4}$$

The estimation for the state vector at the moment k-1:

$$\hat{x}_{k-1}, \sum \hat{x}_{k-1} \tag{5}$$

It is obvious, based on the above five equations, that the burden of computations in the case of Kalman filter is seriously increased in comparison to the 10 statistical methods of estimating shoreline change rate. Still, according to the application results of the original Kalman filter in cases of dynamic measurement systems, the particular method produced reliable results, using primordial data with such high levels of "noise" that the volume and complexity of computations is not taken in consideration (especially since all statistical computations are carried out by computers). Therefore, comparing and combining the predicted (by the Kalman filter) shoreline positions with actual measurements, it is possible to produce new, corrected position estimators. Based on these experimental results and the quality of the predictions, the method of the adapted Kalman filter can be characterized as a statistical tool which undoubtedly improves the quality of the results and offers a better estimation of the future position of the shoreline. During the Kalman filter process, every time that new measurements are made, the validity of the prediction model is tested and in the same time the kinematic parameters of the coastal area, in reference to the time of the last measurements, are computed. Consequently, the prediction model of the future positions of the shoreline on each coastal area will have to be evaluated and tested, considering the reliability of each prediction. In general, that particular test is implemented in cases where the solution improvement of the vector of the unknown determinative parameters is required, using new additional measurements, without combining and "re-correcting" old and new measurements, all together again [12].

III. Results and Discussion

In Table I, the results of the study that is, shoreline change rates (signals) and their uncertainties (noises) of the 10 known methods are presented in comparison to the respective Kalman filter estimations in a Greek coastal area. Two assumptions were made, namely, 2 mm/scale and 5 mm/scale uncertainty of the extraction of the shorelines. Kalman filter results in Table I were obtained using one at a time of the 10 methods as far the shoreline change rate to be used as the start signal of the change rate e.g. Kalman filter and OLS.

According to the magnitude of the "noise" of the measurements but also through the comparison of the estimated shoreline change rates, the 10 available methods were evaluated. Some of them were poorly ranked and considered as completely inappropriate for the estimation of shoreline change rates. Their results either were followed systematically by large uncertainties or even worst, the calculated shoreline change rates presented high deviations from the mean value of all the other estimations of the 10 methods. The AER, RLS, LAD and WLAD were judged as the most inappropriate methods, taking into consideration the specific nature and amount of the original available data of the study. As far as the EPR, AOR, WLS and RWLS methods, despite the considerable uncertainties or deviations of the calculated shoreline change rates in many cases, there were certain applications and specific coastlines where relatively reliable results were produced, which could be used for further studies. In the coastal area that was examined here, the methods that constantly produced reliable results and estimated shoreline change rates with minimum uncertainties ("noise" over the signal) were the OLS and JK [16].

After the statistical analysis of all the results and their evaluation, similar tests and applications were carried out with the exact same original data (historical shoreline positions), using only this time the modified Kalman filter. The new, calculated shoreline change rates were compared to each other and also with the shoreline change rates derived from the 10 known methods.

Derived from the results of Table I it is evident that the modified Kalman filter in comparison to the 10 existing methods is a mathematical tool which processes certain advantages regarding its structure and the reliability of its final predictions, namely:

i. The Kalman filter is characterized by a far more complete mathematical configuration concerning low dynamic systems (e.g. coastal), in comparison to the 10 known methods.

ii. In the Kalman filter method it is possible to check the intermediate stages of the prediction process – including "a priori/a posteriori" predictions for every observation time interval [13], [14] and [15].

iii. The uncertainties of the original data are being incorporated in the shoreline change rates that are computed by the modified Kalman filter.

iv. The produced results present "endurance" and are rather "robust" as far as concerning the existence of severe outliers in the original measurements.

Additionally, one of the most important advantages of the modified Kalman filter is that it can trace the possible discrepancy between the estimated and the actual shoreline position on every coastal area.

The circular, computational process of the Kalman filter offers the opportunity to compute every time an a priori estimator before each measurement and subsequent to that, a corrected a posteriori estimator, which incorporates the new observation with its uncertainties [17]. That way, measurements with high observation errors (such as measured positions of historical shorelines of the year 1945 or even 1960) when they are incorporated in the model, they considerably effect the whole computational process concerning the corrected vector estimators, degrading the reliability and quality of the final prediction. Still, the Kalman filter is capable of incorporating all measurements and produce a relatively reliable prediction, regardless to the size and eminence of the partial position uncertainties. In fact, whenever an original measurement is followed by high uncertainties or an outlier appears on the middle of the distribution of the values, the Kalman filter may incorporate the current problematic value. The a posteriori estimator that is computed for that particular time interval may present decreased reliability, but in the next intervals and given the fact that the following measurements will have lesser uncertainties, the new, vector estimators will be piecemeal improved, regaining the reliability and the accuracy of the filter predictions. All the above conclude that the Kalman filter displays a relative flexibility and adaptability concerning the existence of possible uncertainties in the original data, contrary to the 10 known statistical methods where the quality and reliability of the resulted predictions of shoreline positions are severally influenced by primal errors.

In conclusion, the proposed modification of the Kalman filter that is presented here, can improve the quality and reliability of the future shoreline position predictions, in comparison to the 10 known existing methods that are used worldwide. The Kalman filter offers the opportunity for more precise computations of shoreline change rates, with lesser errors, approximating better the physical reality and coastal evolution by modelling the dynamics of coastal systems and incorporating the uncertainties of the original available data.

TABLE I
SHORELINE CHANGE RATES

The 10 methods of estimating shoreline change rate (m/yr) compared to Kalman filter					
	Non statistical methods			Linear regression methods	
	EPR	AOR	AER	OLS	JK
10 methods [5mm/scale]	-0.21 ±0.43	-0.43 ±1.29	-0.35	-0.42 ±0.001	-0.46
10 methods [2mm/scale]	-0.21 ±0.19	-0.43 ±0.52	-0.35	-0.42 ±0.001	-0.46
Kalman filter [5mm/scale]	-0.22	-0.44	-0.37	-0.43	-0.48
Kalman filter [2mm/scale]	- 0.22	- 0.45	- 0.37	- 0.44	- 0.49
	Weighted Linear regression methods			Methods with the deviation criterion	
	RLS	WLS	RWLS	LAD	WLAD
10 methods [5mm/scale]	0.74 ±0.12	0.01 ±0.03	0.01 ±0.03	-0.3	0.44
10 methods [2mm/scale]	0.74 ±0.12	-0.16 ±0.01	-0.15 ±0.01	-0.3	0.44
Kalman filter [5mm/scale]	0.75	0.01	0.01	-0.31	I
Kalman filter [2mm/scale]	0.78	- 0.17	- 0.16	- 0.32	0.44

The 10 known methods compared to the Kalman filter estimations using original data from a Greek coastal area. 5 and 2mm/scale accuracy are considered with regard to the obtained shoreline extraction.

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