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Formation Control of Mobile Robots

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***Abstract*—**In this paper, we study the formation control problem for car-like mobile robots. A team of nonholonomic mobile robots navigate in a terrain with obstacles, while maintaining a desired formation, using a leader-following strategy. A set of artificial potential field functions is proposed using the direct Lyapunov method for the avoidance of obstacles and attraction to their designated targets. The effectiveness of the proposed control laws to verify the feasibility of the model is demonstrated through computer simulations.

***Keywords*—**Control, Formation, Lyapunov, Nonholonomic

I. Introduction

FLOCKING is a coordinated and cooperative motion of groups (flocks) of entities or beings, ranging from simple bacteria to mammals [1]. Formation behaviors seen in nature, like flocking and schooling, benefit individuals like animals that use them in various ways. Common examples include schools of fishes, flocks of birds, and herds of land animals, to name a few. This outstanding behavior is based on the principle that there is safety and strength in numbers [2, 3]. There are various approaches found in literature in relation to the strict observance of a prescribed formation of a flock during motion [4]. One is the split/rejoin maneuver and the other considers tight formations as can be required in many engineering applications. The control mechanism of formation control can be divided into three layers; formation shape, formation type and robotic control.

The problem of controlling and coordinating multiple mobile robots in a desired formation has gained popularity over the last decade. Formation control problems arise when groups of mobile robots are employed to jointly perform certain tasks [5]. The benefits of coordination and control of multiple autonomous robots outweigh that of a single robot or human especially when the environment is hazardous, or when considering distributed tasks. Various applications of formation control of mobile robots have been considered in literature and some of them are simultaneous transportation of vehicles or delivery of payloads, mapping, aircraft and satellites [6, 7, 8].

In recent literature, a variety of approaches have been proposed for formation or cooperative control of mobile robots and have been roughly categorized into virtual structure, leader-follower and behavior based. The basic idea of the behavior based approach is to assign a behavior (e.g. obstacle avoidance, collision avoidance, target attraction) to each individual mobile robot [9]. This strategy is suitable to a large group of mobile robots, however, it is difficult to analyze its behavioral performance mathematically and guarantee its system stability.

The second approach in cooperative control is the virtual structure approach which considers the formation as a single structure i.e., the formation does not have any leaders or followers. In [5], a virtual structure approach is considered for the formation control of unicycle mobile robots. The virtual structure can evolve as a rigid body in a given direction with some given orientation and maintain a rigid geometric relationship among multiple mobile robots, however, requiring the formation to act as a virtual structure limits the class of potential applications of this approach [10].

In the leader-following approach, some robots take the responsibility of a leader, while the follower robots will follow their designated leader. The follower robots position themselves relative to the leader.

Many researchers have utilized various techniques to create swarming and flocking behaviors for multi agent systems. Recently, Sharma [4] and Vanualailai et al. [11] proposed control algorithms that considered motion planning and control of mobile robots within a constrained environment cluttered with obstacles. In [12], the authors considered the autonomous control of a flock of six 1-trailer robots in an arrowhead formation, while in [13], we utilized a leader-following approach to consider various formation types of 1-trailer mobile robots using split/rejoin maneuvers.

An extension to our previous work, this paper presents a new set of control laws so that a desired formation is maintained even when the group of robots encounters an obstacle with minimum error. The scheme uses Cartesian coordinate's representation to avoid any singular points as encountered when using polar coordinates. The direct method of Lyapunov is then used to derive continuous acceleration-based controllers which render our system stable.

This paper is organized as follows: in Section II the robot model is defined and the proposed scheme to achieve a desired formation; in Sections III and IV the artificial potential field functions are defined; in Section V the dynamic constraints are defined; in Section VI the acceleration-based control laws are derived; in Section VII we illustrate the effectiveness of the proposed controllers via simulations and in Section VIII we conclude the paper and outline future work in the area.

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II. Vehicle Model

In this section, we drive a new kinematic model for the leader- following based formation control of two car-like mobile robots.

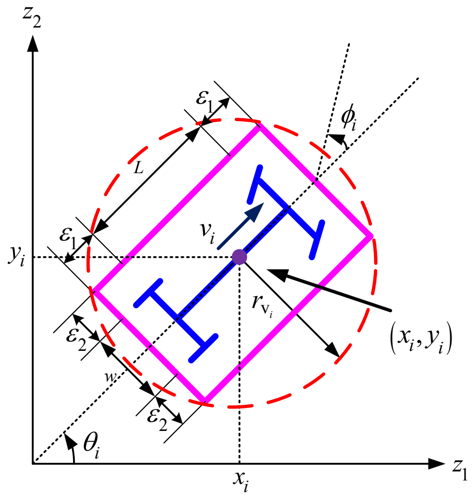


Fig. 1 Kinematic model of the car-like mobile robot

With reference to Fig. 1, (x_i, y_i) for $i = 1, 2$, represents the Cartesian coordinates and gives the reference point of each mobile robot, and θ_i gives the orientation of the ith car with respect to the z_1 axis, while ϕ_i gives the ith robot's steering angle with respect to its longitudinal axis. For simplicity, the dimensions of the two mobile robots are kept the same. Therefore, L is the distance between the center of the rear and front axles of the ith car and w is the length of each axle.

Next, to ensure that each robot safely steers past an obstacle, we adopt the nomenclature of [4] and construct circular regions that protect the robot. With reference to Fig. 1, given the clearance parameters, $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, we enclose the vehicle by a protective circular region centered at (x_i, y_i) with radius $r_v = \sqrt{\dfrac{(L + 2\varepsilon_1)^2 + (w + 2\varepsilon_2)^2}{4}}$.

We next assign a Cartesian coordinate system (X-Y) fixed on the leaders body, as shown in Fig. 2. When the leader rotates, we have a rotation of the X-Y axes. Thus, given the leader's position and its orientation, as long as (r_{12}, α) is fixed, the follower's position will be unique. This gives then the polar coordinate representation of the follower's relative position with respect to the leader. However, such representations using polar coordinates lead to certain singularities in the controller.

To eliminate such singular points, we consider the position of the follower by considering the relative distances of the follower from the leader along the X and Y directions.

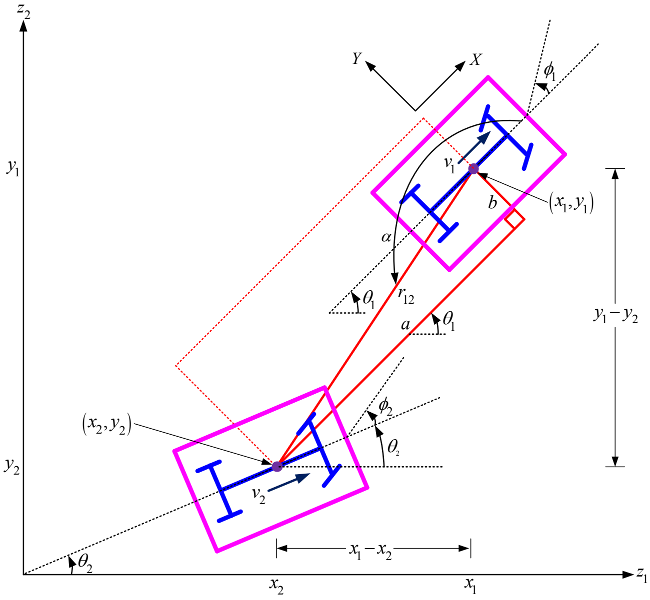


Fig. 2 The proposed scheme utilizing a rotation of axes with the axes fixed at the leader

Thus, we have:

$$A_X = -(x_1 - x_2)\cos\theta_1 - (y_1 - y_2)\sin\theta_1,$$
$$B_Y = (x_1 - x_2)\sin\theta_1 - (y_1 - y_2)\cos\theta_1,$$

where A_X and B_Y are the followers relative position with respect to the X-Y coordinate system. If A_X and B_Y are known and fixed, the follower's position will be unique. Thus, to obtain a desired formation, one needs to know distances a and b, the desired relative positions along the X-Y directions, such that the control problem would be to achieve $A_X \to a$ and $B_Y \to b$ .i.e., $r_{12} \to r_{12}{}^d$, where $r_{12}{}^d = \sqrt{a^2 + b^2}$. The model of the kinematic model of the system, adopted from [13] is

$$\left.\begin{aligned} \dot{x}_i &= v_i \cos\theta_i - \tfrac{L}{2}\omega_i \sin\theta_i\,, \\ \dot{y}_i &= v_i \sin\theta_i + \tfrac{L}{2}\omega_i \cos\theta_i\,, \\ \dot{\theta}_i &= \omega_i, \\ \dot{v}_i &= \sigma_{i1}, \\ \dot{\omega}_i &= \sigma_{i2}, \end{aligned}\right\} \qquad (1)$$

where v_i and ω_i are, respectively, the instantaneous translational and rotational velocities, while σ_{i1} and σ_{i2} are the instantaneous translational and rotational accelerations of the ith vehicle. Without loss of generality, we assume $\phi_i = \theta_i$. The state of the ith mobile robot is then described by $\mathbf{x}_i := (x_i, y_i, \theta_i, v_i, \omega_i)$, where $i = 1$ represents the leader and $i = 2$ the follower.

III. Attractive Potential Field Functions

This section formulates collision free trajectories of the robot system under kinodynamic constraints in a fixed and bounded workspace. It is assumed that the car-like robots have *a priori* knowledge of the whole workspace. We want to design the acceleration controllers, σ_{i1} and σ_{i2}, so that the pair moves safely towards its leaders target while maintaining a desired formation.

A. Attraction to Target

A target is assigned to the leader. While the leader moves towards its defined target the follower robots move with the leader, maintaining a desired formation. For the leader, we define a target

$$T = \left\{(x,y) \in \mathbb{R}^2 : (x_1 - t_1)^2 + (y_1 - t_2)^2 \le rt_1^{\,2}\right\},$$

with center (t_1, t_2) and radius rt_1. For the attraction of the leader to its designated target, we consider an attractive potential function

$$V_1(\mathbf{x}) = \tfrac{1}{2}\left[(x_1 - t_1)^2 + (y_1 - t_2)^2 + v_1^{\,2} + \omega_1^{\,2}\right]. \tag{2}$$

To maintain the desired formation, we utilize the following potential function

$$V_2(\mathbf{x}) = \tfrac{1}{2}\left[(A_x - a)^2 + (B_y - b)^2 + v_2^{\,2} + \omega_2^{\,2}\right]. \tag{3}$$

B. Auxiliary Function

To guarantee the convergence of the mobile robots to their designated goals, we design an auxiliary function defined as:

$$G_1(\mathbf{x}) = \tfrac{1}{2}\left[(x_1 - t_1)^2 + (y_1 - t_2)^2 + (\theta_1 - t_3)^2\right], \tag{4}$$

and

$$G_2(\mathbf{x}) = \tfrac{1}{2}\left[(A_X - a)^2 + (B_Y - b)^2 + (\theta_2 - t_4)^2\right]. \tag{5}$$

where t_3 and t_4 represent the desired final orientation of the leader and follower respectively. These potential functions are then multiplied to the repulsive potential functions to be designed in the following sections.

IV. Repulsive Potential Field Functions

We desire the mobile robots to avoid all stationary obstacles intersecting their paths. For this, we construct the obstacle avoidance functions that merely measure the distances between each robot and the obstacles in the workspace. To obtain the desired avoidance, these potential functions appear in the denominator of the repulsive potential field functions. This creates a repulsive field around the obstacles.

Let us fix q solid obstacles within the workspace and assume that the lth obstacle is circular with center (o_{l1}, o_{l2}) and radius ro_l. For the ith robot with a circular avoidance region of radius r_v to avoid the lth obstacle, we adopt

$$FO_{il}(\mathbf{x}) = \frac{1}{2}\left[(x_i - o_{l1})^2 + (y_i - o_{l2})^2 - (ro_l + r_v)^2\right], \tag{6}$$

for $i = 1,\ 2$ and $l = 1, 2, \ldots, q$.

V. Dynamic Constraints

Practically, the steering and bending angles of the ith mobile robot are limited due to mechanical singularities while the translational speed is restricted due to safety reasons. Subsequently, we have; **(i)** $|v| \le v_{max}$, where v_{max} is the maximal speed of the ith robot; **(ii)** $|\phi_i| \le \phi_{max} < \frac{\pi}{2}$, where ϕ_{max} is the maximal steering angle. Considering these constraints as artificial obstacles, we have the following potential field functions:

$$U_{i1}(\mathbf{x}) = \tfrac{1}{2}\left[(v_{\max} - v_i)(v_{\max} + v_i)\right], \tag{7}$$

$$U_{i2}(\mathbf{x}) = \tfrac{1}{2}\left[\left(\frac{v_{\max}}{|\rho_{\min}|} - \omega_i\right)\left(\frac{v_{\max}}{|\rho_{\min}|} + \omega_i\right)\right], \tag{8}$$

for $i = 1,\ 2$. These potential functions guarantee the adherence to the above restrictions placed upon the translational velocity v_i, and steering angle ϕ_i.

VI. Control Laws

Combining all the potential functions $(2-8)$, and introducing constants, denoted as the control parameters, $\alpha_{il} > 0$ and $\beta_{is} > 0$, for $i, s \in \mathbb{N}$, we define a candidate Lyapunov function

$$L(\mathbf{x}) = \sum_{i=1}^{2}\left\{V_i(\mathbf{x}) + G_i(\mathbf{x})\left[\sum_{l=1}^{q}\frac{\alpha_{il}}{FO_{il}(\mathbf{x})} + \sum_{s=1}^{2}\frac{\beta_{is}}{U_{is}(\mathbf{x})}\right]\right\} \tag{9}$$

Clearly, $L(\mathbf{x})$ is locally positive and continuous on the domain $D(L) = \left\{\mathbf{x} \in \mathbb{R}^{5\times 2} : FO_{il}(\mathbf{x}) > 0,\ U_{is}(\mathbf{x}) > 0\right\}$. We define $\mathbf{x}_e := (t_1, t_2, t_3, 0, 0)$ as an equilibrium point of system (1). Thus, we have $L(\mathbf{x}_e) = 0$.

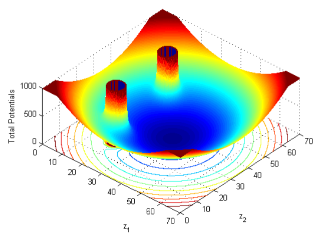


Fig. 3 A three-dimensional view of the total potentials.

The total potentials, as shown in Fig. 3 and the corresponding contour plot in Fig. 4 are generated for target attraction and avoidance of two stationary disk-shaped obstacles. For better visualization the target of the leader is located at $(t_1, t_2) = (35, 35)$, and the disks are fixed at $(o_{11}, o_{12}) = (20, 20)$, $(o_{21}, o_{22}) = (20, 50)$ with radii of $ro_1 = ro_2 = 2.5$, while $\alpha_{1l} = 20$, $l = 1, 2$. Also, the velocity and angular components of the lead robot have been treated as constants such that $v_1 = 0.5$, $\omega_1 = 0$, and $\theta_1 = 0$.

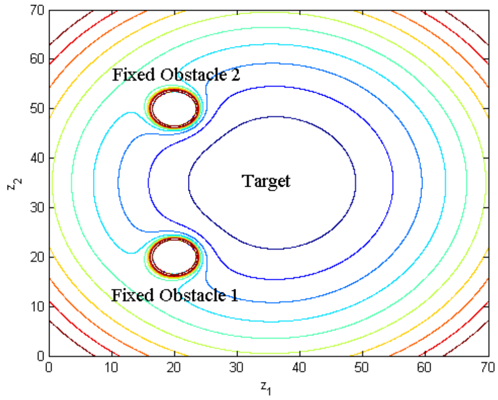


Fig. 4 The corresponding contour plot of the total potential

To extract the control laws, we differentiate the various components of $L(\mathbf{x})$ separately and carry out the necessary substitutions from (1). The nonlinear control laws for system (1) will be designed using Lyapunov's Direct Method. The process begins with the following theorem:

Theorem: Consider a pair of car-like mobile robots whose motion is governed by the ODE's described in system (1). The principal goal is to establish and control a prescribed formation, facilitate maneuvers of the robots within a constrained environment and reach the target configuration while maintaining a desired formation. The subtasks include; restrictions placed on the workspace, convergence to predefined targets, and consideration of kinodynamic constraints. Utilizing the attractive and repulsive potential field functions, the following continuous time-invariant acceleration control laws can be generated, that intrinsically guarantees stability,in the sense of Lyapunov, of system (1) as well:

$$\sigma_{i1} = -\tfrac{1}{g_{i1}}\left[\delta_{i1}v_i + f_{i1}\cos\theta_i + f_{i2}\sin\theta_i\right], \qquad (10)$$

and

$$\sigma_{i2} = -\tfrac{1}{g_{i2}}\left[\delta_{i2}\omega_i + \tfrac{L}{2}\left(f_{i2}\cos\theta_i + f_{i1}\sin\theta_i\right) + f_{i3}\right], \qquad (11)$$

for $i = 1,\ 2$.

Proof: The time derivative of our Lyapunov function $L(\mathbf{x})$ along a particular trajectory of system (1) is:

$\dot{L}_{(1)}(\mathbf{x}) = -\sum_{i=1}^{n}\left(\delta_{i1}v_i^2 + \delta_{i2}\omega_i^2\right) \le 0$ for all $\mathbf{x} \in D(L)$, and $\dot{L}_{(1)}(\mathbf{x}_e) = 0$, where the functions f_{ik} to g_{ij}, for $i, j = 1,\ 2$ and $k = 1,..,3$ are defined as (upon suppressing $\mathbf{x}$):

$$f_{11} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}} + \sum_{s=1}^{2}\frac{\beta_{1s}}{U_{1s}}\right](x_1 - t_1) - G_1\sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}^{\,2}}(x_1 - o_{l1}),$$

$$f_{12} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}} + \sum_{s=1}^{2}\frac{\beta_{1s}}{U_{1s}}\right](y_1 - t_2) - G_1\sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}^{\,2}}(y_1 - o_{l2}),$$

$$f_{13} = \left[\sum_{l=1}^{q}\frac{\alpha_{1l}}{FO_{1l}} + \sum_{s=1}^{2}\frac{\beta_{1s}}{U_{1s}}\right](\theta_1 - t_3),$$

$$g_{11} = 1 + G_1\frac{\beta_{11}}{U_{11}^{\,2}},\ g_{12} = 1 + G_1\frac{\beta_{12}}{U_{12}^{\,2}},$$

$$f_{21} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}} + \sum_{s=1}^{2}\frac{\beta_{2s}}{U_{2s}}\right]\left[(-A_X - a)\cos\theta_1 + (B_Y - b)\sin\theta_1\right] - G_2\sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}^{\,2}}(x_2 - o_{l1}),$$

$$f_{22} = \left[1 + \sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}} + \sum_{s=1}^{2}\frac{\beta_{2s}}{U_{2s}}\right]\left[(-A_X - a)\sin\theta_1 - (B_Y - b)\cos\theta_1\right] - G_2\sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}^{\,2}}(y_2 - o_{l2}),$$

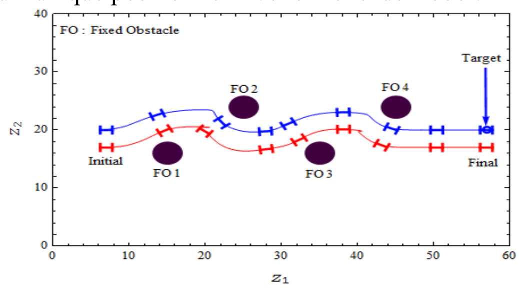
$$f_{23} = \left[\sum_{l=1}^{q}\frac{\alpha_{2l}}{FO_{2l}} + \sum_{s=1}^{2}\frac{\beta_{2s}}{U_{2s}}\right](\theta_2 - t_4),$$

$$g_{21} = 1 + G_2\frac{\beta_{21}}{U_{21}^{\,2}},\ g_{22} = 1 + G_2\frac{\beta_{22}}{U_{22}^{\,2}}.$$

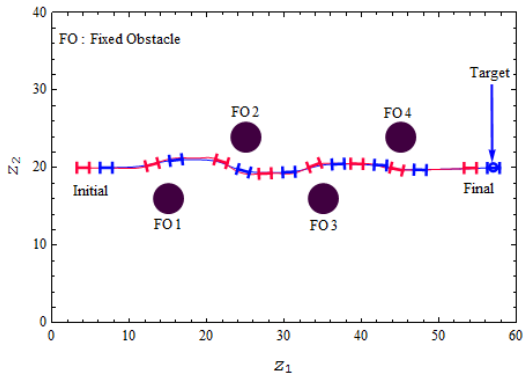
A careful scrutiny of the properties of our scalar function reveals that $\mathbf{x}_e$ is an equilibrium point of system (1) in the sense of Lyapunov and $L(\mathbf{x})$ is a legitimate Lyapunov function guaranteeing stability. This is in no contradiction with Brockett's result [14] as we have not proven asymptotic stability.

VII. Simulation

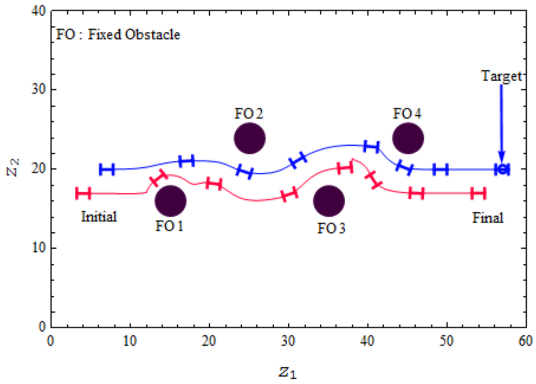
To illustrate the effectiveness of the proposed controllers, we present two car-like mobile robots. The follower robot is assigned a unique position relative to the leader robot.



(a) Scenario 1



(b) Scenario 2



(c) Scenario 3

Fig. 5 The different formations using two car-like mobile robots.

As such, while the leader moves towards its intended target, the follower maintains the desired formation. Upon encountering an obstacle, the formation of the articulated robots does not change, but the robots are able to move around the obstacle. Fig. 5 shows three different formation type scenarios using our control laws.

Graphs in Figs 6, 7, 8, and 9 show details for Scenario 1. Fig. 6 shows the time derivative of the Lyapunov function and Fig. 7 shows the evolution of the translational and rotational velocities of the leader and follower.

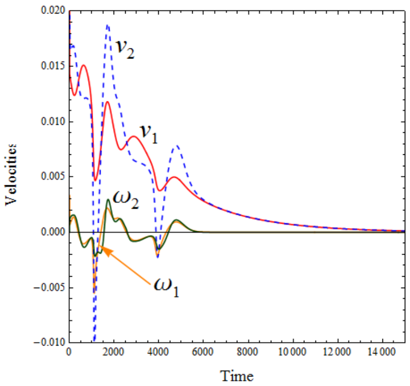


Fig. 6 Translational and rotational velocities

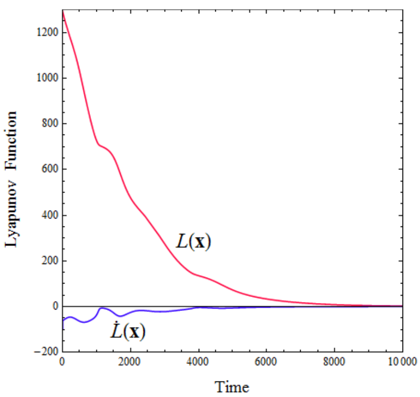


Fig. 7 Evolution of $L(\mathbf{x})$ and its derivative

Fig 8 shows the orientations of the leader and follower, while Fig. 9 shows the convergence and boundedness of the variables at the final state, implying the effectiveness of the control laws. The corresponding initial and final states and other details for the simulation are listed in Table I (assuming that appropriate units have been taken into account).

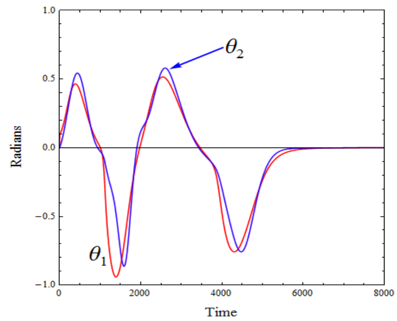


Fig. 8 Orientations of the leader and follower

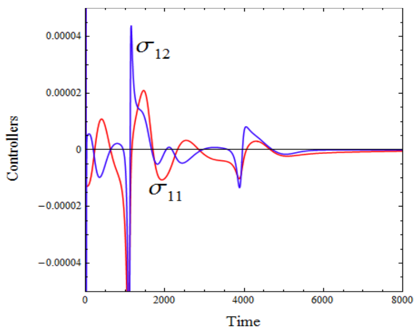


Fig. 9 Accelerations σ_{11} and σ_{12}.

TABLE I
NUMERICAL VALUES OF INITIAL STATES, CONSTRAINTS AND PARAMETERS FOR SCENARIOS 1,2 AND 3.

	Initial Conditions
Rectangular positions	$(x_1, y_1) = (7, 20)$
Angular Positions and velocities	$v_1 = 0.5$, $\omega_i = 0$, $\theta_i = 0$ for $i = 1, 2$
	Constraints and Parameters
Final Orientations	$t_3 = t_4 = 0$
Leader Target	$(t_1, t_2) = (57, 20)$, $rt_1 = 0.5$
Dimensions of Robots	$L = 1.6$, $w = 1.2$
Relative Positions — Scenario 1	$(a, b) = (0, 3)$;
Relative Positions — Scenario 2	$(a, b) = (3, 0)$;
Relative Positions — Scenario 3	$(a, b) = (3, 3)$
Fixed Obstacles (o_{i1}, o_{i2})	$(o_{11}, o_{12}) = (15, 16)$, $(o_{21}, o_{22}) = (25, 24)$; $(o_{31}, o_{32}) = (35, 16)$, $(o_{41}, o_{42}) = (45, 24)$; $ro_1 = ro_2 = ro_3 = ro_4 = 2$
Max. translational speed	$v_{\max} = 5$
Min. turning radius	$\rho_{\min} = 0.14$
Clearance parameters	$\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.05$
	Control and Convergence Parameters

Obstacle avoidance	$\alpha_{il} = 0.1$ for $i = 1,2$ and $l = 1$ to 4
Dynamics constraints	$\beta_{is} = 0.001$ for $i = 1,2$ and $s = 1,2$
Convergence	$\delta_{i1} = 500$, $\delta_{i2} = 50$ for $i = 1,2$

VIII. CONCLUSION

This paper presents a set of artificial field functions derived using Lyapunov's direct method, for formation control of mobile robots, using a leader-following strategy. By using Cartesian coordinates to uniquely assign a position to a follower, we can achieve a desired formation of car-like mobile robots with bounded distance error and heading angle. The derived controllers produced feasible trajectories and ensured a nice convergence of the system to its equilibrium state while satisfying the necessary kinematic and dynamic constraints. We note here that convergence is only guaranteed from a number of initial states of the system.

Future research will address more general applications with more than two mobile robots, and tractor trailer systems.

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