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Buckling Analysis of Rectangular Plates under the Combined Action of Shear and Uniaxial Stresses

V. Piscopo

Abstract**—**In the classical buckling analysis of rectangular plates subjected to the concurrent action of shear and uniaxial forces, the Euler shear buckling stress is generally evaluated separately, so that no influence on the shear buckling coefficient, due to the in-plane tensile or compressive forces, is taken into account.

In this paper the buckling problem of simply supported rectangular plates, under the combined action of shear and uniaxial forces, is discussed from the beginning, in order to obtain new project formulas for the shear buckling coefficient that take into account the presence of uniaxial forces.

Furthermore, as the classical expression of the shear buckling coefficient for simply supported rectangular plates is considered only a "rough" approximation, as the exact one is defined by a system of intersecting curves, the convergence and the goodness of the classical solution are analyzed, too.

Finally, as the problem of the Euler shear buckling stress evaluation is a very important topic for a variety of structures, (e.g. ship ones), two numerical applications are carried out, in order to highlight the role of the uniaxial stresses on the plating scantling procedures and the goodness of the proposed formulas.

Keywords**—**Buckling analysis, Shear, Uniaxial Stresses.

I. INTRODUCTION

THE buckling problem of simply supported rectangular plates, under the action of shearing forces uniformly distributed along the edges, was discussed by several authors, such as Timoshenko [1], M. Stein and J. Neff [2], S. Bergmann and H. Reissner [3]. To evaluate the Euler shear stress at which the buckling of plates occurs, the energy method is generally adopted and the deflection surface of the buckled plate is expressed into an appropriate double sine trigonometric series, whose terms satisfy the plating boundary conditions along all edges.

The energy method was successfully applied, considering only few terms of the series, for plates with aspect ratio $\alpha \leq 1.5$, as for long narrow plates a larger number of harmonics is necessary to obtain a consistent value of the shear buckling coefficient, due to a low convergence of solution. An approximate method for an infinitely long strip with simply supported edges was adopted, considering a different approach, by R.V. Southwell [4]. So, having the exact values of the buckling coefficients for an infinitely long plate and for plates with $\alpha \leq 1.5$, a parabolic curve can be obtained to approximately evaluate it also for other proportions of plates.

V. Piscopo is, since December 2009, Ph.D. in Aerospace, Naval and Quality Engineering at the University of Naples "Federico II", Department of Naval Architecture and Marine Engineering, Via Claudio 21 - 80125 Napoli Italy (e-mail: vincenzo.piscopo@ unina.it).

Denoting by t the plating thickness, E and ν the Young and Poisson modulus respectively, a and b the longer and shorter sides of the panel and by $\alpha = a/b$ the plating aspect ratio, the Euler shear buckling stress is obtained by the following formula:

$$\tau_E = \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 K_s \tag{1}$$

in which K_s is the shear buckling coefficient, defined as follows:

$$K_s = \begin{cases} 5.34 + \dfrac{4.00}{\alpha^2} & if \quad \alpha > 1 \\ 4.00 + \dfrac{5.34}{\alpha^2} & if \quad \alpha \leq 1 \end{cases} \tag{2}$$

The parabolic curve given by (2) is generally used for practical applications, but it doesn't permit to take into account the presence of uniaxial forces, if applied. As the problem of buckling analysis of rectangular plates under the combined action of shearing and uniaxial forces is a widely diffused topic for the scantling of platings of a variety of structures, such as ship ones, this problem is discussed from the beginning, to obtain some new project formulas, similar to the equations given in (2) and so slightly different from those ones given by Wagner [8] for long plates (α>2). The classical solution of the buckling problem of plates under pure shear is also investigated and discussed.

Finally, two numerical applications are proposed, considering a panel subjected to pure shear and combined shear and uniaxial forces: some comparisons with the relevant results obtained by a FEM analysis, carried out by *ANSYS*, are also proposed.

II. THEORETICAL DEVELOPMENT

Let us refer to the coordinate system of Fig.1 and assume that the plate buckles slightly under the action of forces applied in its middle plane. Denoting by w the vertical displacement field, normal to the plate middle plane, and assuming that there are no body forces and lateral loads, the differential equation for the buckled plate becomes:

$$D\nabla^4 w = N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} \tag{3}$$

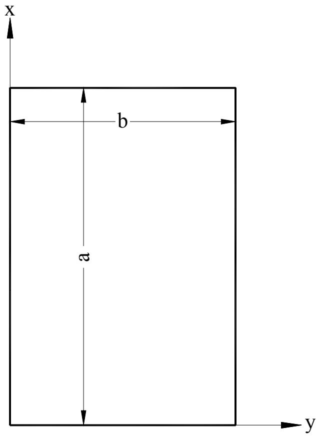

Fig. 1 Plate reference system

where N_x and N_y are the in-plane forces per unit of length, directed along the x and y axes respectively, N_{xy} is the shear in-plane force per unit of length and $D = \frac{Et^3}{12(1-\nu^2)}$ is the plate flexural rigidity. Assuming that: these forces are constant throughout the plate, $N_y = 0$ and there is a given ratio γ between the remaining terms, so that $N_x = \gamma N_{xy}$, the eq. (3) can be rewritten as follows:

$$D\nabla^4 w = N_{xy}\left(\gamma \frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 w}{\partial x \partial y}\right) \tag{4}$$

To evaluate the Euler shearing stress τ_E at which buckling of plate occurs, the energy method is adopted, as there isn't a rigorous solution of eq. (4). In applying this method, it is assumed the plate undergoes some small lateral bending, consistent with the given boundary conditions: obviously, if the work done by the in-plane forces is smaller than the strain energy of bending for every possible shape of buckling, the equilibrium of plate is stable, otherwise it is unstable and buckling occurs.

Considering the plate as simply supported along all edges, the boundary conditions are satisfied by taking for the deflection surface of the buckled plate the following double sine trigonometric series:

$$w = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{m,n} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{5}$$

The strain energy of bending of the buckled plate can be expressed as follows:

$$\Delta U = \frac{D}{2}\int_0^a\int_0^b \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} dxdy \tag{6}$$

considering that $\int_0^a\int_0^b \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 dxdy = 0$. The eq. (6) can be further developed as:

$$\Delta U = \frac{D\pi^4 b}{8a^3}\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{m,n}^2 \left(m^2 + \alpha^2 n^2\right)^2 \tag{7}$$

The work done by the external forces during the buckling of plate is:

$$\Delta T = -\frac{1}{2}\int_0^a\int_0^b N_x\left(\frac{\partial w}{\partial x}\right)^2 + 2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dxdy \tag{8}$$

finally becoming:

$$\Delta T = -4N_{xy}\left[\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\sum_{p=1}^{p=\infty}\sum_{q=1}^{q=\infty} w_{m,n} w_{p,q}\frac{mnpq\chi_{mnpq}}{(m^2-p^2)(q^2-n^2)} + \frac{\gamma\pi^2}{32\alpha}\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{m,n}^2 m^2\right] \tag{9}$$

with $\chi_{mnpq} = 1$ if $m \pm p$ and $n \pm q$ are odd numbers, $\chi_{mnpq} = 0$ otherwise. Equating the work produced by the external forces to the strain energy, the following expression is obtained:

$$N_{xy} = -\frac{\pi^4 D}{32\alpha^3 b^2}\frac{I_1}{I_2} \tag{10}$$

with:

$$I_1 = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{m,n}^2\left(m^2 + \alpha^2 n^2\right)^2 \tag{11.1}$$

$$I_2 = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\sum_{p=1}^{p=\infty}\sum_{q=1}^{q=\infty} w_{m,n} w_{p,q}\frac{mnpq\chi_{mnpq}}{(m^2-p^2)(q^2-n^2)} + \frac{\gamma\pi^2}{32\alpha}\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{m,n}^2 m^2 \tag{11.2}$$

The coefficients $w_{m,n}$ of the series (5) must be chosen to make the expression (10) minimum. Using this condition of minimum and considering the M and N partial sums of (5), the following equation system can be derived:

$$\frac{\lambda}{\alpha^2}\frac{\partial I_1}{\partial w_{m,n}} - \frac{\partial I_2}{\partial w_{m,n}} = 0 \quad \forall\ m = 1...M; n = 1...N \tag{12}$$

with:

$$\lambda = -\frac{\pi^4 D}{32\alpha b^2 \tau_E t} \tag{13}$$

The system (12) can be finally developed as follows:

$$\left[\frac{\lambda}{\alpha^2}\left(m^2 + \alpha^2 n^2\right)^2 - \frac{\gamma\pi^2}{32\alpha}m^2\right] w_{m,n} - \sum_{p=1}^{M}\sum_{q=1}^{N}\frac{mnpq\chi_{mnpq}}{(m^2-p^2)(q^2-n^2)} w_{p,q} = 0 \tag{14}$$

Such equations may yield solutions different from zero only if the determinant of (14) is zero. When the determinant is put equal to zero, an equation for determining the critical value of

λ is found. Starting from (13), the classical expression of the Euler shear stress, see eq. (1), is obtained, introducing the shear buckling coefficient as follows:

$$K_s = -\frac{\pi^2}{32\alpha\lambda} \tag{15}$$

It is noticed that σ_x stresses are related to the Euler shear ones by the following relation:

$$\sigma_x = -\frac{\gamma}{\lambda}\frac{\pi^4 D}{32\alpha b^2 t} = \gamma\tau_E \tag{16}$$

so that, if the convention $\gamma < 0$ ($\gamma > 0$) for $\sigma_x < 0$ ($\sigma_x > 0$) is introduced, it always must be $\lambda < 0$.

The problem of finding the minimum value of λ that makes the determinant of (14) null, has been solved by a dedicated program developed in *MATLAB*. The solution is obtained varying the number of harmonics, to assure its convergence for very long narrow plates, too. The panel aspect ratio $\alpha = a/b$ can be varied as well as the ratio between the shear and uniaxial stresses.

III. Buckling of Plates under Pure Shear

The classical buckling problem of rectangular plates under pure shear can be immediately solved putting $\gamma = 0$. In this case the Euler shear buckling stress doesn't depend on its direction and for panels with aspect ratio $\alpha \le 1.5$ Timoshenko furnished the following expression for λ, considering only five terms of the series (5):

$$\lambda^2 = \frac{\alpha^4}{81\left(1+\alpha^2\right)^4}\left[1+\frac{81}{625}+\frac{81}{25}\left(\frac{1+\alpha^2}{1+9\alpha^2}\right)^2+\frac{81}{25}\left(\frac{1+\alpha^2}{9+\alpha^2}\right)^2\right] \tag{17}$$

In the actual analysis, the parameter λ is evaluated applying the energy method for different values of α, as shown in table 1, and the convergence of the solution is also investigated. The number $M = N$ of harmonics has been varied from 3 to 30, in order to obtain a number of terms comprised between 9 and 900. It is possible to verify that if the harmonics' number is >100, for $\alpha \le 8$ a good convergence in the assessment of λ, and then of the shear buckling coefficient K_s, is obtained for practical purposes, while only for the case of an infinitely long plate a larger number of harmonics may be required.

TABLE I
CONVERGENCE OF λ VALUES AND SHEAR BUCKLING COEFFICIENT COMPARISON

α	λ values					K_s values		
	$M = N$					Timoshenko (T)	Actual (A) ($M=N=30$)	$\frac{T-A}{A}\cdot 100$
	3	5	10	20	30			
0.1	0.00221	0.00225	0.00316	0.00498	0.00532	538.00	579.81	-7.21
0.2	0.00757	0.01080	0.01115	0.01116	0.01116	137.50	138.25	-0.54
0.4	0.01855	0.02033	0.02045	0.02045	0.02045	37.38	37.70	-0.87
0.6	0.02677	0.02708	0.02713	0.02713	0.02713	18.83	18.95	-0.60
0.8	0.03145	0.03171	0.03177	0.03177	0.03177	12.34	12.14	1.72
1.0	0.03274	0.03302	0.03308	0.03308	0.03308	9.34	9.32	0.18
1.2	0.03187	0.03214	0.03219	0.03220	0.03220	8.12	7.98	1.70
1.4	0.02991	0.03018	0.03023	0.03024	0.03024	7.38	7.29	1.31
1.6	0.02757	0.02786	0.02791	0.02791	0.02791	6.90	6.91	-0.06
1.8	0.02520	0.02557	0.02562	0.02562	0.02562	6.57	6.69	-1.70
2.0	0.02297	0.02384	0.02390	0.02390	0.02390	6.34	6.45	-1.74
2.5	0.01855	0.02033	0.02045	0.02045	0.02045	5.98	6.03	-0.87
3.0	0.01541	0.01754	0.01760	0.01761	0.01761	5.78	5.84	-0.92
3.5	0.01277	0.01529	0.01536	0.01537	0.01537	5.67	5.73	-1.17
4.0	0.01064	0.01362	0.01371	0.01371	0.01371	5.59	5.62	-0.61
5.0	0.00757	0.01080	0.01115	0.01116	0.01116	5.50	5.53	-0.49
6.0	0.00559	0.00864	0.00870	0.00939	0.00939	5.45	5.47	-0.42
7.0	0.00427	0.00699	0.00810	0.00810	0.00810	5.42	5.44	-0.39
8.0	0.00335	0.00575	0.00711	0.00712	0.00712	5.40	5.41	-0.22
∞	0.00058	0.00115	0.00228	0.00281	0.00288	5.35	5.35	0.02

From table 1 it is also possible to verify that the classical expressions (2) for the shear buckling coefficient define a very good approximation of the exact values, so that they cannot be considered as only a "rough" expression for practical purposes. In fig. 2 the convergence of λ for different values of the aspect ratio α is shown.

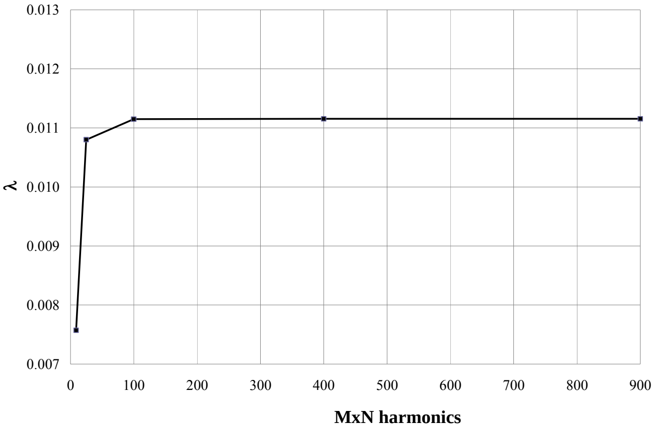

Fig. 2(a) $\alpha = 0.2$

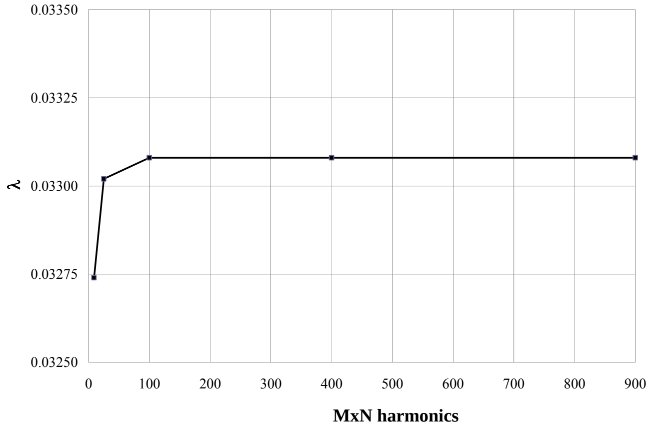

Fig. 2(b) $\alpha = 1.0$

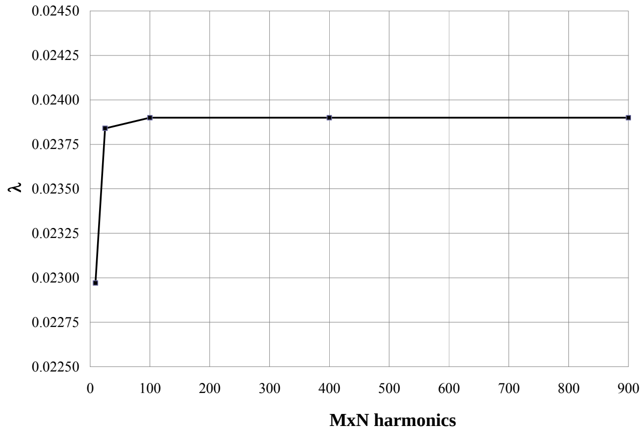

Fig. 2(c) $\alpha = 2.0$

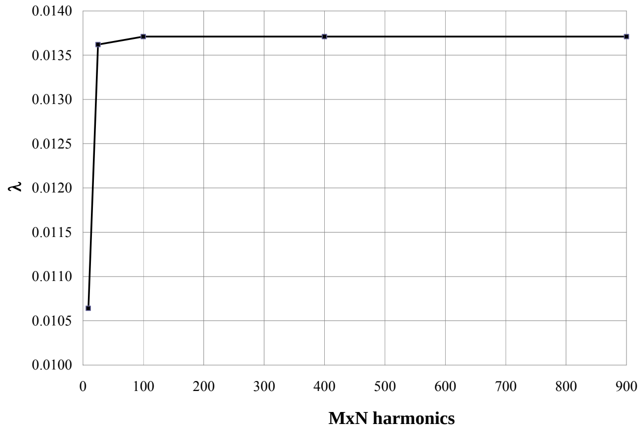

Fig. 2(d) $\alpha = 4.0$

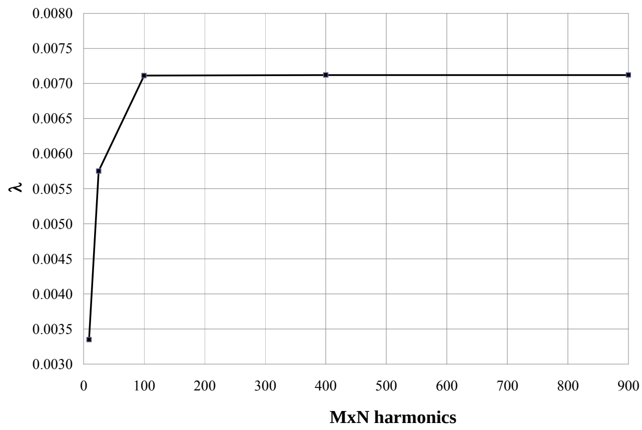

Fig. 2(e) $\alpha = 8.0$

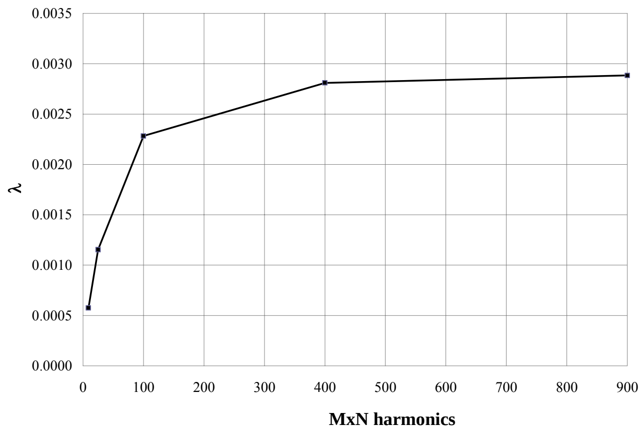

Fig. 2(f) $\alpha = \infty$

Fig. 2 Convergence of λ for different values of the aspect ratio α

IV. Buckling of Plates under Combined Shear and Uniaxial Stresses

The critical λ values have been determined varying the ratio $\gamma = N_x / N_{xy}$ in the range $[-1,1]$. According to the previously assumed convention, in all cases with $\gamma < 0$ ($\gamma > 0$) the uniaxial stresses are negative (positive) and so the panel is compressed (in traction). As it will be subsequently verified (see tab.2), it appears quite clear that when $\gamma < 0$ ($\gamma > 0$) the shear buckling coefficient K_s is less (higher) than the one obtained by (2).

In the table below the shear buckling coefficients, for different values of α and γ are shown; the convergence of the solution, in terms of critical λ values, has also been studied varying the number of harmonics from 9 to 900.

Starting from the data summarized below, the following equations have been obtained to evaluate, for practical purposes, the shear buckling coefficient for simply supported rectangular plates under the combined action of shear and uniaxial stresses. Obviously for $\gamma = 0$ (pure shear) the eq. (2) are obtained again.

$$-1 \le \gamma \le 0;\, N_x \le 0 \quad \begin{cases} K_s = 5.34e^{0.626\gamma} + \dfrac{4.00}{\alpha^2} e^{1.960\gamma} \;\; if\; \alpha \ge 1 \\ K_s = 4.00e^{0.439\gamma} + \dfrac{5.34}{\alpha^2} e^{1.805\gamma} \;\; if\; \alpha < 1 \end{cases}$$

$$0 < \gamma \le 1;\, N_x > 0 \quad \begin{cases} K_s = 5.34e^{0.639\gamma} + \dfrac{4.00}{\alpha^2} e^{1.620\gamma} \;\; if\; \alpha \ge 1 \\ K_s = 4.00e^{0.626\gamma} + \dfrac{5.34}{\alpha^2} e^{1.306\gamma} \;\; if\; \alpha < 1 \end{cases} \tag{18}$$

From the analysis, it follows that a consistent variation of the shear buckling coefficient is found also for low values of the ratio between the axial and shear stresses.

In fig. 3 some project curves, based on the eq. (18), are presented: the thick curve refers to the classical case of pure shear. The curves below (over) the thick one refer to the case of shear with uniaxial compressive (tensile) stresses.

TABLE II
SHEAR BUCKLING COEFFICIENTS FOR DIFFERENT VALUES OF α AND γ

α	Shear + Compression						Pure shear	Shear + Traction					
	γ						γ	γ					
	-1.00	-0.80	-0.60	-0.40	-0.20	-0.10	0.00	0.10	0.20	0.40	0.60	0.80	1.00
0.1	102	127	169	252	396	465	580	629	717	930	1214	1550	1971
0.2	26.8	33.4	44.1	64.4	101.5	119.2	138.2	159.0	181.6	236.9	302.7	387.0	492.7
0.4	8.17	10.05	12.98	17.97	26.98	33.03	37.70	43.31	49.81	63.67	81.81	103.36	131.24
0.6	4.83	5.86	7.38	9.72	13.43	15.96	18.95	22.28	25.14	31.58	40.32	50.95	63.27
0.8	3.80	4.54	5.56	7.04	9.16	10.52	12.14	14.00	16.12	21.06	25.70	31.73	39.64
1.0	3.45	4.06	4.86	5.94	7.39	8.29	9.32	10.52	11.86	15.14	19.29	23.65	28.51
1.2	3.39	3.92	4.60	5.46	6.57	7.23	7.98	8.83	9.79	12.10	15.04	18.77	23.23
1.4	3.46	3.94	4.53	5.26	6.17	6.70	7.29	7.95	8.69	10.45	12.66	15.46	18.97
1.6	3.37	3.87	4.51	5.19	5.96	6.41	6.91	7.46	8.07	9.50	11.28	13.48	16.22
1.8	3.19	3.66	4.24	4.97	5.85	6.25	6.69	7.18	7.71	8.94	10.44	12.27	14.50
2.0	3.10	3.54	4.08	4.75	5.57	6.04	6.45	7.00	7.49	8.60	9.92	11.50	13.39
2.5	3.09	3.48	3.95	4.52	5.21	5.60	6.03	6.51	7.02	8.17	9.29	10.56	12.01
3.0	3.00	3.40	3.88	4.46	5.11	5.46	5.84	6.26	6.70	7.71	8.89	10.21	11.49
3.5	2.98	3.35	3.80	4.34	4.98	5.34	5.73	6.16	6.58	7.50	8.56	9.76	11.13
4.0	2.96	3.33	3.79	4.31	4.91	5.25	5.62	6.03	6.46	7.42	8.42	9.53	10.76
5.0	2.93	3.30	3.73	4.24	4.83	5.16	5.53	5.92	6.34	7.22	8.21	9.32	10.50
6.0	2.92	3.28	3.70	4.20	4.79	5.12	5.47	5.86	6.26	7.13	8.12	9.17	10.29
7.0	2.92	3.26	3.68	4.18	4.76	5.09	5.44	5.82	6.21	7.09	8.05	9.08	10.21
8.0	2.90	3.25	3.67	4.17	4.75	5.07	5.41	5.79	6.19	7.05	8.00	9.04	10.13
∞	2.88	3.21	3.67	4.12	4.73	5.01	5.35	5.71	6.14	7.01	7.91	8.91	9.95

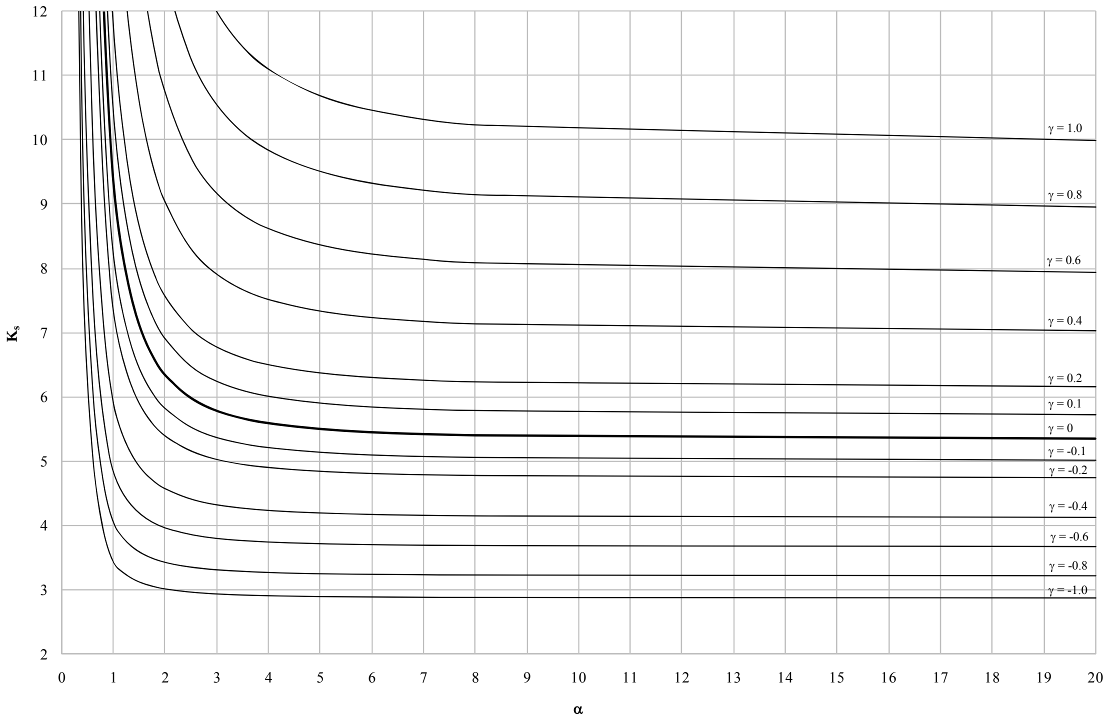

Fig. 3 Shear buckling coefficients

V. Numerical applications

In order to verify the goodness of the presented results, two applications are proposed: the first one is relative to the buckling analysis of a rectangular plate under pure shear, the second one, instead, under the combined action of shear and uniaxial stresses. It is noticed, for the second application, that the most representative case is that one with $\gamma < 0$, as the Euler shear buckling stress is less than the one relative to pure shear.

Some comparisons with the relevant results obtained by a FEM analysis, carried out by *ANSYS,* are also presented to validate the theoretical analysis. The convergence of the model has been studied by thickening the mesh (the last one with a mean element length of 0.010 m); the chosen element is the 4-node finite strain SHELL181, suitable for analyzing thin to moderately thick structures and well-suited for linear, large rotation, and/or large strain nonlinear applications. The following panels have been considered:

1. Case 1: a=1 m; b=1 m; t = 10 mm;
2. Case 2: a=3 m; b=1 m; t = 10 mm;
3. Case 3: a=8 m; b=1 m; t = 10 mm.

Concerning the material properties, it was assumed E=2.06E11 Pa, ν=0.3.

A. Plates under pure shear

In tables III.A, III.B and III.C the Euler shear buckling stress τ_E in N/mm^2 is shown. The convergence of the solution obtained by *ANSYS* is also studied verifying that in all cases it is quite quickly achieved and a very good accordance with the theoretical values is always found.

TABLE III.A
CASE 1 – α = 1, γ = 0

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	400	177.1	173.5	-2.033
0.025	1600	174.1		-0.345
0.015	4489	173.1		0.231
0.010	10000	172.7		0.463

TABLE III.B
CASE 2 – α = 3, γ = 0

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	1200	109.9	108.7	-1.092
0.025	4800	108.8		-0.092
0.015	13600	108.5		0.184
0.010	30000	108.4		0.277

TABLE III.C
CASE 2 – $\alpha = 8, \gamma = 0$

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	3200	102.0		-2.353
0.025	12800	100.9	99.6	-1.288
0.015	35778	100.7		-1.092

B. Plates under combined shear and uniaxial stresses

In tables IV and V the Euler shear buckling stresses τ_E in N/mm^2 are evaluated for $\gamma = -0.6$ and $\gamma = -1.0$. The convergence of the solution obtained by *ANSYS* has been studied and also in this case a very good accordance with the theoretical values is found.

TABLE IV.A
CASE 1 – $\alpha = 1, \gamma = -0.6$

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	400	91.4		-0.985
0.025	1600	91.1	90.5	-0.659
0.015	4489	90.8		-0.330
0.010	10000	90.7		-0.221

TABLE IV.B
CASE 2 – $\alpha = 3, \gamma = -0.6$

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	1200	75.5		-4.371
0.025	4800	73.5	72.2	-1.769
0.015	13600	72.8		-0.824
0.010	30000	72.5		-0.414

TABLE IV.C
CASE 3 – $\alpha = 8, \gamma = -0.6$

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	3200	68.7		-0.582
0.025	12800	68.2	68.3	0.147
0.015	35778	68.1		0.294

TABLE V.A
CASE 1 – $\alpha = 1, \gamma = -1.0$

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	400	65.4		-1.835
0.025	1600	64.8	64.2	-0.926
0.015	4489	64.4		-0.311
0.010	10000	64.2		0.000

TABLE V.B
CASE 2 – $\alpha = 3, \gamma = -1.0$

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	1200	59.5		-6.050
0.025	4800	57.5	55.9	-2.783
0.015	13600	56.7		-1.411
0.010	30000	56.3		-0.710

TABLE V.C
CASE 3 – $\alpha = 8, \gamma = -1.0$

Mean element length	Elements	*ANSYS*	Theoretical	$\frac{T-A}{A}\cdot 100$
m	---	N/mm^2	N/mm^2	%
0.050	300	54.2		-0.554
0.025	12800	53.9	53.9	0.000
0.015	35778	53.8		0.186

VI. CONCLUSIONS

In this paper the problem of the buckling analysis of simply supported rectangular plates under the combined action of uniaxial and shear stresses has been analyzed. It was found that uniaxial stresses have a great influence on the shear buckling coefficient and so on the platings' scantling procedures. The relevant results have been obtained by a dedicated program developed in *MATLAB* and the convergence of the solution was studied, too. Two applications have been proposed to validate the numerical analysis by a comparison between the shear buckling stresses evaluated by a FEM analysis carried out by *ANSYS* and the theoretical ones: a very good accordance is always found.

Starting from the classical formulas for the buckling coefficient of platings under pure shear, new design expressions have been obtained for it, as function of the ratio γ between the axial and shear stresses. These formulas may be of practical aid for the scantling of platings of a variety of structures, e.g. ship ones, where these problems are often encountered. The present analysis can be also extended to platings under the combined action of shear and biaxial stresses; the influence of the plating thickness on the shear buckling coefficient for thick plates can be studied, too. These topics will be the subject of future works.

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V. Piscopo. Bachelor Degree in Naval Engineering in 2004, Master Degree in Naval Engineering in 2006, Ph.D. in Aerospace, Naval and Quality Engineering at the University of Naples "Federico II" in 2009