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Mixed Convection with Radiation Effect over a Nonlinearly Stretching Sheet

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Abstract**—**In this study, an analysis has been performed for free convection with radiation effect over a thermal forming nonlinearly stretching sheet. Parameters n, k0, Pr, G represent the dominance of the nonlinearly effect, radiation effect, heat transfer and free convection effects which have been presented in governing equations, respectively. The similarity transformation and the finite-difference methods have been used to analyze the present problem. From the results, we find that the effects of parameters n, k0, Pr, Ec and G to the nonlinearly stretching sheet. The increase of Prandtl number Pr, free convection parameter G or radiation parameter k0 resulting in the increase of heat transfer effects, but increase of the viscous dissipation number Ec will decrease of heat transfer effect.

Keywords**—**Nonlinearly stretching sheet, Free convection, Finite-difference, Radiation effect.

I. INTRODUCTION

THE study of viscous fluids pass a thermal forming nonlinearly stretching sheet has been become of increasing importance in the lately. Qualitative analyses of these studies have significant bearing on several industrial applications such as polymer sheet extrusion from a dye, drawing of plastic films, etc. When the manufacturing product of a thermal forming stretching sheet at high temperature and need cooling, it needed a good efficient fluid flow to reduce the heat form the sheet. And also, the fluid flow may have processed radiation effect, buoyancy effect or other kinds of effects couple with the fluid flow and stretching sheet and have become a hybrid system which need to analysis by many different ways.

The pioneering work of Sakiadis [1], extensive literature is available on this topic for a linearly stretching sheet. A broad effort has been made to gain information regarding the stretching flow problems in various situations. Such situations include consideration of non-Newtonian fluids, MHD fluid, heat transfer; mass transfer, porous medium, slip effects, etc. A vast body of literature is now available on the topic. Some very recent attempts in this direction have been made in the investigations. Sadeghy [2] studied realistic viscoelastic fluid models such as a Maxwell model should be invoked in the analysis. Indeed, this fluid model has recently been used to study the flow of viscoelastic fluids. Arial et al. [3] studied an elastic-viscous non-Newtonian fluid past a stretching sheet with partial slip effect, but the stretching sheet was linearly. Liao [4] had studied unsteady boundary-layer flows caused by an impulsively stretching plate. In his work, it used the homotopy analysis method (HAM) to find an analytical solution. Xu [5] had studied an explicit analytic solution for convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. The study considered the electrical force adding to momentum equation and obtained its heat transfer effect. Cortell [6,7] had studied the effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet, and considered with internal heat generation or absorption. All the above mentioned investigations deal with the flows over a linearly stretching sheet. Very little attention has been given to the flows over a nonlinearly stretching sheet. Cortell [8] studied the effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. On one side, the effects of thermal radiation are included in the energy equation, on the other hand, the prescribed wall heat flux case (PHF case) is also analyzed. Kechil and Hashim [9] studied series solution of flow over nonlinearly stretching sheet with chemical reaction and magnetic field, it is investigated by employing the Adomian decomposition method (ADM). Bataller [10] studied similarity solutions for flow and heat transfer of a quiescent fluid over a nonlinearly stretching surface. In this study, the viscous dissipation was considered in the energy equation. Cortell [11] studied viscous flow and heat transfer over a nonlinearly stretching sheet, the paper had employed a novel numerical procedure and two cases are studied, namely, (i) the sheet with constant surface temperature (CST case) and (ii) the sheet with prescribed surface temperature (PST case). Vajravelu [12] studied viscous flow over a nonlinearly stretching sheet and Abbas and Hayata [13] studied radiation effects on MHD flow in a porous space. From above, there are still not considering the free convection couple with radiation effect over a nonlinearly stretching sheet. In the present investigation, a study for free convection with radiation effect problem pass a nonlinearly stretching sheet has been processed.

II. THEORETICAL AND ANALYSIS

A. Flow Field Analysis

In this study, consider the flow of an incompressible viscous fluid past a thermal forming stretching sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis, so that the wall is

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stretched keeping the origin fixed. The well-known Boussinesq approximation is used to represent the buoyancy mixed term. The steady two-dimensional boundary layer equations for this fluid, in the usual notation, are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g_x\beta(T-T_\infty) \tag{2}$$

Where *(x, y)* denotes the Cartesian coordinates along the sheet and normal to it, u and v are the velocity components of the fluid in the *x* and *y* directions, respectively, g_x is the magnitude of the gravity, β is the coefficient of thermal expansion and υ is the kinematic viscosity. The boundary conditions to the problem are:

$$u_w(x)=Cx^n,\ v=0 \text{ at } y=0, \tag{3}$$

$$u\to 0 \text{ as } y\to\infty,$$

Where *C* and *n* are parameters related to the surface stretching speed. Defining new similarity variables as:

$$\eta=y\sqrt{\frac{C(n+1)}{2v}}x^{\frac{n-1}{2}},\quad u=Cx^n f'(\eta), \tag{4}$$

$$v=-\sqrt{\frac{Cv(n+1)}{2}}x^{\frac{n-1}{2}}\left[f+\frac{n-1}{n+1}\eta f'\right]$$

Defining the non-dimensional temperature $\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty}$ and $T_w=T_\infty+Ax=\text{constant}$, substituting into Eqs. (1) and (2) give:

$$(f')^2\left(\frac{2n}{n+1}\right)-ff''-f'''-G\theta=0 \tag{5}$$

$G=\frac{2Ag_x\beta}{C^2(n+1)}$ is the free convection parameter, where a prime denotes differentiation with respect to the independent similarity variable *η*. The boundary conditions (3) becomes :

$$f=0, f'=1 \text{ at } \eta=0 \tag{6}$$

$$f'\to 0 \text{ as } \eta\to\infty$$

For the linearly stretching boundary problem (i.e., n = 1) the exact solution for the velocity field f is:

$$f(\eta)=1-\exp(-\eta) \tag{7}$$

and this exact solution is unique, while for the nonlinearly stretching boundary problem (i.e., n ≠ 1) there is no exact solution. The shear stress at the stretched surface is defined as:

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_w \tag{8}$$

and we obtain equation (9) from equations (4) and (8)

$$\tau_W=C\mu\sqrt{\frac{C(n+1)}{2\upsilon}}x^{\frac{3n-1}{2}}f''(0) \tag{9}$$

Where μ is the viscosity of the fluid.

B. Heat Transfer Analyses

By using usual boundary layer approximations, the equation of the energy for temperature T in the presence of radiation and viscous dissipation, is given by:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{10}$$

where *k* is the thermal conductivity, *ρ* is the density, c_p is the specific heat of a fluid at constant pressure and q_r is the radiative heat flux. Using the Rosseland approximation for radiation [14], the radiative heat flux is simplified as

$$q_r=-\frac{4\sigma^*}{3k}\frac{\partial T^4}{\partial y} \tag{11}$$

Where σ^* and k^* are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. It assumes that the temperature differences within the flow such as that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms we get

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{12}$$

In view to Eqs. (11) and (12), Eq. (10) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\alpha}{k_0}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{cp}\left(\frac{\partial u}{\partial y}\right)^2 \tag{13}$$

Where $\alpha=\frac{k}{\rho cp}$ is the thermal diffusivity; $k_0=\frac{3N_R}{3N_R+4}$ and $N_R=\frac{k_T k^*}{4\sigma^* T_\infty^3}$ is the radiation parameter. Similarity solutions of Eq. (13) can be found by choosing appropriate boundary conditions. It is of a certain interest to consider separately the characteristics of the following two cases of main practical interest. For prescribed surface temperature (PST case), the boundary conditions are

$$T=T_W\left(=T_\infty+Ax^k\right) \text{ as } y=0; \tag{14}$$

$$T\to T_\infty \text{ as } y\to\infty$$

Where k is the surface temperature parameter. Defining the non-dimensional temperature $\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty}$ and using Eqs. (4) and (14) into Eq. (13), we get

$$\theta''+\sigma k_0 f\theta'-\left(\frac{2k}{n+1}\right)\sigma k_0 f'\theta=-\sigma k_0 E_c x^{2n-k}\left(f''\right)^2 \tag{15}$$

$$\theta(0)=1,\quad \theta(\infty)\to 0, \tag{16}$$

Where $E_C=\frac{C^2}{Ac_p}$, and $\sigma=\frac{\upsilon}{\alpha}$ is the Prandtl number.

If k=2n, we find form (18)

$$\theta''+\sigma k_0 f\theta'-\left(\frac{4n}{n+1}\right)\sigma k_0 f'\theta=-\sigma k_0 E_C\left(f''\right)^2 \tag{17}$$

It is clear from Eq. (17) that all solutions are then of the similar type. On the other hand, for an arbitrary value of *k* and neglecting heat dissipation, we find from Eq. (15)

$$\theta'' + \sigma k_0 f\theta' - \left(\frac{2k}{n+1}\right)\sigma k_0 f'\theta = 0 \quad (18)$$

The local surface heat flux can be expressed as:

$$q_w = -k_T\left(\frac{\partial T}{\partial y}\right)_w + \left(q_r\right)_w \quad (19)$$

$$= -\frac{k}{k_0} A x^{\frac{2k+n-1}{2}} \theta'(0)\sqrt{\frac{C(n+1)}{2\upsilon}}$$

III. Numerical Technique

In the present problem, the set of similar equations (5), (6), (15), (16) and (17) are solved by a finite difference method. These ordinary differential equations have discretized by a accurate finite difference method, and a computer program has been developed to solve these equations. To avoid errors in discretization and calculation processing and to ensure the convergence of numerical solutions, some conventional numerical procedures have been applied in order to choose a suitable grid size $\Delta\eta = 0.01 - 0.05$, a suitable η range and a direct gauss elimination method with Newton's method is used in the computer program to obtain solutions of these difference equations. Hsiao et al. [15-20] and Vajravelu [21] were also using analytical and numerical solutions to solve the related problems. So, some numerical technique methods will be applied to the same area in the future. In this study, the program to compute finite difference approximations of derivatives for equal spaced discrete data. The code employ centered differences of O(h^2) for the interior points and forward and backward differences of O(h) for the first and last points, respectively. See Chapra and Canale, Numerical Methods for Engineers [22].

IV. Results and Discussion

In this study, an analysis has been performed for free convection with radiation effect over a thermal forming nonlinearly stretching sheet. The problem for viscous fluids pass a thermal forming nonlinearly stretching sheet is very important to produce a good product. The paper provides and obtains the related effects of dimensionless parameters which including the nonlinearly number (n), the Prandtl number (Pr), the viscous dissipation number (Ec), the radiation parameter (N_R) and the free convection number (G) helping to produce a good quality thermal forming nonlinearly stretching sheet. The present study is mainly deal with the heat transfer problem for flow and temperature fields of the thermal forming nonlinearly stretching sheet. For the purpose to solve the problem, the whole flow fields are analyzed by utilizing the boundary layer concept to obtain a set of coupled momentum equation, energy equation and mass equation. Because of the governing equations are belong to partial differential equations and very difficult to solve. To simplify the partial differential equations to ordinary differential equations, so that, a similarity variable has been introduced which transforms the momentum and energy equations into a group of nonlinear ordinary differential equations and similarity heat transfer numerical results were found. So that, a similarity transformation is then used to convert the nonlinear coupled partial differential equations to a set of nonlinear ordinary differential equations. On the other hand, a second-order accurate finite difference method is used to solve the problem and to obtain solutions of those equations. From the solving results, it was obtained the influences of the parameters n, Pr, Ec, N_R and G on both velocity profiles and temperature profiles were examined in those analyses.

Table I is a comparison with the values of $-\theta'(0)$ for different values of physical parameters n and Pr, and fixed parameters Ec=0.1 and N_R =0.1with Ref. [8]. Table I points out that the results from the current study have a good agreement with the previous work (Cortell, 2005) that does not include the free convection effect, because of it is a novel made by this study. From the comparison has a good agreement result for present study, for convenience comparison for others adding the Table II and including the free convection effect. Table II shows that the unknown values of skin friction $-f''(0)$ and Nusselt number $-\theta'(0)$ are obtained by present study for different values of physical parameters n, Pr, Ec and Sc.

TABLE I
Comparison the Values of $-\theta'(0)$ for Different Values of Physical Parameters and Fixed Parameters Ec=0.1 and N_R =0.1with [8]

n	Pr	$-\theta'(0)$ Ref.[8]	$-\theta'(0)$ Present
1.5	1	0.8234	0.8240
3.0	1	0.9138	0.9142
10.	1	1.0016	1.0018
1.5	2	1.2806	1.2807
1.5	5	2.1788	2.1788

TABLE II
The Different Values of Skin Friction $-f''(0)$, Nusselt Number $-\theta'(0)$ for Different Values of Physical Parameters n, Pr, G and Fixed Parameter N_R =1 or k0=3/7, Ec=0.1

n	Pr	G	$-f''(0)$	$-\theta'(0)$
0.2	1.0	0.0	0.7668	0.5017
0.5	3.0	0.1	0.8455	1.2973
1.5	5.0	0.2	1.0031	2.1944
3.0	7.0	0.2	1.1017	2.8992
10.	10.	0.3	1.1785	3.8247

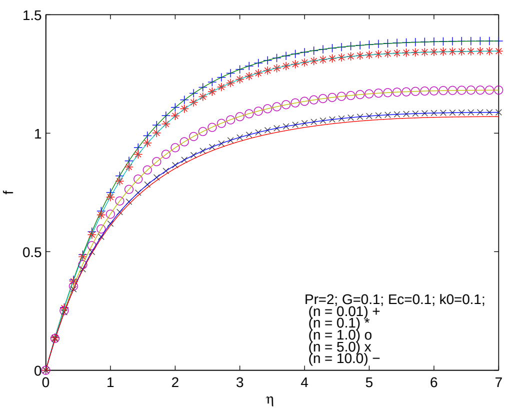

Fig. 1 f vs. η for varies parameters

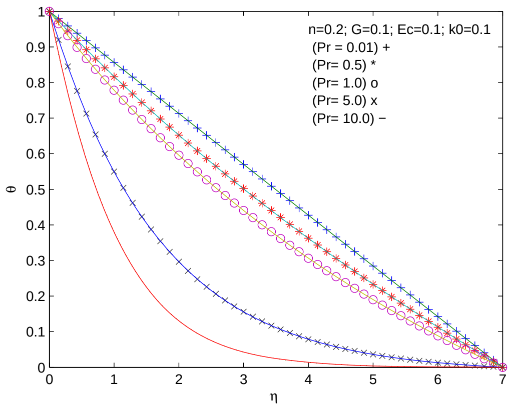

Fig. 2 θ vs. η for varies parameters

Fig. 1 depicts dimensionless velocity profiles f vs. η as Pr=2.0, G=0.1, Ec=0.1, k0=0.1 and n=0.01, 0.1, 1.0, 5.0, 10.0. From figure 1 reveals that the increase of nonlinearly number n results in the decrease of velocity distribution at a particular point of the flow region. This is because there would be a decrease of the fluid boundary layer thickness with the increase of values of nonlinearly number n. Fig. 2 depicts dimensionless temperature profiles θ vs. η as n=0.2, G=0.1, Ec=0.1, k0=0.1 and Pr=0.01, 0.5, 1.0, 5.0, 10.0. The Fig. 2 reveals that the increase of Prandtl number Pr results in the decrease of temperature distribution. This is because there would be a decrease of the thermal boundary layer thickness with the increase of values of Prandtl number Pr. On the contrary, the Fig. 3 reveals that the increase Ec will increase of temperature distribution and the physical phenomena are different to the Fig. 2. The Fig. 3 depicts dimensionless temperature profiles θ vs. η as n=0.2, G=0.1, Pr=2, k0=0.1 and Ec=0.1, 1.0, 3.0, 5.0, 7.0. Fig. 4 depicts dimensionless temperature profiles θ vs. η as n=0.2, G=0.1, Pr=2, Ec=0.1 and k0=0.01, 0.5, 1.0, 2.0, 3.0. The Fig. 4 reveals that the increase the radiation parameters k0 results in the decrease of temperature distribution. This is because there would be a decrease of the thermal boundary layer thickness with the increase of value of k0.

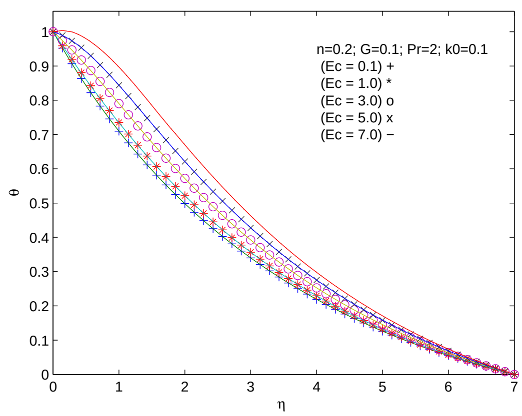

Fig. 3 θ vs. η for varies parameters

From Fig. 2 to 4 are the dimensionless temperature distribution for different parameters n, Ec, Pr and k0 along the thermal boundary layer η. These figures indicate that θ decrease when Pr, k0 increased. According to the equation (19), the value $-\theta'(0)$ increases for Pr or k0 increases. Therefore, the heat transfer rate is positive proposing to Pr, k0 and negative proposing to Ec clearly. The parameter Ec will produce a contrary result from to Prandtl number or radiation parameter.

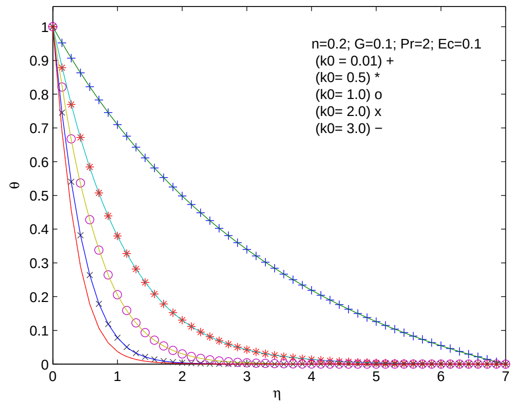

Fig. 4 θ vs. η for varies parameters

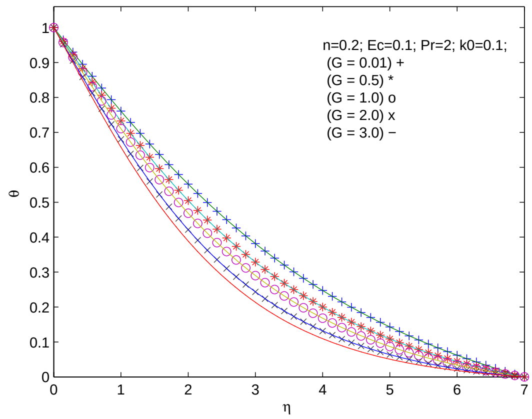

Fig. 5 θ vs. η for varies parameters

Fig. 5 depicts dimensionless concentration profiles θ vs. η as n=0.2, Ec=0.1, Pr=2.0, k0=0.1 and G = 0.01, 0.5, 1.0, 2.0, 3.0. The effect of free convection parameter G on heat transfer process may be analysis from Fig. 5 for the case of prescribed temperature and prescribed heat transfer phenomena. Fig. 5 shows that the increase of value of free convection parameter G results in the decrease of dimensionless temperature distribution as a result of decrease of the thermal boundary layer thickness with the increased values of G.

Although that the studied problem are not easy to obtain, it had been solved by present theoretical and numerical solution methods. Finally, the flow quantities have been discussed through graphs and tables. The physical results are interesting, an excellent agreement of the present results with existing limiting results is shown. The main contribution of this study considers the radiation effect in a mixed convection for a viscous fluid flow past a nonlinearly stretching sheet thermal forming heat transfer system. From the figures provide more physical insights, and use to a thermal forming engineering problem, as follow:

1. These graphs 2, 4, 5 reveal that the increase of Pr, k0 or G result in the increase of dimensionless temperature distribution to lower. This is because there would be a decrease of the thermal boundary layer thickness with the increase of value of Pr, k0 or G. But the heat transfer phenomenon is good at these physical conditions.
2. On the contrary, from the graph 3 reveal that the increase of Ec results in the increase of temperature gradient on the wall and let to higher. This is because there would be an increase of the thermal boundary layer thickness with the increase of values of Ec. The heat transfer phenomenon is not good at these physical conditions.

V. Conclusion

In this study, it has been analyzed boundary-layer flow and heat transfer in a viscous fluid over a moving flat surface which is nonlinearly stretched in the presence of thermal radiation and the Rosseland approximation for the radiative heat flux is used. At the same time, Boussinesq approximation is used to represent the buoyancy mixed convection. At last, it obtained some important results as following. The increase of Prandtl number Pr, free convection parameter G or radiation parameter k0 resulting in the increase of the heat transfer effects. On the other hand, the increase of Ec results in the decrease of the heat transfer effect.

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