Quantum Ion Acoustic Solitary and Shock Waves in Dissipative Warm Plasma with Fermi Electron and Positron

Hamid Reza Pakzad

Abstract**—**Ion-acoustic solitary and shock waves in dense quantum plasmas whose constituents are electrons, positrons, and positive ions are investigated. We assume that ion velocity is weakly relativistic and also the effects of kinematic viscosity among the plasma constituents is considered. By using the reductive perturbation method, the Korteweg–deVries–Burger (KdV-B) equation is derived.

Keywords**—**Ion acoustic shock waves; Quantum plasmas

I. INTRODUCTION

QUANTUM plasmas, where the dominated wave nature of electrons gives rise to collective effects, have received a lot of attention because of their potential applications in nanoscale systems [1], microelectronic devices [2], laser fusion plasmas [3], dense plasma particularly in astrophysical and cosmological studies [4–7] and next generation high intensity light sources [2,8]. Quantum plasma is studied by the quantum hydrodynamic (QHD) model [9–14] as well as the Wigner–Poisson system [15], which is the integrodifferential system. The advantages of the QHD model over the Wigner–Poisson system are its numerical efficiency [16, 17], the direct use of the macroscopic variables of interest, and the easy way the boundary conditions are implemented. Electron-positron plasmas have been observed in active galactic nuclei [18], in pulsar magnetospheres [19], in the polar regions of neutron stars [20], as well as in the intense laser fields [21]. Electron-positron plasma is also believed to exist in the early universe [22] as well as at the center of our own galaxy [23]. Since in many astrophysical environments there exist a small number of ions along with the electrons and positrons, therefore, it is important to study linear and nonlinear behavior of plasma waves in electron-positron-ion (*e-p-i*) plasmas. A lot of research has been carried out to study the *e-p* and *e-p-i* plasmas in the past few years [24-27]. For instance, Nejoh [24] investigated the effect of ion temperature on the large amplitude ion-acoustic waves in *e-p-i* plasma and observed that the ion temperature decreased the amplitude and increased the maximum Mach number of the ion acoustic wave. Ali *et al.* [28] investigated the linear and nonlinear ion-acoustic waves in an unmagnetized electron-positron-ion quantum plasma. It is well known that in a nonlinear dispersive media, shock-like solutions are formed. This happens due to the balance between the nonlinearity (causing wave steepening) and dissipation (e.g., caused by viscosity, collisions, wave particle interaction, etc.). However, when a medium has both dispersive and dissipative properties, then the propagation of small amplitude perturbations can be adequately described by the Korteweg–deVries–Burgers (KdVB) equation. The equation has a dissipation term (Burger's term) in addition to the nonlinear and dispersive terms. The dissipation could be produced by many mechanisms, such as, wave-particle interaction, dust charge fluctuation, anomalous viscosity, etc. In this paper, the dissipation in the KdVB equation arises by taking into account the kinematic viscosity among the plasma constituents [29]. In the KdVB equation, the wave braking due to nonlinearity is balanced by the combined effects of dispersion and dissipation resulting into a monotonic or oscillatory dispersive shock wave in a plasma [30]. Several authors have investigated different aspects of quantum ion acoustic solitary and shock waves in electron–ion (ei) [29-34] and electron-ion-positron (epi) [35-42] plasmas. To the best of our knowledge, quantum ion acoustic shock waves in dissipative, weakly relativistic and warm plasmas have never been addressed in the e-p-i plasma literature. Therefore, we are interested to study the behavior of ion acoustic waves in such plasma. The organization of the paper is as follows. In Sec. 2, the basic set of equations is given and the KdV- Burgers equation is derived for ion acoustic shock waves. Section 3 is kept for the results and discussion, while Sec. 4 is kept for the conclusion.

H. R. Pakzad is with the Department of physics, Bojnourd Branch, Islamic Azad University, Bojnourd, Iran. (phone: 9153846142; fax: +985842241200; (E-mail:pakzad@bojnourdiau.ac.ir & ttaranomm83@yahoo.com).

II. THE KDV-B EQUATION FOR QIAWS

Let us consider a quantum dense plasma consisting of inertial warm weakly relativistic ions, inertialess electrons and positrons. Moreover, the kinematic viscosity among the plasma constituents is considered. The basic equations describing the nonlinear dynamics in the quantum plasma system are [43,44]

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0, \tag{1}$$

$$\frac{\partial^2 \phi}{\partial x^2}=\mu n_e-(\mu-1)n_p-n, \tag{2}$$

$$\frac{\partial (\gamma u)}{\partial t}+u\frac{\partial (\gamma u)}{\partial x}+\sigma n\frac{\partial n}{\partial x}+\frac{\partial \phi}{\partial x}-\eta\frac{\partial^2 u}{\partial x^2}=0, \tag{3}$$

$$0=\frac{\partial \phi}{\partial x}-2n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2\mu}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_e}}{\partial x^2\sqrt{n_e}}\right), \tag{4}$$

$$0 = -\frac{\partial \phi}{\partial x} - 2\delta n_p \frac{\partial n_p}{\partial x} + \frac{H^2}{2\mu}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_p}}{\partial x^2 \sqrt{n_p}}\right), \quad (5)$$

In above equations, the ion density (n), electron density (n_e) and positron density (n_p) are normalized by unperturbed ion density ($n_\circ$). U and ϕ, are the ion fluid velocity, the pressure and electrical potential. These quantities are normalized by the sound velocity $\sqrt{\frac{K_B T_{Fe}}{m}}$ and $(\frac{K_B T_{Fe}}{e})$, respectively, where K_B is Boltzmann's constant, m is the ion mass and e is the charge of electron. The coefficient of kinematic viscosity η is incorporated in the parameter, $\eta = \mu\omega_{pi}/c_s^2$ where $c_s = \sqrt{2K_B T_{Fe}/m}$ is the ion acoustic speed in terms of Fermi temperature and $\omega_{pi} = \sqrt{4\pi n_\circ e^2/m}$ is the ion plasma frequency. Also, we considered that the electrons and positrons obey the equation of state pertaining to a one-dimensional zero-temperature Fermi gas

$$p_\alpha = \frac{m_\alpha v_{F\alpha}^2}{3n_{\alpha\circ}^2} n_\alpha^3 \quad (6)$$

where $v_{F\alpha} = \sqrt{\frac{2k_B T_{F\alpha}}{m}}$ with α = e, p is the Fermi thermal speed, $T_{F\alpha}$ is the particle Fermi temperature, k_B is Boltzmann's constant and $n_{\alpha\circ}$ is the equilibrium particle number density. The quantum statistical effects can be seen through the dimensionless parameters $\sigma = T_{Fi}/T_{fe}$ and $\delta = T_{Fp}/T_{Fe}$. We introduce the following notations

$$\delta = \frac{T_{Fp}}{T_{Fe}} = \left(1 - \frac{1}{\mu}\right)^{2/3},\ \eta = \mu\omega_{pi}/c_s^2,\ H = \hbar\omega/(k_B T_{Fe}),$$

$$\mu = 1/(1-p), \quad p = \frac{n_{p0}}{n_{e0}} \quad (7)$$

Integrating once Eqs. (4) and (5) with boundary conditions $n_e = 1$, $n_p = 1$ and $\phi = 0$ at infinity, we have

$$\phi = -1 + n_e^2 - \frac{H^2}{2\mu}\frac{\partial^2 \sqrt{n_e}}{\partial x^2 \sqrt{n_e}},$$

$$\phi = \delta - \delta n_p^2 + \frac{H^2}{2\mu}\frac{\partial^2 \sqrt{n_p}}{\partial x^2 \sqrt{n_p}}. \quad (8)$$

(9) In order to investigate the propagation of QIASWs and to derive the KdVB equation in relativistic *e-p-i* plasma, the independent variables are stretched as $\xi = \varepsilon^{1/2}(x - \lambda t)$, $\tau = \varepsilon^{3/2} t$, $\eta = \varepsilon^{1/2}\eta_\circ$, while $\eta_\circ$ is a finite quantity of the order of unity, and the dependent variables are expended as

$$\begin{aligned}
n_e &= 1 + \varepsilon\, n_{e1} + \varepsilon^2 n_{e2} + \varepsilon^3 n_{e3} + \ldots, \\
n_p &= 1 + \varepsilon\, n_{p1} + \varepsilon^2 n_{p2} + \varepsilon^3 n_{p3} + \ldots, \\
n &= 1 + \varepsilon\, n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 + \ldots, \\
u &= u_\circ + \varepsilon\, u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \ldots, \\
\phi &= \varepsilon\, \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \ldots.
\end{aligned} \quad (10)$$

where ε is the small nonzero parameter proportional to the amplitude of the perturbation. Now, substituting the expressions from equation (10) along with the stretching coordinates into equations (1-3) and collecting the terms in different power of ε , the lowest order of ε yields the following dispersion relation

$$\left[(\lambda - u_\circ)^2 - 3\sigma\right]\gamma_1 = \frac{2\delta}{\mu(\delta + 1) - 1} \quad (11)$$

where $\gamma_1 = 1 + \frac{3u_\circ^2}{2c^2}$. Now if $\gamma_1 \to 1$, then this dispersion relation has the same form as that derived by the Roy *et. al* [45]. Now, in the next higher order of ε, we eliminate the second order perturbed quantities from the set of equations using standard procedure, to obtain the required KdVB equation for QIAWs:

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} - C\frac{\partial^2 \phi_1}{\partial \xi^2} = 0. \quad (12)$$

where

$$A = \frac{(\lambda - u_\circ)2^{-3/2}\ \delta^{-1/2}}{\left\{\mu(\delta+1) - 1 + 3\sigma\gamma_1\left[\mu(\delta+1) - 1\right]^2\right\}^{3/2}} \times \left\{\begin{aligned}&3\mu^2(\delta+1)^2 + \mu(\delta-7)(\delta+1) + 4 + \\ &\mu\left[2\{1 - (\gamma_2/\gamma_1)(\lambda - u_\circ)\} + \gamma_1(\lambda - u_\circ)^2 + 9\sigma\gamma_1^2\right]\end{aligned}\right\}$$

$$B = \frac{(\lambda - u_\circ)2^{1/2}\ \delta^{3/2}}{\left[\mu(\delta+1) - 1 + 3\sigma\gamma_1(\mu(\delta+1) - 1)^2\right]^{3/2}} \times \left[1 - (1 + \delta^{-1/2})\frac{H^2}{16}\right],$$

$$C = \frac{\eta_\circ}{2\gamma_1} \quad (13)$$

where $\gamma_2 = \frac{3u_\circ}{2c^2}$. Relativistic, quantum, dissipative effects, ion temperature, Fermi temperature and relative density influence, respectively, on the equations through the parameter $h = u_0/c$, H, $\eta_\circ$, σ, δ, μ. Equation (12) is the well known KdV Burger equation describing the nonlinear propagation of the ion acoustic shock waves in a warm quantum plasma with inertialess electrons and positrons and relativistic ions. In this equation A and B are the nonlinear coefficient and dispersive term and the Burger term (C) arises due to the effect of ion kinematic viscosity. The above results in (13) are congruent with the observations in [49,50] made in relativistic e-p-i plasmas. The Burger term implies the possibility of the existence of a shock-like solution. In the absence of viscosity, the equation (12) reduces to KdV equation for QIAW. In this situation, the solitonic structure will be established by balancing the effects of dispersive and nonlinear terms. On the other hand, if the coupling becomes very strong the shock waves will appear. The nature of these shock structures depends on the relative values between the dispersive and dissipative coefficients B and C, respectively. The KdV-Burger equation is widely used in plasma physics. The tangent hyperbolic method seems to be a powerful tool for the computation of exact traveling wave solutions. The stationary solution of Eq. (12) can also be obtained analytically by the so-called "tanh method" [46-48]. The solution of the KdVB equation reads

$$\phi_1 = \frac{12B}{A}[1 - \tanh^2 \chi] - \frac{36C}{15A}\tanh \chi \tag{14}$$

where $\chi = \kappa(\xi - v\tau)$ (where κ and v are the wave number and the wave velocity, respectively). It is clear that for C=0, the solitonic solutions appear. Now, using this stationary solution, we study.

The effects of the quantum parameter (*H*), ion temperature (σ), dissipative factor ($\eta_\circ$) and relativistic factor (*h*) on the soltary and shock waves can be studied numerically.

REFERENCES

[1] H. G. Craighead, Science 290, 1532 (2000).
[2] M. Marklund and P. K. Shukla, Rev. Mod. Phys. 78, 591 (2006).
[3] S. H. Glenzer, O. L. Landen, P. Neumayer, R. W. Lee, K. Widmann, S. W. Pollaine, R. J. Wallace, G. Gregori, A. Holl, T. Bornath, R. Thiele, V. Schwarz, W.-D. Kraeft, and R. Redmer, Phys. Rev. Lett. 98, 065002 (2007).
[4] G. Chabrier, F. Douchin, and A. Y. Potekhin, J. Phys.: Condens. Matter 14, 9133 (2002).
[5] M. Opher, L. O. Silva, D. E. Dauger, V. K. Decyk, and J. M. Dawson, Phys. Plasmas 8, 2454 (2001).
[6] Y. D. Jung, Phys. Plasmas 8, 3842 (2001).
[7] D. Kremp, Th. Bornath, M. Bonitz, and M. Schlanges, Phys. Rev. E 60, 4725 (1999).
[8] G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. 78, 309 (2006).
[9] E. Madelung, Z. Phys. 40, 332 (1926).
[10] M. G. Ancona and G. J. Iafrate, Phys. Rev. B 39, 9536 (1989).
[11] M. V. Kuzelev and A. A. Rukhadze, Phys. Usp. 42, 603 (1999).
[12] C. Gardner and C. Ringhofer, VLSI Des. 10, 415 (2000).
[13] I. Gasser and P. A. Markowich, Asymptotic Anal. 14, 97 (1997).
[14] I. Gasser, C. K. Lin, and P. Markowich, Taiwan. J. Math. 4, 501 (2000).
[15] P. A. Markowich, C. A. Ringhofer, and C. Schmeiser, SemiconductorEquations (Springer, Vienna, 1990).
[16] C. Gardner, IEEE Trans. Electron Devices 38, 392 (1991).
[17] C. Gardner, J. Jerome, and D. Rose, IEEE Trans. Comput.-Aided Des. 8, 501 (1989).
[18] H. R. Mille and P. Witta, Active Galactic Nuclei (Springer, Berlin, 1987), p. 202.
[19] F. C. Michel, Rev. Mod. Phys. 54, 1 (1982).
[20] F. C. Michel, Theory of Neutron Star Magnetosphere (Chicago University Press, Chicago, 1991).
[21] V. Berezhiani, D. D. Tskhakaya, and P. K. Shukla, Phys. Rev. A 46, 6608 (1992).
[22] W. Misner, K. Thorne, and J. A. Wheeler, Gravitation (Freeman, SanFrancisco, 1973), p. 763.
[23] M. L. Burns, Positron-Electron Pairs in Astrophysics, edited by M. L.Burns, A. K. Harding, and R. Ramaty (American Institute of Physics, Melville, NY, 1983).
[24] Y. N. Nejoh, Phys. Plasmas 3, 1447 (1996).
[25] F. Verheest, M. A. Hellberg, G. J. Gray, and R. L. Mace, Astrophys. Space Sci. 239, 125 (1995).
[26] A. Mushtaq and H. A. Shah, Phys. Plasmas 12, 072306 (2005).
[27] I. Kourakis, F. Verheest, and N. Cramer, Phys. Plasmas 14, 022306 (2007).
[28] S. Ali, W. M. Moslem, P. K. Shukla, and R. Schlickeiser, Phys. Plasmas 14, 082307 (2007).
[29] B. Sahu and R. Roychoudhury, Phys. Plasmas 14, 072310 (2007).
[30] J-K. Xue, Phys. Plasmas 10, 4893 (2003).
[31] B. Sahu, R. Roychoudhury, Phys. Plasmas 14, 012304 (2007).
[32] P. Chatterjee, K. Roy, S. V. Muniandy, S. L. Yap and C. S. Wong, 16, 042311 (2009)
[33] W. Masood and H. Rizvi, Phys. Plasmas 14, 022303 (2010).
[34] S. A. Khan, S. Mahmood and H. Saleem, Phys. Plasmas 15, 082303 (2008)
[35] W. Masood,1 Arshad M. Mirza,2 and M. Hanif, Phys. Plasmas 15, 072106 (2008)
[36] W. Masood, A. Mushtaq, and R. Khan, Phys. Plasmas 14, 123702 (2007)
[37] S. A. Khan1 and W. Masood, Phys. Plasmas 15, 062301 (2008)
[38] Saeed-ur-Rehman, Phys. Plasmas 17, 062303 (2010)
[39] S.A. Khan, S. Masood, S. Ali, Phys. Plasmas 16, 044505 (2009)
[40] Sheng-Chang Li, Phys. Plasmas 17, 082307 (2010)
[41] U.M. Abdelsalam, W.M. Moslem, P.K. Shukla, Phys. Lett. A 372 (2008) 4057
[42] A. Mushtaq, S. A. Khan, Phys. Plasmas 14, 052307 (2007)
[43] S.A. Khan, W. Masood, Phys. Plasmas 15 (2008) 062301.
[44] F. Haas, L.G. Garcia, J. Goedert, G. Manfredi, Phys. Plasmas 10 (2003) 3858.
[45] Roy K, Misra A P and Chatterjee P Phys. Plasmas 15 032310 (2008)
[46] R. Roychoudhury, G. C. Das, and J. Sarma, Phys. Plasmas 6, 2721 (1999).
[47] A.M. Wazwaz, Nonlinear Sci. Numer. Simul. 13 (2008) 584.
[48] Asif Shah, R. Saeed, Physics Letters A 373 (2009) 4164.
[49] T.S. Gill, A. Singh, H. Kaur, N.S. Saini, P. Bala, Phys. Lett. A 361 (2007) 364.
[50] H.R. Pakzad, Astrophys Space Sci (2011) 331: 169–174.