

# Neutral to Earth Voltage Analysis in Harmonic Polluted Distribution Networks with Multi-Grounded Neutrals

G. Ahmadi<sup>1</sup>, and S.M. Shahrtaash<sup>2</sup>

**Abstract**—A multiphase harmonic load flow algorithm is developed based on backward/forward sweep to examine the effects of various factors on the neutral to earth voltage (NEV), including unsymmetrical system configuration, load unbalance and harmonic injection. The proposed algorithm composes fundamental frequency and harmonic frequencies power flows. The algorithm and the associated models are tested on IEEE 13 bus system. The magnitude of NEV is investigated under various conditions of the number of grounding rods per feeder lengths, the grounding rods resistance and the grounding resistance of the in feeding source. Additionally, the harmonic injection of nonlinear loads has been considered and its influences on NEV under different conditions are shown.

**Keywords**—NEV, Distribution System, Multi-grounded, Backward/Forward Sweep, Harmonic Analysis

## I. INTRODUCTION

THE use of nonlinear loads and power switching devices is increasing because of their high efficiency and convenience for control. In the meantime, they induce harmonic pollution problems. As harmonics propagate through a system, they result in losses increase, communication networks interference, possible equipment loss of life and elevated neutral to earth voltage. The existence of NEV causes to unbalance three phase voltage for three phase customers and to reduce phase to neutral voltage for single phase customers.

The objective of this paper is to advance an appropriate harmonic load flow algorithm and the associated power system modeling technique for NEV profile calculation, through which a fast harmonic analysis method for unbalanced distribution systems is proposed. The mentioned algorithm is a constitution of fundamental frequency power flow and harmonic frequencies power flow. The results obtained from these power flows are gathered and the desired results are achieved. In order to calculate neutral wire and ground currents and voltages on different harmonic frequencies, the three-phase backward/forward procedure [1] is generalized,

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where the  $3 \times 3$  network representation is expanded to a  $5 \times 5$  representation; considering three-phase wires, neutral wire, and ground equivalent wire. The proposed method is developed based on exact three-phase models, and a commonly used forward/backward sweep technique. The backward sweep is used to build the relationship between branch currents and bus current injections of a distribution system, and then the forward sweep can use the branch currents to calculate bus voltages.

## II. MODELING OF RADIAL DISTRIBUTION NETWORKS

In fact, the presentation method in this paper for radial distribution system analysis is an improvement in the method of [1]. In this improvement the distribution feeder is modeled by a five wire model to reveal the profile of neutral to earth voltages.

In this method, the branches and nodes need to be numbered to describe the radial topology of the distribution systems. The procedure is better understood by following the example network shown in Fig.1. The source usually is denoted as node 0, and the two nodes of each branch are numbered as  $N_1$  and  $N_2$  respectively, where the node closer to the root node is  $N_1$  and the other node is  $N_2$ . In the meantime, the  $N_2$  node of the corresponding branch is assigned with a number similar to the branch number.

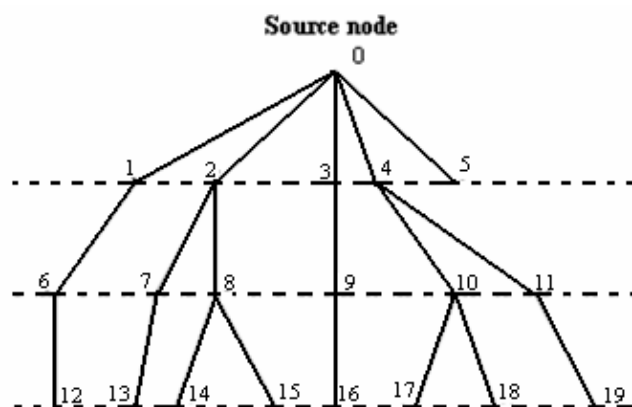


Fig. 1. The numbered radial distribution network

In the proposed method, in addition to usage of five wire model for multi-grounded distribution feeders, the parameters of this model are derived according to Carson's theory with considering the skin effect for different harmonics, based on the formulation given in [6] and assigning the self and mutual

impedance of ground equivalent wire according to [2]. A multi-grounded distribution feeder may be divided to  $\pi$  segments, where the three phase feeder with two segments is illustrated in Fig. 2.

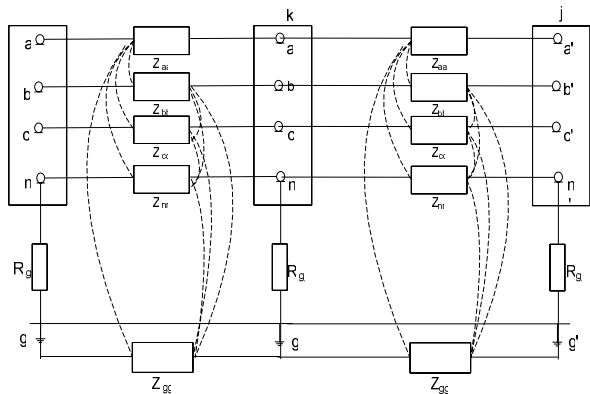


Fig. 2. A three phase multi-grounded feeder

In power system analysis, the distributed loads connected to the transmission lines are usually aggregated at the nodes of interest. Thus for each feeder, the nodes in the middle need to be eliminated from the final equivalent circuit. The Kron reduction can be applied to simplify the above circuit with multiple  $\pi$  segments to one  $\pi$  segment for each feeder. The resulting equivalent circuit is shown in Fig. 3.

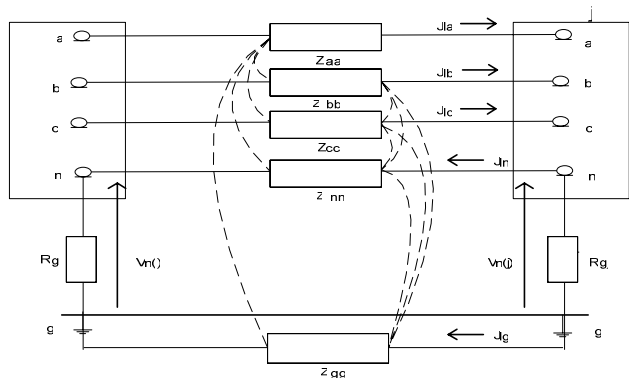


Fig. 3. The  $\Pi$  equivalent circuit of a multi-grounded feeder

### III. HYBRID POWER FLOW ALGORITHM

The proposed backward/forward sweep algorithm to solve the radial system can be divided into two parts: 1) backward current sweep and 2) forward voltage sweep.

**Backward Current Sweep:** For a radial feeder, the branch current can be calculated by summing the injection currents from the receiving bus toward the sending bus of the feeder. The general equation can be expressed as:

$$\begin{bmatrix} J_{la} \\ J_{lb} \\ J_{lc} \\ J_{ln} \\ J_{lg} \end{bmatrix}^{(h),k} = - \begin{bmatrix} I_{ja} \\ I_{jb} \\ I_{jc} \\ I_{jn} \\ I_{jg} \end{bmatrix}^{(h),k} + \sum_{m \in \Omega_j} \begin{bmatrix} J_{ma} \\ J_{mb} \\ J_{mc} \\ J_{mn} \\ J_{mg} \end{bmatrix}^{(h),k} \quad (1)$$

where the relation is derived for  $h^{th}$  harmonic order and  $k^{th}$  iteration and;

$J_l^{abcng}^{(h)}$  is harmonic current flows on line section l,

$I_j^{abcng}^{(h)}$  is current injections at bus j,

$\Omega_j$  is set of line sections connected downstream to bus j.

For a radial distribution system if branch currents were calculated, the bus voltages can be found from the sending bus toward the receiving bus of the feeder. The general equation can be expressed as:

$$\Delta V_j^{(h),k} = \begin{bmatrix} V_{ia} \\ V_{ib} \\ V_{ic} \\ V_{in} \\ V_{ig} \end{bmatrix}^{(h),k} - \begin{bmatrix} V_{ja} \\ V_{jb} \\ V_{jc} \\ V_{jn} \\ V_{jg} \end{bmatrix}^{(h),k} \quad (2)$$

$$= \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} & Z_{ag} \\ Z_{ab} & Z_{bb} & Z_{bc} & Z_{bn} & Z_{bg} \\ Z_{ac} & Z_{bc} & Z_{cc} & Z_{cn} & Z_{cg} \\ Z_{an} & Z_{bn} & Z_{cn} & Z_{nn} & Z_{ng} \\ Z_{ag} & Z_{bg} & Z_{cg} & Z_{ng} & Z_{gg} \end{bmatrix}^{(h)} \times \begin{bmatrix} J_{la} \\ J_{lb} \\ J_{lc} \\ J_{ln} \\ J_{lg} \end{bmatrix}^{(h),k}$$

where, as in (1), the relation is derived for  $h^{th}$  harmonic order and  $k^{th}$  iteration and;

$V_i^{abcng}^{(h)}$  is the harmonic voltage of bus i,

$Z_{ij}^{abcng}^{(h)}$  is the branch impedance from bus i to bus j.

The harmonic power flow calculation is started with an initial voltage for all nodes. It is necessary to assume that the initial voltage for all nodes at fundamental frequency is equal to the source node voltage, with taking the initial voltage of neutral wire and ground equal conductor as zero, i.e.

$$\begin{bmatrix} V_{ia} \\ V_{ib} \\ V_{ic} \\ V_{in} \\ V_{ig} \end{bmatrix}^{(0)} = \begin{bmatrix} V_{ref} \\ a^2 \cdot V_{ref} \\ a \cdot V_{ref} \\ 0 \\ 0 \end{bmatrix} \quad a = e^{j2\pi/3} \quad (3)$$

The initial voltages for all nodes at harmonic frequencies are assumed zero.

There may be three types of loads or current injections at each node [5].

- Constant current loads, where the current injection is directly known,
- Constant impedance loads, where the current injection equals to the ratio of relevant node voltage to load impedance,
- Constant power loads (only at fundamental frequency), where the current injection equals to the ratio of load consuming power to node voltage.

According to this procedure,  $[I^{(h),k}]$ , the total vector of nodal current injections for  $h^{th}$ , and similarly for any harmonics can be found.

In [7], the resulting branch current vector is explained and its formation procedure is presented. Thus, it can be written:

$$J_{ij}^{(h),k} = [A_{ij}^{(h)}] [I^{(h),k}] \quad (4)$$

where  $[A_{ij}^{(h)}]$  is the coefficient vector of harmonic currents for the branch between bus  $i$  and  $j$  at the  $h^{th}$  harmonic order.

The next step is to calculate the bus voltage variations with respect to the harmonic current injection vector, which can be calculated by combining (2) and the forward voltage sweep and expressed as:

$$[(V_s - V)^{(h),k}] = [HA^{(h)}] [I^{(h),k}] \quad (5)$$

where  $V_s$  is the source node voltage, and  $V$  is the bus voltage vector.  $[HA^k]$  is the relationship matrix between bus voltage vector and harmonic current injection vector, that the formation procedure is explained in [7].

#### A. Fundamental frequency power flow

In this power flow, the current injections in buses are built from injections by each of three load types. The constant impedance loads are considered as parallel branches that their currents are calculated by:

$$I^{(1),k} = Y^{(1)} \times V^{(1),k} \quad (6)$$

where  $Y^{(1)}$  is the admittance matrix which contains constant impedance loads and  $V^{(1),k}$  is the bus voltage vector as defined before, both at fundamental frequency.

Now, equation (5) can be rewritten as below:

$$[(V_s - V)^{(1),k}] - [HA]^{(1)} [Y]^{(1)} [V_i]^{(1),k} = [HA_f]^{(1)} [I_f]^{(1),k} \quad (7)$$

where  $[HA_f]^{(1)}$  is composed of the column vectors with respect to the buses of constant power and constant current loads;

$[I_f]^{(1),k}$  is the fundamental injection currents by these type of loads.

The new bus voltage vector fundamental frequency can be expressed as:

$$[V^{(1),k+1}] = [V^{(1),0}] + [V^{(1),k}] \quad (8)$$

The iteration will stop, if the following criterion is fulfilled:

$$|V_i^{(1),k+1} - V_i^{(h),k}| \leq \varepsilon \quad \text{for} \quad i = 1 \dots N \quad (9)$$

where  $\varepsilon$  is the predetermined tolerance and  $N$  is the bus number.

#### B. Harmonic frequencies power flow

In harmonic frequencies, the current injections of constant power loads are zero. Thus, the harmonic current injections in buses are only from constant current loads, and (7) becomes as:

$$[(V_s - V_i)^{(h)}] - [HA]^{(h)} [Y]^{(h)} [V_i]^{(h)} = [HA_h]^{(h)} [I_h]^{(h)} \quad (10)$$

where  $[HA_h]^{(h)}$  is composed of the column vectors with respect to the buses of constant current loads;  $[I_h]^{(h)}$  is the harmonic injection currents by these loads.

The harmonic power flow finishes at single iteration, because  $[I_h]^{(h)}$  is constant at each harmonic.

### IV. TEST SYSTEM SIMULATION

In this paper, the IEEE 13 bus test distribution system [4] is made less complex [3] for modeling and simulating as shown in Fig.4, and the effects of system parameters variations on NEV have been studied.

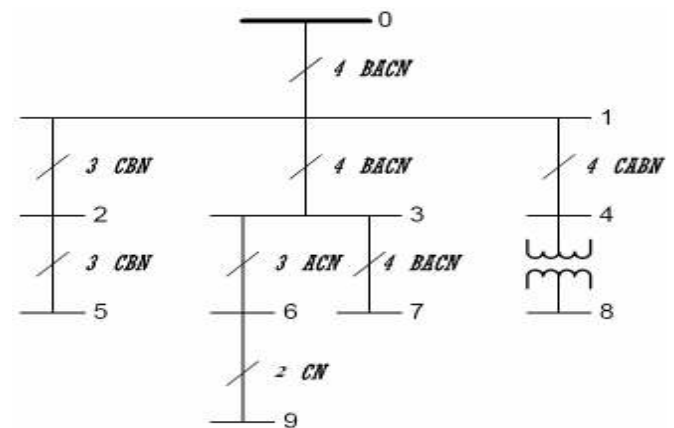


Fig. 4. Test distribution system

## V. RESULTS

Results obtained by performing the proposed power flow method on IEEE example system are shown in next steps.

### A. Effect of harmonic loads on NEV

To examine the effect of harmonic distortion on NEV, two kinds of simulation were performed with two structures for loads. The first one was simulated with constant power only (w/o harmonic load), and the second was simulated with all types of load that mentioned previously. The results obtained for these simulations are shown in Fig.5.

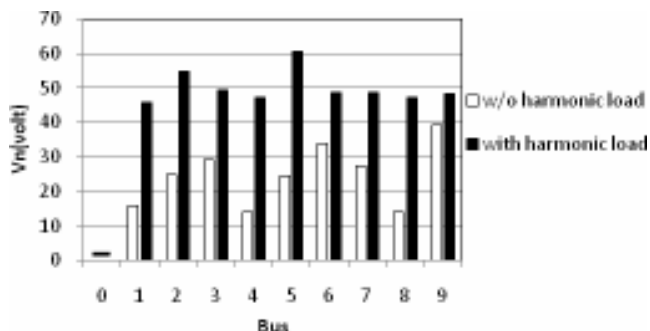


Fig. 5. Effect of harmonic distortion on NEV

### B. Variation of grounding rod resistance $R_g$

In the next simulation, to analyze the effect of  $R_g$  variations on NEV, the source grounding resistance was assumed zero. Test results for three values of  $R_g$  are illustrated in figs.6 and 7. The neutral current is rapidly decreasing as  $R_g$  increases. So, the THD of NEV that it is mainly a function of the third current harmonic will be decreased.

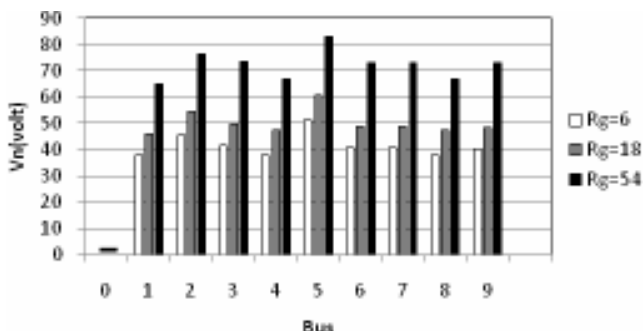


Fig. 6. NEV variation for various  $R_g$

In Fig.6, it can be seen that NEV increases as  $R_g$  increases, because the current flows through the earth decreases and, therefore, the return current and drop voltage in neutral wire is increased.

### C. Number of grounding rods per length unit

In this part, the resistance of source grounding is set on zero and the resistance of rods is taken as 18 ohms. The results of three simulations are shown in figs.8 and 9.

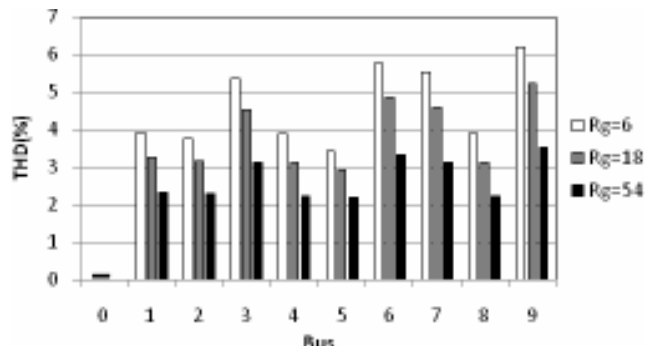


Fig. 7. THD of neutral voltage with various  $R_g$

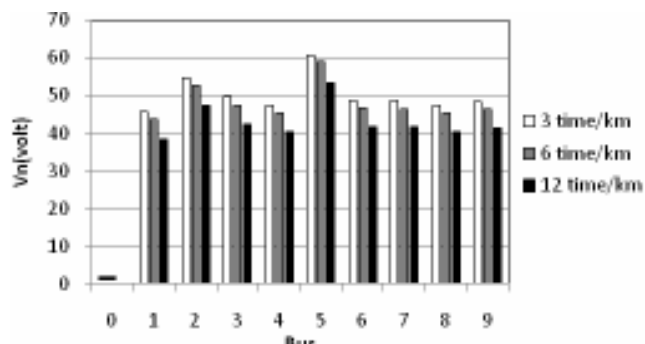


Fig. 8. NEV variation for various numbers of rods per length unit

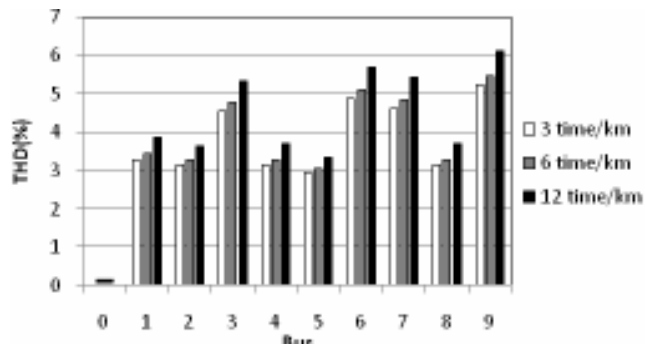


Fig. 9. THD of neutral voltage for various numbers of rods per length unit

It can be seen, when the number of  $R_g$  increases, or in other words a better connection of neutral wire to earth, generally, there is a trend of decreasing in NEV. The variation of NEV depends on the resistance of grounding rods, where the larger  $R_g$ s cause smaller variation in NEV (not shown).

### D. Effect of source grounding resistance

The resistance-to-earth at the source of a distribution feeder ( $R_n$ ) can have a significant effect upon the level of neutral-to-earth voltage for source. The results show that an increase in source resistance will result in neutral-to-earth voltage increase at source (Fig.10).

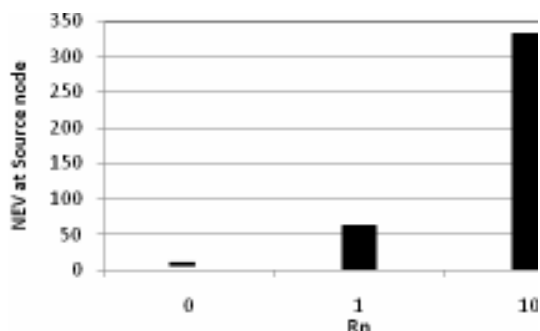


Fig. 10. Neutral voltage of source with different  $R_n$

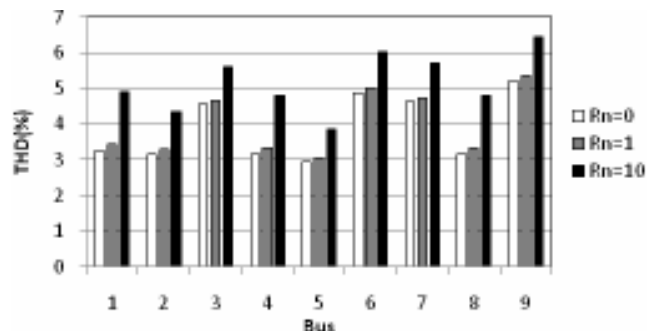


Fig. 13. THD of neutral voltage with different  $R_n$

Increasing of source neutral grounding leads to the decrease in the neutral current, especially at the fundamental frequency. Therefore, the fundamental frequency NEV decreases at source and its THD increases (Fig.11).

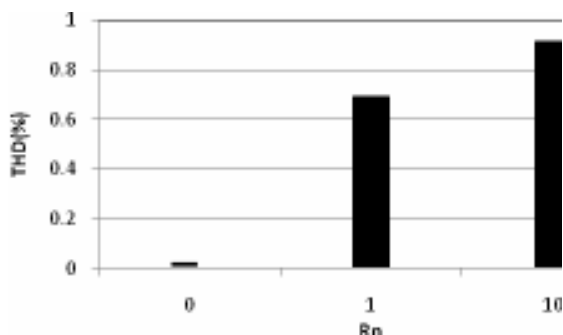


Fig. 11. THD of neutral voltage of source with different  $R_n$

By increasing  $R_n$ , the resistance of neutral network (neutral and ground resistance) increases. So, the current through neutral and ground wires decreases, which may cause in decreasing the NEV and increasing the voltage THD in other buses (figs.12 and 13).

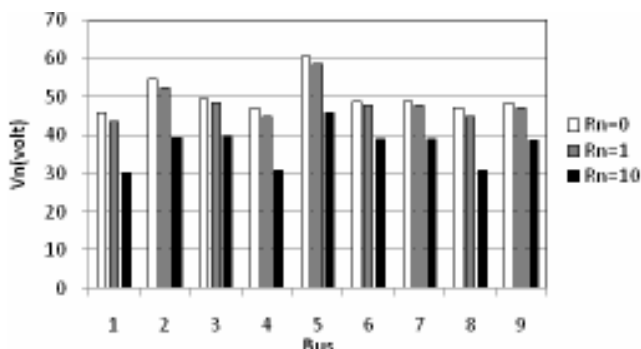


Fig. 12. NEV with different  $R_n$

## VI. CONCLUSION

In this paper, a power flow method for harmonic polluted radial distribution system is presented, which is an improvement in the method of [1]. In this improvement the distribution feeder is modeled by a five wire model to reveal the profile of neutral to earth voltages. The proposed power flow method is composed from fundamental frequency and harmonic frequencies power flows that the harmonic power flow finishes at single iteration. The magnitude of NEV and its THD are investigated under various conditions of the number of grounding rods per feeder lengths, the grounding rods resistance the grounding resistance of the in feeding source and the harmonic injection of nonlinear loads and the results are given.

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