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Data Envelopment Analysis under Uncertainty and Risk

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Abstract—Data Envelopment Analysis (DEA) is one of the most widely used technique for evaluating the relative efficiency of a set of homogeneous decision making units. Traditionally, it assumes that input and output variables are known in advance, ignoring the critical issue of data uncertainty. In this paper, we deal with the problem of efficiency evaluation under uncertain conditions by adopting the general framework of the stochastic programming. We assume that output parameters are represented by discretely distributed random variables and we propose two different models defined according to a neutral and risk-averse perspective. The models have been validated by considering a real case study concerning the evaluation of the technical efficiency of a sample of individual firms operating in the Italian leather manufacturing industry. Our findings show the validity of the proposed approach as ex-ante evaluation technique by providing the decision maker with useful insights depending on his risk aversion degree.

Keywords—DEA, Stochastic Programming, Ex-ante evaluation technique, Conditional Value at Risk.

I. Introduction

DATA Envelopment Analysis (DEA) is a well known technique introduced in [1] for evaluating the relative efficiency of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. Its widespread use in production contexts mainly comes from its appealing nonparametric nature that avoids the need of functional specifications of the production technology. A major weakness of the conventional DEA models is the implicit assumption of perfect information of input and/or output parameters. It is widely recognized that in real settings input and output data may be subject to significant uncertainty deriving from different sources (e.g. random noise, outliers observations, external effects). Indeed, the whole surrounding economic system has a stochastic nature. As a consequence, ignoring data uncertainty might lead to biased or wrong evaluations. This consideration becomes even more relevant when DEA is used as predicting performance evaluation technique. In this context, we mention the critical issues of the credit risk evaluation and bankrupt assessment which represent one of the most modern areas of application of the DEA technique (see for example, [2], [3] and the references therein).

Acknowledging the need of explicitly assessing uncertainty in the DEA approach, a considerable effort has been devoted by the scientific community in the direction of deriving suitable extensions of the classical deterministic models. More specifically, different stochastic formulations have been derived by adopting the paradigm of chance constraints [4]. According to this paradigm, random input and/or output parameters are represented as random variables defined on a given probability space and the stochastic constraints are required to be satisfied with a given reliability level. Land, Lovell and Thore analyzed in [5] the case of deterministic inputs and random outputs and proposed a stochastic DEA formulation where chance constraints are individually imposed on each random output so to guarantee that the best practice outputs are not exceeded with high probability. We also mention the contribution of Olesen and Petersen who proposed in [6] a formulation where separate chance constraints are imposed individually on each DMU. We observe that the two formulations mentioned above might be considered complementary since they are defined by imposing separate chance constraints on the primal and on the dual form of the classical DEA formulation, respectively. Later on, Olesen combined the two approaches and investigated in [7] the effectiveness of their integration. Additional aspects of the stochastic DEA approach have been also investigated in [8], [9], [10], to name a few. In all the aforementioned contributions, the random variables representing the stochastic input and/or output parameters are assumed to have a continuous nature. Under the assumption of normal distribution function a deterministic reformulation of the problem can be obtained through the introduction of nonlinear constraints (see, for example, [5]).

Extension of the paradigm of separate chance constraints to the joint case have been recently proposed in [11]. This more general framework allows to simultaneously deal with inter and intra DMU correlation. Under the assumption of discrete random variables, the authors derived a deterministic reformulation which was successfully applied to the breast screening evaluation activity. In this paper we propose a new stochastic DEA model defined within the stochastic programming framework (see, for example, [12], and the references therein). As common in many stochastic formulations, we assume that inputs are deterministic whereas outputs are uncertain. In particular, we suppose that output variables are represented by discretely distributed random variables. We observe that discrete distributions arise very often in real applications, either directly or as empirical approximations of the continuous ones derived, for example, by taking a Monte Carlo sample from a general distribution.

The proposed stochastic formulation has been derived from the classical Charnes, Cooper and Rodhes (CCR, for short) deterministic model [1]. The relative evaluation accounts also for all the unknown circumstances implied by random fluctuations of output levels. Under this respect, the new DEA model should provide the decision maker with recommendations less biased by random effects.

The basic model is further refined to explicitly assess risk measured in terms of Conditional Value at Risk (see, for example, [13]). Such a measure allows to focus on a specified size (quantile) of the worst case performance that a given

DMU can achieve for a prescribed reliability value. The proposed models have been validated by considering a real case study related to a sample of firms operating in the Italian leather manufacturing industry.

The rest of the paper is organized as follows. Section 2 introduces the proposed stochastic programming formulation in its basic form. Risk is explicitly assessed in Section 3 by a refined formulation including CVaR constraints. Extensive numerical results are presented and analyzed in Section 4. Some concluding remarks are reported in Section 5.

II. The Stochastic DEA Model

Let us denote by K the set of DMUs to evaluate. We assume that each DMU $k \in K$ is characterized by a set of m inputs and n outputs. In particular, we denote by x_{ik}, with $i = 1, \ldots, m$, the i-th input level and by y_{jk}, with $j = 1, \ldots, n$ the j-th output level of DMU k, $\forall k \in K$. Assuming that input and output parameters are known, the relative efficiency of a given DMU k can be measured by the ratio:

$$E^k = \frac{\sum_{j=1}^{n} y_{jk} t_j}{\sum_{i=1}^{m} x_{ik} w_i}$$

where t_j and w_i denote the output and input weights, respectively. Thus, the relative efficiency of each DMU $k \in K$ can be obtained by solving a deterministic DEA model that in its CCR version can be formulated as:

$$\max \sum_{j=1}^{n} y_{jk} t_j$$

$$\sum_{i=1}^{m} x_{ik} w_i = 1$$

$$\sum_{j=1}^{n} y_{jl} t_j - \sum_{i=1}^{m} x_{il} w_i \leq 0 \quad \forall l \in K$$

$$w_i \geq \delta \quad i = 1, \ldots, m$$

$$t_j \geq \delta \quad j = 1, \ldots, n$$

where $\delta > 0$ is a non-Archimedean positive number introduced to force all the output and input levels to be taken into account.

In what follows, we relax the hypothesis of perfect information on model parameters and we assume that uncertainty affects output levels, whereas inputs are considered deterministic. Under this assumption the problem of determining the relative efficiency becomes an optimization problem under uncertainty. To deal with this more complex class of mathematical programming problems, we adopt the stochastic programming (SP, for short) framework where uncertain parameters are represented in probabilistic terms by random variables defined on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Roughly speaking, the aim of the stochastic formulation is to determine an optimal solution able to properly hedge (according to the different paradigms defined within the SP framework) against all the situations that may occur in the future.

In our model we assume that the random variables associated with the uncertain outputs follow a discrete distribution and we denote by y^s_{jk} the s-th realization (scenario) of the j-th output of DMU k. We denote by $\mathcal{S}$ the scenario set and by p_s the probability of occurrence of scenario $s \in \mathcal{S}$.

By borrowing the classical notation of the recourse paradigm, we partition the decision variables of the DEA model into two different subsets: scenario-invariant variables (first-stage variables) are related to the input weights, whereas scenario-dependent variables (second-stage variables) are associated with the output weights. Thus, the stochastic DEA model can be formulated as follows:

$$\max \sum_{s \in \mathcal{S}} p_s \sum_{j=1}^{n} y^s_{jk} t^s_j \tag{1}$$

$$\sum_{i=1}^{m} x_{ik} w_i = 1 \tag{2}$$

$$\sum_{j=1}^{n} y^s_{jl} t^s_j - \sum_{i=1}^{m} x_{il} w_i \leq 0 \quad \forall l \in K \quad \forall s \in \mathcal{S} \tag{3}$$

$$w_i \geq \delta \quad i = 1, \ldots, m \tag{4}$$

$$t^s_j \geq \delta \quad j = 1, \ldots, n \quad \forall s \in \mathcal{S} \tag{5}$$

where t^s_j denote the weight associated with the output j under scenario s. Model (1)-(5) aims at maximizing the expected efficiency of the DMU k under investigation by properly selecting the input weights so to hedge against all the future unknown circumstances produced by the random output levels, allowing, at the same time, to account for specific scenario effects by second stage variables. Besides providing an efficiency score for each DMU, the model solution also suggests the input radial reduction required to improve the efficient level of inefficient DMUs. We stress that during period of crisis efficiency improvement by operating on the input side may represent the most practicable alternative since outputs are seldom controllable because influenced by external factors (e.g. market conditions, product demands). Under this respect, the proposed SP model should provide the decision maker with reliable recommendations which are less biased by the unknown situations that may occur in the future.

In evaluating the DMU performance, the decision maker can take different positions toward risk. The expected value introduced in (1) mathematically translates a risk neutral position. Such a choice may result unwise in periods of crisis when changes in market conditions may lead to severe losses. To address this issue, in the next Section we explicitly assess risk from a modelling side.

III. Risk assessment

Risk measurement and management both represent a very critical issue in changing uncertain environments. In effect, whatever the applicative context, reliable solutions should explicitly account for possible negative outcomes which may produce undesired effects or might lead to significant losses.

Since the seminal Markowitz's contribution in the field of the financial optimization [14], a growing attention has been devoted by the scientific community toward the definition of suitable risk measures. For long time, the term risk has been associated with the statistical measure of variance. Although this measure is easy to compute, simple counterexamples might show its fallacy in discriminating over-performance against under-performance, leading to incorrect dominance relations.

With the aim of overcoming this main drawback, several risk measures have been proposed in the scientific literature during the last decades. Among the others, we mention the Conditional Value at Risk (CVaR) which, as demonstrated by the results presented in [15], allows a more accurate measure of tails of the density function. In addition, CVaR is a "choerent" measure in the terminology of Artzner et al. [16] and enjoys nice computational properties (see [17]). In our approach we shall consider CVaR as "safety" measure. Typically, a decision maker may be concerned about the worst case performance that a given DMU can achieve under specific scenarios. Eventually, he may wish to determine the worst case scenario, even though this information is well known to be over-conservative, especially if the worst situation occurs with very low probability. Much useful information could be gained by focusing on the mean of a specified size (quantile) of worst realizations, i.e. on the CVaR.

From a mathematical standpoint, CVaR represents the normalized second quantile function defined, for a given nonnegative random variable ξ and a tolerance level $\beta \in (0, 1]$, as:

$$F_\xi^{-2}(\beta) = \frac{1}{\beta}\int_0^\beta F_\xi^{-1}(\alpha)d(\alpha)$$

where $F_\xi^{-1}(\alpha) = inf\{\eta : F_\xi(\eta) \geq \alpha\}$ is the left-continuous inverse of the cumulative distribution function of the random variable ξ. In the case of discrete random variables, CVaR admits a linear reformulation ([18]) and the risk DEA model is obtained by replacing the objective function (1) with:

$$\max \eta - \frac{1}{\beta}\sum_{s\in\mathcal{S}} p_s d_s \tag{6}$$

and by adding to basic model the CVaR constraints:

$$\eta - \sum_{j=1}^{n} y_{jl}^s t_j^s \leq d_s \qquad \forall s \in \mathcal{S} \tag{7}$$

$$\eta \geq 0, \quad d_s \geq 0 \qquad \forall s \in \mathcal{S} \tag{8}$$

where η represents the β-quantile, while variables d_s measure, under each scenario s, the deviation of DMU efficiency from η provided that η is greater.

The explicit risk assessment in the DEA model provides the decision maker with a powerful and reliable tool to evaluate firm efficiency under uncertainty. By varying the required reliability level β, the decision maker may obtain a spectrum of possible efficiency scores to systematically use in the evaluation process.

IV. Numerical illustration

This section reports on the computational experiments carried out to validate the proposed stochastic models. They have been applied to evaluate ex-ante the efficiency of a set of individual firms operating in the Italian leather manufacturing industry. In what follows, we first introduce the considered case study and then we present and discuss the numerical results. The proposed models have been implemented in the General Algebraic System Gams[1] and solved by using ILOG CPLEX 10.1[2]. The computational experiments have been carried out on a computer equipped with an Intel Core 2 Duo CPU at 2.00 GHz and 3 GByte RAM.

A. Case Study

The computational experiments have been carried out by considering actual data related to a sample of 20 Italian firms. The proposed stochastic models have been applied with the aim of determining an efficiency level that the decision maker might wish to use as ex-ante indicator of the future firm performance in a credit risk evaluation perspective. It is widely accepted that the higher is the efficiency level, the more healthy is the firm under evaluation and the lower the risk to default. The literature on the application of the DEA approach in credit risk evaluation is rich. We remind the interested readers to the recent contribution [3] and to the references therein. In the present paper, we use this meaningful case study for illustrative purpose, but we underline that the proposed models have a general validity and may be applied to other different interesting contexts.

In our models we have used two input and two output variables. According to our approach, the indicators selected as inputs have to be minimized whereas those chosen as outputs have to be maximized. More specifically, on the input side we have considered liabilities and average duration of accounts receivable. The former indicator represents the total liabilities of the firm, whereas the latter is defined as the amount of accounts receivable divided by the average daily sales and gives the average extension of payments received from customers. As far as output parameters are concerned, we have considered earnings before interest, taxes, depreciation and amortization and cash flow. The first parameter concerns the ability to make profit by the firm at an operating level, whereas the second one is the simplest form of cash flow. All the data have been derived from the Balance Sheet written according to the Italian Civil Code. Numerical results have been collected by considering historical data available for the time horizon $2001 - 2009$. For the considered firms all the parameters take nonnegative values. In our experiments, all but the last year output data have been elaborated to derive statistical measures used in the scenario generation, leaving year 2009 as benchmark for validating the proposed models.

B. Scenario generation

Preliminary to the solution of the proposed stochastic models, is the scenario generation phase. As mentioned above, in our study we have assumed that input parameters are known in advance, whereas output values are uncertain. This is a reasonable assumption since typically input data refer to the firm status eventually defined on the basis of a prescribed management policy, whereas outputs are generally influenced by external factors (i.e. sales, production costs) which are out of firm control. We observe that scenario generation represents a very critical issue in any optimization problem under

[1] www.gams.com.
[2] www.ilog.com/products/cplex.

uncertainty. The literature on scenario generation is rich and different techniques have been proposed over the past decades. They range from sampling methods (e.g. importance sampling, bootstrapping), to simulation (as the classical Brownian motion and its variants belong to this class), from statistical methods (such as principal component analysis technique, regression methods, moment matching) to other methods (e.g. clustering approaches, neural networks). Interested readers are referred to [19] and references therein. The choice of the most suitable technique is problem dependent. In this paper, scenarios have been generated by adapting to the one period case the general integrated approach proposed in [19]. Roughly speaking, such an approach can be seen as a variant of the well-known moment matching technique defined with the aim of overcoming the computational difficulties related to the solution of nonlinear problems. More specifically, the basic idea is to integrate simulation and optimization techniques. First, a simulation technique is used to determine the realizations of the uncertain parameters. They provide the input data of an optimization model defined with the aim of minimizing, by the optimal choice of the probability levels, the distance between the moments associated with the generated scenarios and some target moments defined by the end-user on the basis of the historical data and/or his preferences. In our experiments, we have adopted a Monte Carlo simulation technique for generating random parameters. More specifically, on the basis of the historical data, we have computed, for each firm in the sample, the yearly relative changes for the years 2001-2008. Such data have been used to determine the weighted averages and variances: lower weights have been associated with older informations so to reflect the lower impact determined by less recent observations. The statistical analysis has further showed that, for a given output, the yearly relative changes of the different DMUs have a low correlation level. On the contrary, for a given DMU the two output parameters show an average correlation level equal to 0.78, with a standard deviation of 0.25.

Starting from the known output values y_{jk} referring to the year 2008, scenarios for the incoming year 2009 $\tilde{y_{jk}}$ have been generated by considering a random change ξ_{jk} assumed to be normally distributed with known expected value μ_{jk} and standard deviation σ_{jk} (used data are reported in [20]). By taking into account, for each DMU k, correlation ρ_k between the two output parameters, the values of the random changes have been determined by using the coefficients of the the classical Cholesky matrix, as:

$$\xi_{1k} = \mu_{1k} + \sigma_{1k}\epsilon_1$$
$$\xi_{2k} = \mu_{2k} + \sigma_{2k}(\rho_k\epsilon_1 + \sqrt{1-\rho_k^2}\epsilon_2)$$

where ϵ_1 and ϵ_2 are standard random variables.

Different test cases with an increasing number of scenarios have been generated by considering a different number of extractions from the standard normal distribution. In particular, we have randomly generated instances with 100, 200 and 500 scenarios, respectively.

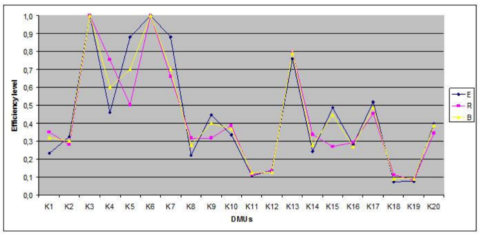

Fig. 1. Deterministic vs basic model: comparison with the real efficiency levels

C. Results and discussion

With the aim of validating the proposed SP models and assessing their performance, different computational experiments have been carried out.

The first set of experiments has been aimed at evaluating the so called ‘in sample” stability [21]. This analysis allows the decision maker to measure how sensible is the model solution with respect to scenario samples used. To this aim, different scenario sets with the same cardinality have been generated and the corresponding instances have been solved. The numerical results have been collected by considering, for each test case, ten different instances. The analysis of the results have empirically shown that the solutions are quite stable. For example, for the instances with 100 scenarios, the average DMU standard deviation from the mean value is around 0.10. We observe that this value tends to slightly decrease when the number of scenarios is increased. In what follows, for all the test cases we shall present the averaged DMU performances.

A second set of experiments has been carried out to compare the performance of the basic model with its deterministic counterpart obtained by replacing the random variables with their expected value computed on the basis of the generated scenarios. Evaluation has been performed by an “out of sample” analysis. To this aim, we have considered as benchmark the actual efficiency scores obtained by running the deterministic DEA model on the last year of the historical data set. The following Figure 1 compares the efficiency levels obtained in the three cases (model (1)-(5) has been solved with 500 scenarios). More specifically, B refers to efficiency levels obtained by solving model (1)-(5), E refers to the levels obtained by replacing in the deterministic CCR output data with their expected values and R reports the actual efficiency levels observed for the year 2009. As evident, the basic model outperforms the expected value model, confirming the high quality of the efficiency evaluation process thanks to the explicit assessment of uncertainty in the model parameters.

A deeper analysis has been carried out by running the basic SP model as function of the scenario set cardinality. The following Figure 2 compares the results with the real efficiency levels.

Accuracy of our model have been measured in terms of absolute percentage errors averaged on all the DMUs. Such a value is typically limited, being around 6.73%, 5.84% and 4.8% for the test cases with 100, 200 and 500 scenarios, re-

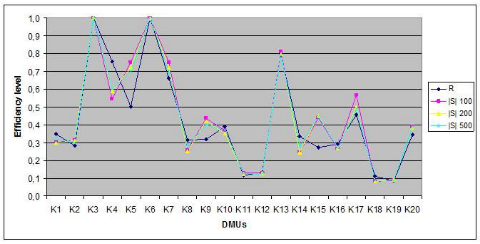

Fig. 2. Comparison of efficiency level as function of the scenario number

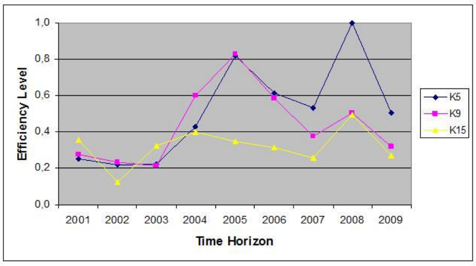

Fig. 3. Historical efficiency level of critical DMUs

TABLE I
η VALUES AS FUNCTION OF THE RELIABILITY LEVEL

DMU	0.85	0.90	0.95
K1	0.128	0.159	0.182
K2	0.192	0.224	0.259
K3	1	1	1
K4	0.236	0.267	0.285
K5	0.490	0.502	0.502
K6	1	1	1
K7	0.589	0.608	0.615
K8	0.124	0.144	0.155
K9	0.300	0.317	0.317
K10	0.205	0.247	0.262
K11	0.065	0.079	0.088
K12	0.090	0.103	0.109
K13	0.534	0.602	0.666
K14	0.041	0.070	0.097
K15	0.265	0.272	0.272
K16	0.166	0.193	0.215
K17	0.442	0.448	0.451
K18	0.042	0.048	0.053
K19	0.050	0.059	0.067
K20	0.301	0.335	0.342

spectively, thus slightly decreasing as the number of scenarios is increased. The results obtained for some specific DMUs, namely $K5$ $K9$ and $K15$, deserve some special attention. For such DMUs the efficiency level provided by the model solution is below the actual one of about $15-20\%$. In effect, the analysis of the efficiency scores (see Figure 3), computed on the basis of historical data available, shows that such DMUs exhibit a wavering behavior with high variation of the efficiency level over the time horizon and a sudden decrease in 2009. High volatility in market conditions calls for a morjuhjuhe appropriate risk control strategy trying to prevent undesired evaluation errors. This consideration has aimed our next set of computational experiments, carried out by running the risk based model for different values of the reliability level β. The solution of this model provides the decision maker with important information. First of all, the model gives the efficiency level η that with a given value β will be maintained also in averse conditions. Roughly speaking, with a probability of β the efficiency score will be greater than η and, as a consequence, with a probability of $(1-\beta)$ the efficiency will be lower than this bound. The following Table reports the η values collected by considering the case of 500 scenarios and three different β values.

Furthermore, the model solution allows to determine the expected value of the efficiency score below the η value. The richness of information provided by the risk based model can further support the decision maker in the difficult process of evaluation under uncertainty. As confirmed by the results, higher probability values lead to less risky, but more conservative, solutions. The proper calibration of the reliability value is up to decision maker who could find the best trade-off on the basis of his experience, taking also into account the past history of the specific DMU under investigation. Eventually, the decision maker might decide to tune the reliability value depending on DMUs specific characteristics. For example, the choice of high reliability value for the critical DMU might drastically reduce (or even eliminate) the evaluation error.

V. Concluding Remarks

Conventional DEA models rely on the assumption of perfect information of input and output parameters. In this paper we overcome this major weakness by proposing a new DEA model relying on the Stochastic Programming paradigm. The model, in its basic form, aims at maximizing the relative expected efficiency level by properly selecting weights to account for the unknown circumstances implied by random fluctuations of output levels. The basic formulation is further refined to explicitly account for risk measured in terms of Conditional Value at Risk. More specifically, the risk based model provides the decision maker with a tool to control a specified size (quantile) of worst performance realizations. The two stochastic programming formulations have been validated by considering a meaningful case study related to the evaluation of the future performance of a sample of firms operating in the Italian leather manufacturing industry. Extensive computational experiments have been carried by an "ot of sample" simulation. Our findings have shown the efficacy of the proposed models also as ex-ante evaluation technique.

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