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# A Frequency Dependence of the Phase Field Model in Laminar Boundary Layer with Periodic Perturbations

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***Abstract***—The frequency dependence of the phase field model(PFM) is studied. A simple PFM is proposed, and is tested in a laminar boundary layer. The Blasius's laminar boundary layer solution on a flat plate is used for the flow pattern, and several frequencies are imposed on the PFM, and the decay times of the interfaces are obtained. The computations were conducted for three cases: 1) no-flow, and 2) a half ball on the laminar boundary layer, 3) a line of mass sources in the laminar boundary layer. The computations show the decay time becomes shorter as the frequency goes larger, and also show that it is sensitive to both background disturbances and surface tension parameters. It is concluded that the proposed simple PFM can describe the properties of decay process, and could give the fundamentals for the decay of the interface in turbulent flows.

***Keywords***—Phase field model, two phase flows, Laminar boundary Layer



## I. Introduction

THE research aims to describe a complex multi-phase flow with the phase field model (PFM) in boundary layer since the boundary layer flow appears in the most common in fluid machinery(Ref.[1],[2],[3]). The traditional computational methods of interfaces yield very complicate computational steps to crate the shape of the interface and its locations. In the phase filed model, however, the situation is different, the system of the differential equations is complicate, but the computation step is simpler. It includes several additional pseudo physical parameters and functions which also need to be adjusted. Thus, the PFM requires a lot of effort to finalize the parameters. Most methods can be used only in the early stage of mixing of the multi-phase flows, and the interface problem in developed turbulence could not be applied. In the present work the objective is to find a method for the interface problem for flows from the laminar to turbulence somehow. Since the PFM has a generalized representation, this method is extended to disturbed flows which are similar to the intermittent region. The responses with several frequencies on the PFM are tested, and it is observed how the decay time of the interface changes in disturbed flows.

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## II. Preliminaries

As the phase field model is treated in chemical engineering, thermo-mechanics, fluid mechanics, the equation of the PFM needs to be defined clearly at the beginning. An order parameter Φ is used to indicate the state of fluid like a state variable in thermodynamics. The phase field model can be defined as the following way.

### *A. Def. Diffusion function ζ*

It is assumed that there is an analytic function ζ such that ζ is a function of the order parameter Φ that has at least three domains

$$\zeta<0 \quad [\Phi1,\Phi2] \tag{1}$$

$$\zeta>0 \quad [\Phi2,\Phi3] \tag{2}$$

$$\zeta<0 \quad [\Phi3,\Phi4] \tag{3}$$

where the order parameter Φ varies in the closed domain [0.2,0.8], and where the order parameter Φ is a spatial function (x,y).

### ***B.*** *Def. The Chemical Potential* Ψ

By integration of ζ with respect to Φ the chemical potential Ψ is defined. The relationship between two variables is shown as

$$\zeta(\phi) = \frac{\partial}{\partial\phi}\Psi \tag{4}$$

### *C. Example of the diffusion function*

If ζ depends on the temperature then the representation is ζ(T, Φ). A typical representation is the Van-der-Walls equation (Ref. [1]) shown below,

$$\zeta(T,\phi) = \frac{\partial}{\partial\phi}\Psi = -2a\phi + \frac{T}{(1-b\phi)^2}, \tag{5}$$

or can be any analytic function which satisfies the definition in A, and can be expanded as

$$\zeta(T,\phi) = \frac{\partial}{\partial\phi}\Psi = a\phi^n + b\phi^{n-1} + c\phi^{n-2} + d\phi^{n-3} \tag{6}$$

The order parameter Φ satisfies the following equation for no-convective flows

$$\frac{\partial\phi}{\partial t} = \Gamma_0(\phi_{,i}\varsigma - k_2\phi_{,iqq})_{,i} \tag{7}$$

where $\Gamma_0$and ck2 are constants.

*D. Existence of a steady solution in the phase field equation*

In a steady state the equation (7) must satisfy the following condition:

$$\frac{\partial\phi}{\partial t} = \Gamma_0(\phi_{,i}\varsigma - k_2\phi_{,iqq})_{,i} = 0 \tag{8}$$

By integration, the Gauss ‘s theorem yields

$$\iint q_i ds_i = Source \tag{9}$$

where the flux q defined as

$$q_i = \phi_{,i}\varsigma - k_2\phi_{,iqq} \tag{10}$$

To preserve the system, a source term is necessary. This means that it is necessary to have a source at the boundary or somewhere in the computation domain to preserve the interface.

## III. Bench Mark Test for Phase Field Model :

*A. Test of sensitivity of Temperature*

Two phase field models are tested. The first model is

$$\zeta(T_n,\phi) = -2\phi + \frac{T_n}{(1-\phi)^2} , \tag{11}$$

where the temperature is normalized Ref.[1]. In Fig.1 four curves are shown with equation(11) in different normalized temperature. For the temperature Tn=0.195, the order parameter is split into two domains [Φ:<0.1 (ζ: positive)] and [0.6>Φ>0.1 where ζ is negative. Since it means that at the interface the contraction force happens, the interface forms steeper for Φ > 0.1 in the most area. The first model is computed for the two dimensional problem with the following condition:

BC: [Φ =0.02 at x=0, any y]
[Φ =0.8 at x=1(I=50) for any y ]

The computational results with different temperatures are shown in Figs.2, 3, 4, and 5. There, they show that the lowest temperature gives the steepest slope.

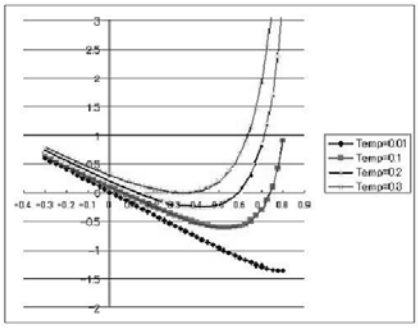


Fig.1 An example of diffusion function

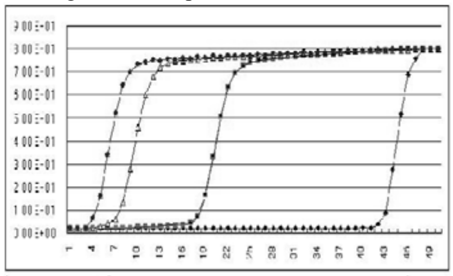


Fig.2 Time variation of Φ in different (Tn=0.193)

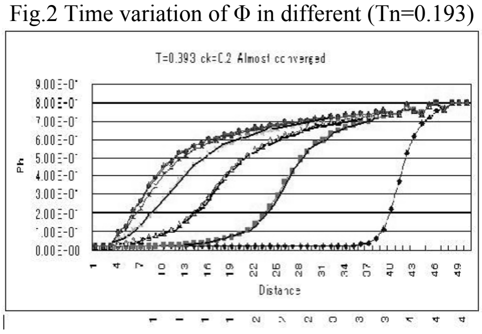


Fig. 3(Tn=0.243)

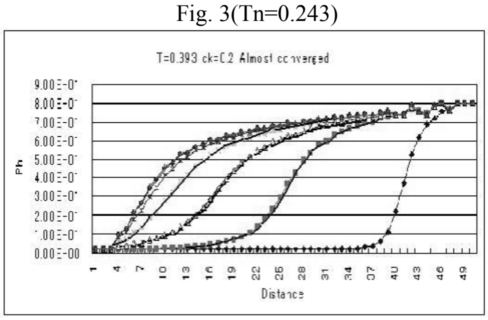


Fig.4(Tn=0.393)

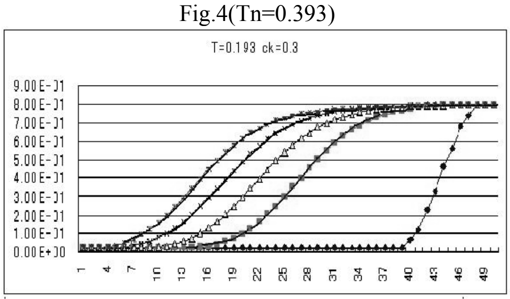


Fig.5 (Tn=0.593)

*B. Sensitivity test of surface tension Ck2*

The parameters Ck2 of 0.003, 0.3 are used instead of using Ck2=0.02. The results shows that Ck=0.3 gives wider interfaces than Ck=0.02. The computation conditions are following:

Boundary Conditions

1)X=Domain Ω [0,1]

Φ [0.02, 0.8]

X =0 at Φ=0.02, and X =1 at Φ=0.8

2) Initial Condition

Φ=0.2, in all domain Ω

3) Temperature(normalized)

T=[0.193, 0.243, 0.393,0.593]

Results are following:

1) For all temperatures the interfaces are created. The steepness is stronger as the temperature is lower.

2) For small Ck=0.002, the computation is unstable

3) For a large Ck=0.3, the slope is moderate.

The computations were performed with homogeneous temperature distribution.

### *C. Geometrical Change*

Several geometrical interfaces can be considered.

An example is shown in Fig.6 where there are two interfaces with three different temperatures. The interfaces are vanished at the end of the computation.

*1) Example1:*

The computation conditions are following.

T=0.193 at x=0(negative diffusivity);

T=0.243 at middle part (negative and positive diffusivity);

T=0.593 at x=1(positive diffusivity),

i.e. two thermal interfaces are given as

X=Domain Ω [0,1], Φ [0.02,0.8]

X =0, Φ=0.8, and X=1 , Φ=0.8.

Initial Conditions are Φ=0.2, in all domain Ω.

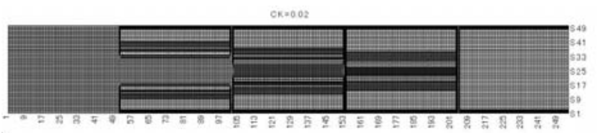


Fig.6 Time change of order parameter

*2) Example2:*

The temperature distribution with two interfaces is changed as follows:

T=[0.193 at x=0]

T=[0.353 at middle part](always positive diffusivity)

T=[0.593 at x=1](always positive diffusivity),

and ,

X=Domain Ω{0,1}

Φ[0.02,0.8]

X =0, Φ=0.8, and X =1 , Φ=0.8

Initial Condition [Φ=0.2, in all domain Ω]

The result is shown in Fig. 7.

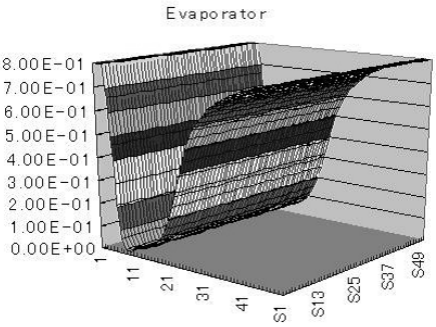



Fig. 7 Computed Order parameter

*3) Example3:*

More sever temperature conditions are tested. The results are shown in Fig. 8. There, it showed a very sustainable interface at T=0.1. The Interface is steep but can be preserved. It means that the liquid is preserved at the end of the computation.

*4) Example4:*

In the comparison of the initial distribution with the stage of the converged order parameter a sustained interfaces were obtained when T=0.1 in which has both negative and positive diffusivity.

*5) Example5:*

A cold cylinder is tested. The initial order parameter is shown Fig.9. The shape of the initial cylinder is preserved, and it was adjusted to the boundary condition at the end of the computation in Fig.10

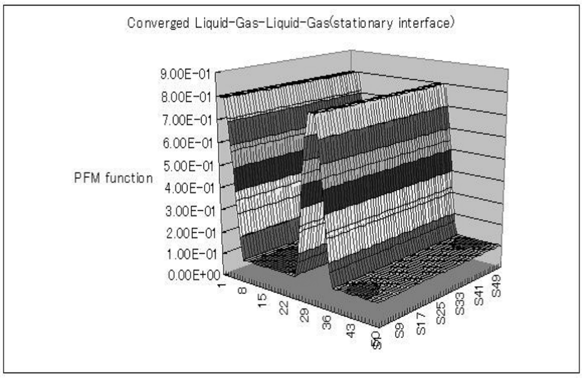



Fig.8 Final results of the order parameter

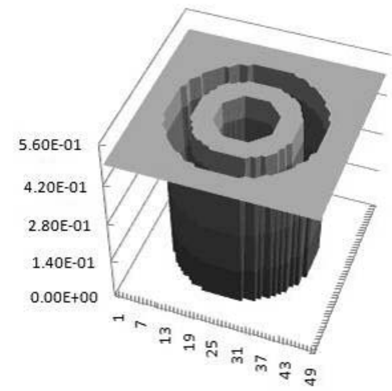



Fig.9 Initial PHM order parameter

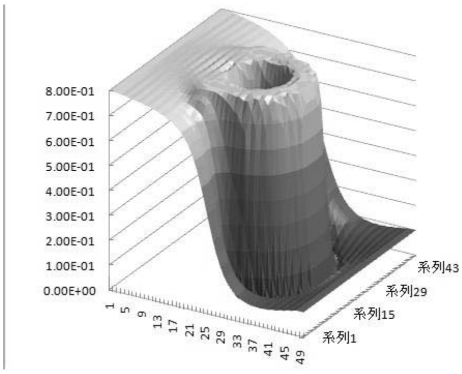



Fig.10 Order parameter at the final stage

*D.Time Variable boundary conditions*

The change of the order-parameters at the boundaries shows how the interface will attain the boundary values. The normalized temperature T is set to 0.2 since 0.1<T< 0.27 where there is a negative diffusion, so the interface will be created near the temperature at T=0.2. From the result at 2000 iteration, Φ is changed from 0.02 to 0.15 shown in Fig 11,12,13,14. It can be inferred that the width of Φ increased, and the Φ at the higher temperature increased. This may be interpreted as the evaporation through the interface.

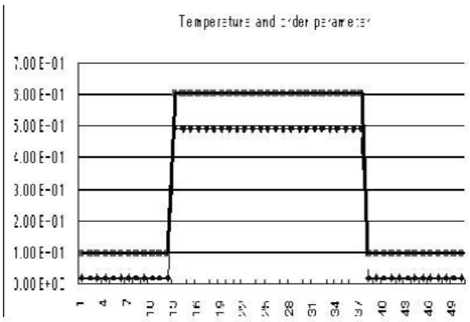


Fig.11 Time sequence of order parameter

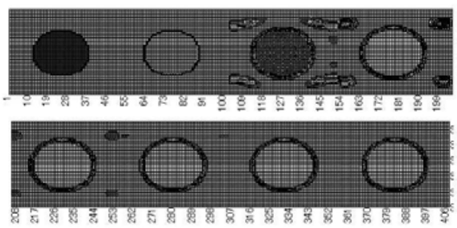


FIg.12. Converged order parameter, there is an interface surface.

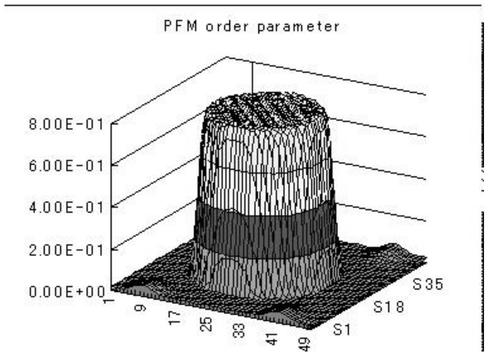



Fig.13 ck=0.03 Initial Order parameter

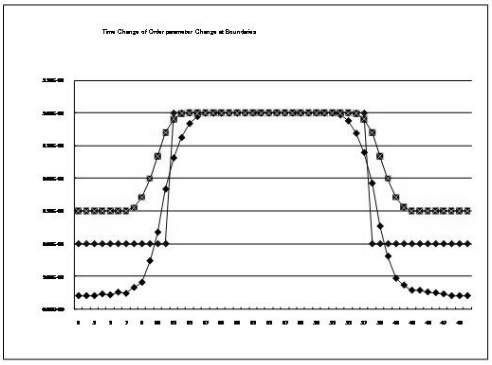


Fig.14 Movement of the boundary conditions

## IV. A SIMPLER DIFFUSION MODEL

It is valuable to test a simpler form only works for a small interval near the certain temperature. That is usual situations when liquids are below the boiling point. The variation of $\zeta$ is shown in Fig.15 and the equation of the diffusion model is as follows:

$$\xi(T,\phi) = cc(\phi - 0.4)^2 - 0.1(1 - \frac{T - T_0}{T_{\max} - T_0}) \tag{12}$$

$$\xi_2(T,\phi) = 10.52(\phi - 0.4)^2 - 0.1(1 - \frac{T - T_0}{T_{\max} - T_0}) \tag{13}$$

where T0=293K and Tmax=373K, cc is an adjusting constant. The decay processes are shown in Fig. 16, Fig. 17, and Fig. 18. For small cc=0.002 the interface was not preserved. However, for greater than cc=2.0 the interface was preserved as shown in Figs.19,20,21,22 , and 23. It is interesting to see Fig.24 where the interface was gone at 9th Images. Though the temperature is recovered, once the interface was vanished the interface does not recover since there is no source terms in the equation.

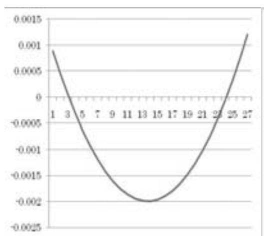


Fig.15 $\xi_2(T,\phi)$

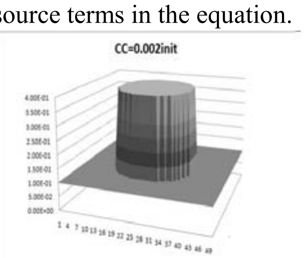



Fig.16 Initial condition

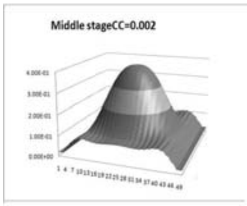



Fig.17 Middle stage

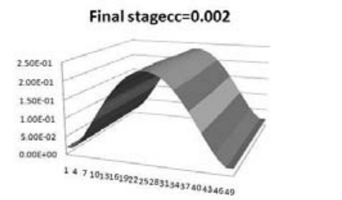



Fig.18. Decay of the initial I interface

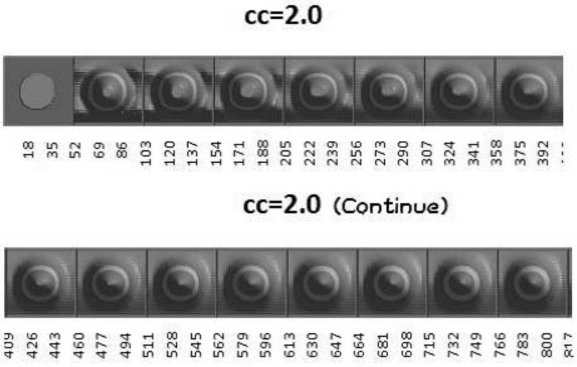
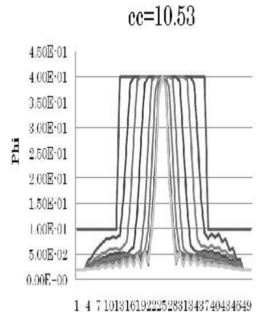



Fig.19 Preservation of shape



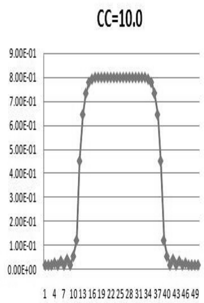




Fig. 20 preservation of shape Fig.21 preservation of shape

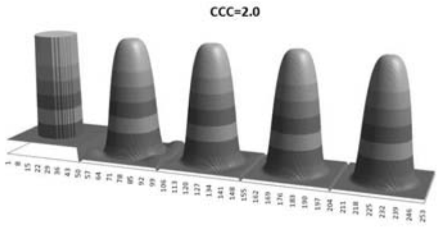



Fig 22 example of preservation of shape

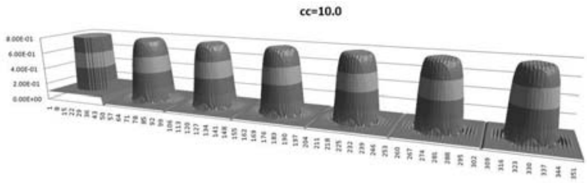



Fig.23 example of preservation of shape

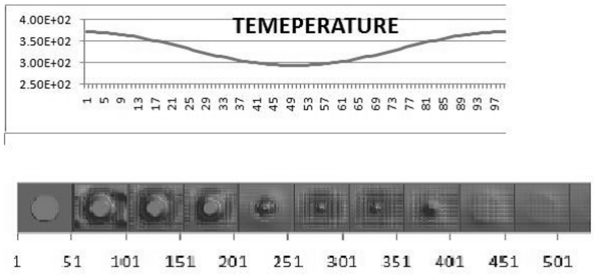



Fig.24 Vanishing the interface

## V. PFM IN PERTURBED FLOWS

The PFM equation can be decomposed as the following form with the perturbation part and mean part as

$$\phi = \hat{\phi} + \phi' \tag{14}$$

then, the PFM equation with no-flows becomes as

$$\frac{\partial \hat{\phi}}{\partial t} = \Gamma_0(\hat{\phi}_{,i}\hat{\varsigma})_{,i} - \Gamma_0 k_2(\hat{\phi}\hat{\phi}_{,qq,i})_{,i} + \Gamma_0(\overline{\phi'_{,i}\varsigma'})_{,i} - \Gamma_0 k_2(\overline{\phi'\phi'_{,qq}})_{,i} \tag{15}$$

The perturbed part can be expanded as

$$F_T = \Gamma_0(\overline{\phi'_{,i}\varsigma'})_{,i} - \Gamma_0 k_2(D^{ii}{}_{jj} + \overline{\phi_{ii}\phi_{jj}} - \frac{1}{2}(\overline{\phi\phi})_{,iijj}) \tag{16}$$

where $D_{ij}$ represent the correlation of the perturbed density gradient or order parameter gradient Ref[6]. Since the perturbed flows are diffusive, Fr must be negative all the time, so the equation must satisfies the following inequality.

$$(\overline{\phi'_{,i}\varsigma'})_{,i} < k_2(D^{ii}{}_{jj} + \overline{\phi'_{ii}\phi'_{jj}} - \frac{1}{2}(\overline{\phi'\phi'})_{,iijj}) \tag{17}$$

## VI. SIMULATION OF PERTURBED FLOW AT REST

To see the essence of the frequency response, first a no-flow case is tested. The computational conditions are as follows: Two fluids are in the computational domain. The outer boundary condition of Φ is set by a lighter gas. Temperature is constant, so the diffusion function depends only on the order parameter. The background perturbation level of the equation (16) is taken as 0.0001 to 0.03. The initial condition of Φ =0.01. The relaxing time $T_R$ can be obtained by changing the perturbation frequency of zero cycle, 2500cycle , and 5000cycle.

## VII. SIMULATION OF A BALL AND SOURCE IN A LAMINAR-BOUNDARY-LAYER ON A FLAT PLATE WITH PERTURBATION

If several equations are necessary to perform the computation if the computation were done exactly, our interest is to find the response property of the PFM, thus the Blasius's solution is used for the mean flows. Besides, for only the computation of the order parameter Φ the TVD scheme is used in Ref.[6],[7]. In the analysis, the effect of the pressure tensor to the flow is supposed to be neglected in the present model.

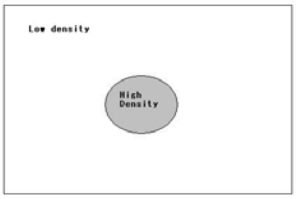



Fig.25 Configuration for No-flow

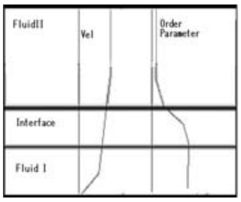



Fig.26 Configuration for Laminar Boundary Layer

## VIII. Results of Simulations

### A. A ball of water at rest

For the at-Rest case shown in Fig.27 the decay time(iteration) is shown iteration =150 with 5000Cyclic with the noise (perturbed) level=0.0150. In Fig.27, the upper line shows the decay time of no-cyclic-time, the second line shows in 5000 cyclic perturbation, the bottom line shows in 10000cyclic perturbation. As the interval of the perturbation increases, the decay time does. Besides, the noise level increases, the decay time also does. Typical variations of decay are shown in Figs.28, 29, and 30.

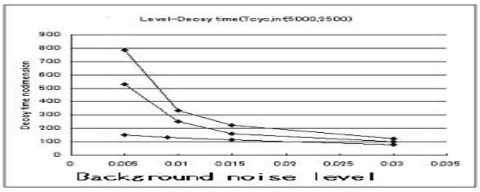



Fig.27 Decay Time (Ck2, Noise level, Decay time)

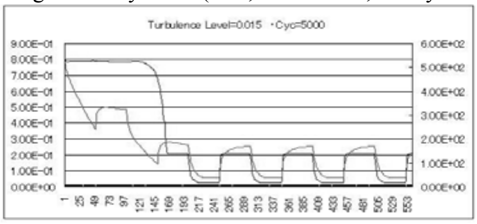



Fig.28 Typical simulation the interface decays

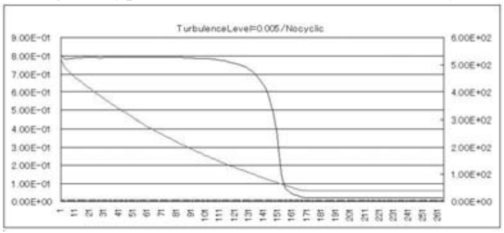



Fig.29 Typical simulation without forced perturbation

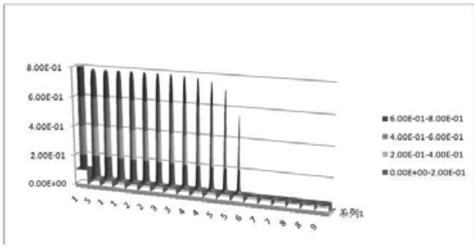


Fig.30 Time sequence of the simulation the last part

### B. A ball in a laminar boundary layer

With the parameter obtained in at-Rest problem, it can be used them for the laminar boundary layer problem. The results in the problem of a ball-in-boundary-layer on a flat plate are shown in Figs. 31, 32, 33, and 34. The ball was drifting and vanished in the 5th image. For a line source of mass in the two dimensional boundary layer, the results are shown in Figs.35, 36 where the interface was created in the boundary layer clearly. For all computations for the boundary layer the parameters Ck is 0.02 and back ground noise level is 0.015 with the cyclic 5000Cyclic. The computations were stable for all cases.

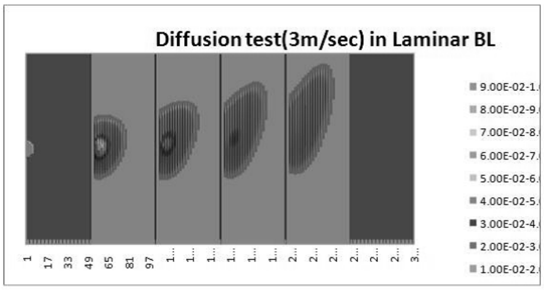



Fig.31Drifting a ball on the LBL

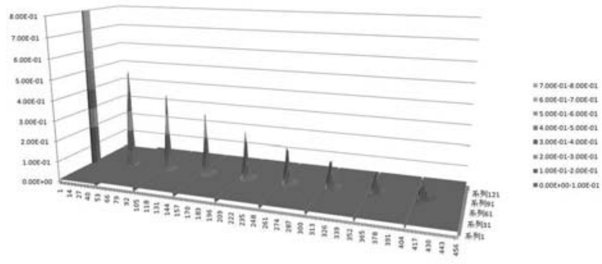


Fig.32 LEVEL 0.001-0.01（No-effect on the diffusion by turbulence）

Fig.33 Laminar velocity on a flat plate

Fig.34 Turbulent Flow Patterns (TBL=0.001)

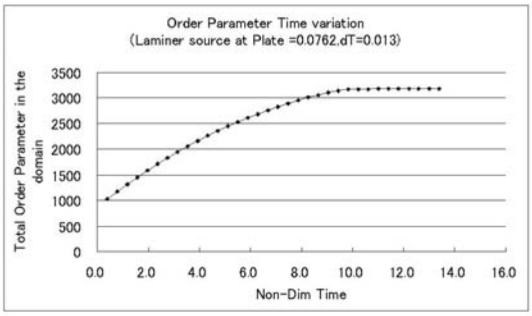



Fig.35Mass or order parameter flux becomes constant

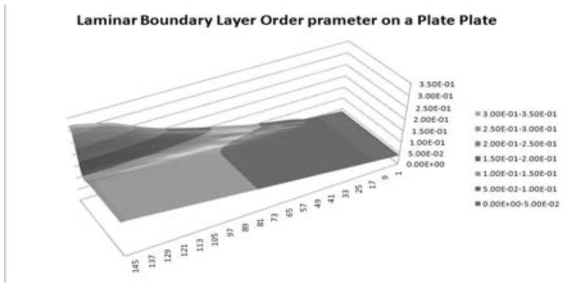



Fig. 36 Surface formation on a flat plate with a line source

## IX. Conclusion

The proposed simpler form of the diffusion function $\zeta$ can well form the interfaces, and it can describe the decay rate qualitatively. This will be a practical model to simulate the physical properties since the multiphase problems contains so many parameters, and the simpler form will reduce the cost for the further application. Since the ultimate goal of the present study is to extend the possibility to use the PFM for the compressible flows which contains the correlation of the density gradient fluctuation, the stable computations encourage using the present model to the future problems.

## Acknowledgment

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