Mathematical Approach for Large Deformation Analysis of the Stiffened Coupled Shear Walls

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Abstract—Shear walls are used in most of the tall buildings for carrying the lateral load. When openings for doors or windows are necessary to be existed in the shear walls, a special type of the shear walls is used called "coupled shear walls" which in some cases is stiffened by specific beams and so, called "stiffened coupled shear walls". In this paper, a mathematical method for geometrically nonlinear analysis of the stiffened coupled shear walls has been presented. Then, a suitable formulation for determining the critical load of the stiffened coupled shear walls under gravity force has been proposed. The governing differential equations for equilibrium and deformation of the stiffened coupled shear walls have been obtained by setting up the equilibrium equations and the moment-curvature relationships for each wall. Because of the complexity of the differential equation, the energy method has been adopted for approximate solution of the equations.

Keywords—Buckling load, differential equation, energy method, geometrically nonlinear analysis, mathematical method, Stiffened coupled shear walls.

I. INTRODUCTION

In high-rise buildings, providing enough resistance and stiffness to withstand lateral forces caused by wind and earthquake is of special importance. Using shear walls is one of the methods of providing stiffness. Creating openings in a vertical row breaks up the shear wall into two or more parallel walls. Such shear walls are called coupled shear walls. The existence of the connecting beams increases the lateral stiffness and decreases the stresses in the wall. As the stiffening beams are usually arranged regularly in the height of the building, considering the regular geometry and the number of the stories, continuous medium method has been used for the analysis of such walls. By this method modeling the behavior of the structure with linear differential equations will be possible which will lead to a closed form solution. Hence, the analysis of the coupled shear walls with constant specifications throughout the height leads to the solution of a linear differential equation with constant coefficients and the exact solution of the governing equation has been computed.

II. THE EQUILIBRIUM EQUATIONS IN THE DIFFERENTIAL FORM

A typical stiffened coupled shear wall is shown in Fig. 1. In this figure, the stiffening beam has been placed at the height of $H_t$ and the other specifications and parameters of
the wall have also been shown. The connecting beams are replaced with a broad continuous medium by using continuous connection medium shown in Fig. 2. Assuming that the inflection points of the connecting beams are in the mid-span and as the vertical displacement of the connecting beam in the middle is zero, the governing differential equations and the axial force in the walls are developed in reference [6]. The moment-curvature relationships for the stiffened coupled shear walls are as follows, [6].

\[
EI \frac{d^4y}{dx^4} = M_e - L \int_a^b q_a \, dz + 2 \int_a^b P_a(\eta_a - y_a) \, dz \\
hs \leq x \leq H
\]

\[
EI \frac{d^4y}{dx^4} = M_e - L \left( \int_a^b q_a \, dz + \int_a^b P_a(\eta_a - y_a) \, dz + Q_b \right) \\
0 \leq x \leq \hs
\]

\[
2 \left[ \int_a^b P_a(\eta_a - y_a) \, dz + \int_a^b P_b(\eta_b - y_b) \, dz \right]
\]

\( P_a \) and \( P_b \) are the uniform gravitational load on the top and bottom of the stiffening beam, respectively, \( Q_b \) is the shear force in the stiffening beam, \( M_e \) is the moment caused by the external loading which calculated having uniform distributed load \( (a) \), triangular distributed load with maximum amount \( (w) \) at top of the structure and concentrated load \( (p) \) at top of the structure and is obtained as follows:

\[
M_e = \frac{\mu}{2} (H - x)^2 + \frac{w}{6H} (2H^2 - 3H^2x + x^2) + p(H - x) \]

\[
T_{a1} = \int_a^b \frac{p_a}{x} \, dz + 2 \int_a^b \frac{P_a(\eta_a - y_a)}{L} \, dz - \frac{EI}{L} \frac{d^2 y_a}{dx^2} \frac{Me}{L} \]

\[
T_{a2} = \int_a^b \frac{p_a}{x} \, dz + 2 \int_a^b \frac{P_a(\eta_a - y_a)}{L} \, dz - \frac{EI}{L} \frac{d^2 y_a}{dx^2} \frac{M_a}{L} \]

\[
T_{b1} = \int_a^b \frac{p_a}{x} \, dz + 2 \int_a^b \frac{P_a(\eta_a - y_a)}{L} \, dz + \int_a^b \frac{P_b(\eta_b - y_b)}{L} \, dz
\]

\[
\frac{EI}{L} \frac{d^2 y_b}{dx^2} + \frac{Me}{L}
\]

\[
T_{b2} = \int_a^b \frac{p_a}{x} \, dz + 2 \int_a^b \frac{P_a(\eta_a - y_a)}{L} \, dz + \int_a^b \frac{P_b(\eta_b - y_b)}{L} \, dz
\]

\[
\frac{EI}{L} \frac{d^2 y_b}{dx^2} + \frac{Me}{L}
\]
III. THE DISPLACEMENT EQUATIONS IN THE DIFFERENTIAL FORM

The displacement equation in the differential form for the top of the stiffening beam for the case that the cross sectional areas of two walls are equal, \((A_1 = A_2)\), can be obtained as follows, [6]:

\[
\frac{d^3 y_a}{dx^3} - \frac{12I_b}{hb^3l} \left[ L^2 + \frac{2I}{A_1} - \frac{hh^3}{6E_l} P_s (H-x) \right] \frac{d^3 y_a}{dx^3} - \frac{4P_s}{EI} \frac{d^3 y_a}{dx^3} = 0
\]

The displacement equation in the differential form for the bottom of the stiffening beam for the case that the cross sectional areas of two walls are equal, \((A_1 = A_2)\), can be obtained as follows, [6]:

\[
\frac{d^3 y_b}{dx^3} - \frac{12I_b}{hb^3l} \left[ L^2 + \frac{2I}{A_1} - \frac{hh^3}{6E_l} P_s (h - x) \right] \frac{d^3 y_b}{dx^3} - \frac{4P_s}{EI} \frac{d^3 y_b}{dx^3} = 0
\]

IV. SOLVING THE EQUATIONS

In order to find the displacements and the stresses in the...
stiffened coupled shear walls through nonlinear analysis explained in the former sections, the equations (8) and (9) must be solved. Because of the complexity of these equations, the exact solution for them has not been found yet. An approximate solution of them using energy method can be found in reference [6].

V. CONCLUSION

In the method proposed in this paper, the geometrically nonlinear analysis of stiffened coupled shear walls has been discussed and a suitable formulation has been suggested for buckling load of the stiffened coupled shear walls under gravitational loading. The effects of axial load and stiffening beam on the behavior of the stiffened coupled shear walls, and also the effects of the axial load on the lateral displacements have been accounted for. By setting up the equilibrium equations and moment-curvature relationships for each wall, and eliminating the laminar shear from the relationships, the governing equation of the stiffened coupled shear walls displacement has been obtained. The equations are very complex and no exact solution has been found for them yet. For Approximate solution, the energy method can be used by choosing a suitable shape function which satisfies the boundary conditions. Then, the total potential energy can be calculated and by minimizing it in terms of the unknown coefficients, the stiffened coupled shear walls deformation equation can be obtained. At the end, the uniform distributed critical load for the stiffened coupled shear walls can be computed by equating the equation coefficients determinant to zero. The explanation of the approximate solution and several numerical examples can be found in reference [6].

REFERENCES