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Bending Gradient Coefficient Correction for I-Beams

H.R. Kazemi Nia[1], A. Yeganeh Fallah[2]
[1] AmirKabir University of Technology , Tehran, Iran
Email: Civil_Kazemi@aut.ac.ir
[2] AmirKabir University of Technology , Tehran, Iran
Email: Yeganeh_arash@aut.ac.ir

Abstract**—**Without uncertainty by applying external loads on beams, bending is created. The created bending in I-beams, puts one of the flanges in tension and the other one in compression. With increasing of bending, compression flange buckled and beam in out of its plane direction twisted, this twisting well-known as Lateral Torsional Buckling. Providing bending moment varieties along the beam, the critical moment is greater than the case its under pure bending. In other words, the value of bending gradient coefficient is always greater than unite. In this article by the use of " ANSYS 10.0" software near 80 3-D finite element models developed for the propose of analyzing beams` lateral torsional buckling and surveying influence of slenderness on beams' bending gradient coefficient. Results show that, presented Cb coefficient via AISC is not correct for some of beams and value of this coefficient is smaller than what proposed by AISC. Therefore instead of using a constant Cb for each case of loading , a function with two criterion for calculation of Cb coefficient for some cases is proposed.

Keywords**—**Beams critical moment, Bending Gradient Coefficient, finite element, Lateral Torsional Buckling

I. INTRODUCTION

I- BEAMS subjected to flexure have much greater strength and stiffness in the plane in which the loads are applied (major axis) than in the plane of minor axis. Unless these member are properly braced against lateral deflection and twisting, they are subjected to failure by lateral torsional buckling prior to attainment of their full in-plane capacity. Lateral torsional buckling is a limit state of structural usefulness where the deformation of a beam changes from predominantly in-plane deflection to a combination of lateral deflection and twisting while the load capacity remains at first constant, before dropping off due to large deflection and yielding. Since beam buckling subjected to pure bending is most critical state and its calculating is easier, AISC consider pure bending for calculating beams' buckling moment. The buckling moment of a I-beam in the case of pure bending is given by (1). [5,3]

$$(M_0)_{cr} = \sqrt{\frac{\pi^4}{L^4} EC_w EI_y + \frac{\pi^2}{L^2} EI_y GJ} \qquad (1)$$

"L" is the unbraced length which is the span length in models. Equation (1) is the result of equilibrium equation which is linear differential equation with constant coefficient of an I-beam with slightly deformation from its primary state. Equation (1) is derived for a beam with equal moments in its two ends, which deformation has one curvature and shear doesn't generate in beam. But in practical condition, beams subjected to various loading and so they are subjected to none uniform bending moment along their length. In such cases, differential equation governs on buckling manners of beam, has nonlinear coefficient, and therefore has no accurate analytical solution. Solving of such equations and calculating critical loads is possible with numerical methods.

Bending moment changing influence on critical moment can easily apply with considering bending gradient coefficient C_b. So critical moment in the case of none uniform moment is achieved by equation (2):

$$M_{cr} = C_b.(M_0)_{cr} \qquad (2)$$

From equation (2) it can be found, bending gradient coefficient is ratio of critical moment in case of none uniform moment to uniform case of it. AISC for calculating bending gradient coefficient propose equation (3):

$$C_b = 1.75 + 1.05\frac{M_1}{M_2} + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3 \qquad (3)$$

It can be seen that AISC's purposed equation for calculating bending gradient coefficient only depends on beam's loading. The purpose of this paper is to investigate the influence of bending moment changing and I-beams slenderness on lateral torsional bending moment. For this purpose by using of ANSYS software some 3-D finite element models with wide slenderness were developed.

II. FINITE ELEMENT MODEL

For the purpose of I-beams' lateral torsional buckling behavior investigation, elastic linear finite element models with ANSYS software were made. In this software, quadrilateral shell element "shell93" was used for modeling web and flanges. “Fig. 1,” Flanges were modeled with four elements in width. Yang's module "E" and Poason's ratio are assumed 210GPa and 0.3 respectively. Residual stress was not considered in this work.

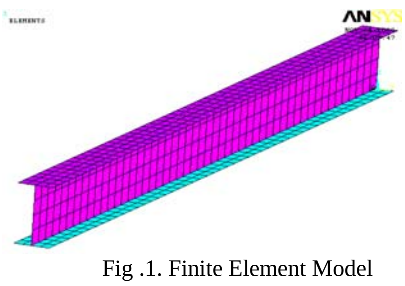

Fig .1. Finite Element Model

III. Geometry, Loading and Restraints

For purpose of calculating I-beams' critical moment and bending gradient coefficient, simply supported beams with IPE140, IPE180, IPE220, IPE300 and span of 1, 2, 3, 5, 10 meters, subjected to four loading cases ($M_1/M_2=-1$, $M_1/M_2=1$, $M_1/M_2=-0.25$, $M_1/M_2=-0.5$ that minus shows creation of simple curvature in beams) are considered. It is not possible for beams to twist in their two ends but warping in their sections is possible. It should be mentioned that in beams' finite element model subjected to ending moments, for preventing web local yielding, moments were applied as couples of uniform pressure to the top and bottom flanges. "Fig. 2."

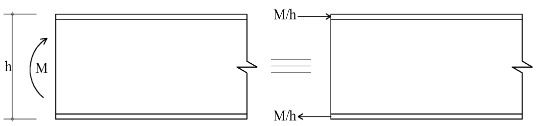

Fig. 2. applying ending moments

IV. Validation of Modeling Technique

In this part we intend to evaluate the accuracy of the finite-element models, four sample of models with length of approximate 2 meters were chosen and the results of these finite element models were compared with the results of theoretical relations proposed by Timoshenko in his book: (moments unit are KN.m)

TABLE I
Comparision Of theoretical And Finite Element Methods

	Length(m)	Theory	F.E.M	Error
IPE140	2.00	27.10	25.60	5%
IPE180	2.00	64.50	61.56	4%
IPE220	2.00	132.71	126.62	4%
IPE300	2.00	528.81	512.19	3%

The differences between finite element methods and theory method can be considered of approximations and simplifying are existed in theory method. Since in this research ratio of moments for calculating bending gradient is used therefore the actual error is smaller than 2%. So models are acceptable and results are valid.

V. Analysis (Eigen Buckling) I-beams

After surveying accuracy of presented finite element models in this research, for investigating influence of slenderness coefficient in value of C_b coefficient, some I-beam models are analyzed. In these analysis's, in addition to loading case of pure bending ($M_1 = -M_2$), three other loading cases ($\frac{M_1}{M_2}=-1$, $\frac{M_1}{M_2}=1$, $\frac{M_1}{M_2}=-0.25$, $\frac{M_1}{M_2}=-0.5$ that minus shows creation of simple curvature in beams) are considered. "Fig. 3."

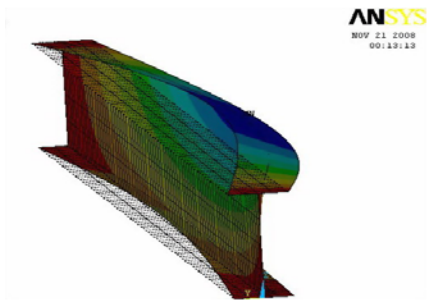

Fig. 3. Lateral Torsional Buckling of beam after applying ending Moment

Simply supported I-beams with different length and sections (IPE140, IPE180, IPE220, IPE300) were analyzed. Beams critical moment under pure bending load case and also under first, second, third load cases are showed respectively in table II,III,IV,V.

TABLE II
Pure Bending Load Case Critical Moment ($M_1=-M_2$)

Length(m)	IPE140	IPE180	IPE220	IPE300
1 m	57.42	147.40	386.32	1091.93
2 m	25.60	61.56	126.62	512.19
3 m	14.57	32.66	70.94	262.82
5 m	8.29	17.83	35.16	104.35
10 m	3.93	8.15	16.00	43.05

TABLE III
Pure Bending Load Case Critical Moment ($\frac{M_1}{M_2}=1$)

Length(m)	IPE140	IPE180	IPE220	IPE300
1 m	121.51	294.99	542.15	915.53
2 m	67.61	156.66	305.31	912.74
3 m	38.91	86.99	185.47	616.77
5 m	21.91	47.54	94.06	277.07
10 m	10.18	21.29	42.14	114.98

TABLE IV
Pure Bending Load Case Critical Moment ($\frac{M_1}{M_2}=-0.5$)

Length(m)	IPE140	IPE180	IPE220	IPE300
1 m	75.21	193.40	501.28	1155.02
2 m	33.76	81.15	166.79	670.09
3 m	19.19	43.05	93.53	346.08
5 m	10.90	23.47	46.32	137.61
10 m	5.16	10.71	21.03	56.67

TABLE IV
PURE BENDING LOAD CASE CRITICAL MOMENT ($\frac{M_1}{M_2}=-0.25$)

Length(m)	IPE140	IPE180	IPE220	IPE300
1 m	86.98	224.37	568.39	1174.95
2 m	39.54	95.05	195.06	772.74
3 m	22.45	50.43	109.59	404.43
5 m	12.71	27.42	54.20	161.33
10 m	5.99	12.46	24.50	66.22

VI. BENDING GRADIENT COEFFICIENT (C_B)

Bending gradient coefficient C_b, is equal to the ratio of each load case critical moment to pure bending critical moment of beam. Cb coefficient proposed by AISC for first, second and third load case are respectively 2.3, 1.3, 1.51. C_b's value for each load case drew in figures "Fig. 4,5,6". Also AISC's C_b are drew in these diagrams for comparing.

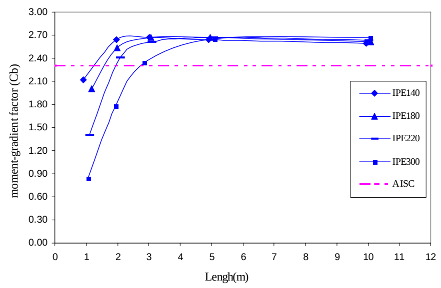

Fig .5. Bending Gradient Coefficient For First L. C.

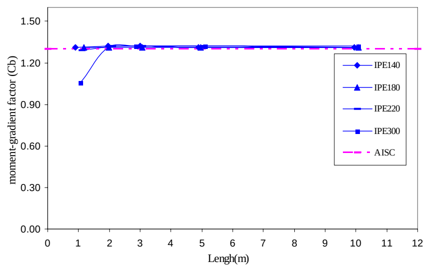

Fig .6. Bending Gradient Coefficient For Second L. C.

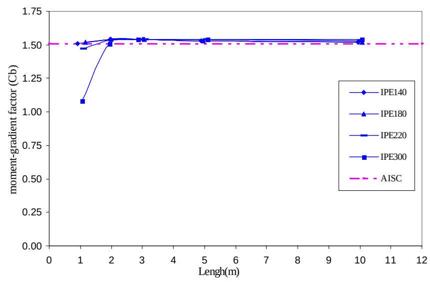

Fig .7. Bending Gradient Coefficient For Third L. C.

It can be seen that, for medium and long length, bending gradient coefficient are equal to AISC's proposed coefficient, but for short length specially IPE300 this coefficient is smaller than AISC's purposed coefficient. Therefore C_b for first load case instead of having constant value of 2.3 it is adjusted into the (4) relation:

$$C_b = \begin{cases} 0.85 & L \le 2.80 \\ 2.30 & L > 2.80 \end{cases} \tag{4}$$

Also C_b for second load case instead of having constant value of 1.3 it is adjusted into the (5) relation:

$$C_b = \begin{cases} 1.00 & L \le 2.00 \\ 1.30 & L > 2.00 \end{cases} \tag{5}$$

And finally C_b for third load case instead of having constant value of 1.51 it is adjusted into (6) relation:

$$C_b = \begin{cases} 1.00 & L \le 2.00 \\ 1.51 & L > 2.00 \end{cases} \tag{6}$$

VII . CONCLUSION

In this research Lateral Torsional Buckling of simply supported I-beams subjected to ending moments in wide changes of slenderness is investigated by finite element methods. In such beams, slenderness has considerable influence on bending gradient coefficient. In other word values of C_b coefficient is not constant versus all quantities of slenderness. While AISC for all quantities of slenderness purpose constant C_b coefficient. Therefore it can be concluded that value of bending gradient coefficient doesn't only depend on loading it also depends on beams' slenderness (length, moment of inertia). What ever beam's has shorter length and larger moment of inertia the difference between AISC's bending gradient coefficient and its actual value is larger. Hence for beams with short span, value of AISC's bending gradient coefficient is larger than result of finite element method and may be used of larger bending gradient coefficient causes an unsafe design.

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