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Tuning a Fractional Order PID Controller with Lead Compensator in Frequency Domain

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***Abstract*—**To achieve the desired specifications of gain and phase margins for plants with time-delay that stabilized with FO-PID controller a lead compensator is designed. At first the range of controlled system stability based on stability boundary criteria is determined. Using stability boundary locus method in frequency domain the fractional order controller parameters are tuned and then with drawing bode diagram in frequency domain accessing to desired gain and phase margin are shown. Numerical examples are given to illustrate the shapes of the stabilizing region and to show the design procedure.

***Keywords*—**Fractional controller, Lead compensator, Stability regions, Stability boundary locus

I.INTRODUCTION

IN recent years finding an increasing number of studies related with the application of fractional calculus in many areas of science and engineering (see, e.g., [1], [2]). Using the differentiation and integration of fractional order or non-integer order in systems control is gaining more and more interests from the systems control community [3].

In what concerns automatic control theory the fractional calculus concepts are adapted to frequency domain based methods. The frequency response and the transient response of the non-integer integral and its application to control systems was introduced by Manabe (see [4]) [5]. In theory, the control systems can include both the fractional order dynamic system or plant to be controlled and the fractional-order controller. However, in control practice, more common is to consider the fractional order controller.

This is due to the fact that the plant model may have already been obtained **as** an integer order model in classical sense and the objective is to apply the fractional order control to enhance the system control performance [3]. Generally, fractional order controllers divided into 4 classes. TID[1] controller, CRONE controller[2], $PI^{\lambda}D^{\mu}$ controller and fractional lead-lag compensator [3]. The purpose of fractional order controller design is determined controller parameters so that the closed-loop system was stable and has optimal performance. There is different ways to designing this class of controllers in the both of time domain and frequency domain that each has own advantages and disadvantages.

For instance in [6] a fractional differentiator by using system linearization in frequency domain for chaos control is designed. In [7] differ-integrators are designed in the time domain with using least square method. In [8] based on definition of piecewise orthogonal functions the parameters for fractional order PID controller are determined. In [9] applying an improved differential evolution method a $PI^{\lambda}D^{\mu}$ controller is designed using discretization. In [10]-[15] fractional order PI, PD and PID controllers are designed using gain margin, cross over frequency and phase margin criteria in frequency domain for integer and fractional order systems. Here, the goal of this paper is using stability boundary locus method to design a class of fractional order PID controller with three adjustable parameters in frequency domain. This method has three conditions in frequency domain to design fractional order controllers and so for a controller with three adjustable parameters can be used well. On the other hand for better analysis and design of fractional order controllers a gain-phase margin tester can be used. Based on *definition 2.3* in [16] this tester can be thought of as a "virtual compensator".

In industrial process control applications, phase-lead/lag compensators are widely used next only to PID controllers [17]. Such controllers are tuned usually with specifications on gain and phase margins which can lead to good performance and robustness [18]. In the tuning of phase lead/lag compensators, knowledge of specific points on the frequency response of the plant are required. Such points are specialized by their frequency, gain and phase and are not readily available without an accurate model of the plant [19]

To the best knowledge, there is no method available to achieve gain and phase margins exactly. Note also that phase-lead compensators have some parameters in the denominator of its transfer function, unlike PID controllers where all the parameters appear linearly, Thus, effective techniques for PID controllers with exact gain and phase margin specifications are not applicable to phase-lead compensators [18].

The traditional tuning method for phase-lead/lag compensator parameters is based on the “trial and error” procedure Attempts towards analytical synthesis have been made since Wakeland in 1976 first proposed a one-step design for phase-lead compensators [17]. The major difficulties for tuning of phase-lead compensators under the gain and phase margin specifications lie in nonlinearity and coupling of all their three parameters [18]. Later, Yeung, Chaid, and Dinh in 1995 developed a series of Bode design charts to allow “non-trial and error” designs of both continuous-time and discrete-time compensators [20]. Wang in 2003 presented the exact

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and unique solution to the design of phase-lead and phase-lag compensation when the desired gains in magnitude and phase are known at a given frequency [21]. Recently, in [22] another tuning method for phase lead compensators was proposed which can achieve the desired gain and phase margins exactly regardless of the plant order, time delay or damping nature.

In [17] a new approach is presented to first determine the stabilizing parameter set of phase-lead/lag compensators for all pole stable plants with time-delay and then to synthesize phase-lead/lag compensators with desired gain and phase margins.

In [18] a simple and effective tuning method for phase-lead compensators which can achieve exact gain and phase margins simultaneously is presented.

Here, at first using the stability boundary locus in the frequency domain parameters of fractional order controller are set and then gain-phase margin tester provides information for plotting the boundaries of constant gain margin and phase margin in the parameter plane [16].

II. Controller Tuning Procedure

Generally an integer order integrating time delay system described by the following dynamic equation:

$$G(s)=\frac{K}{a_n s^n+\ldots+a_1 s+a_0}e^{-\theta s}=\frac{K}{\sum_{i=0}^{n}a_i s^i}e^{-\theta s} \tag{1}$$

Where K open-loop gain, $a_i,(i=0,\ldots,n)$ transfer function denominator coefficients and is θ delay for the system.

Now a class of fractional order controllers with three adjustable parameters is consider as follow:

$$C(s)=K_P+K_I s^{-\lambda}-s^{\mu} \tag{2}$$

Where in "(2)," K_P, K_I are controllers gains and $\mu,\lambda=1-\mu\in(0,1)$ are fractional order. One of the fractional order PID controllers is that they are not sensitive to the system parameters changing and due to fractional order having more flexibility in systems control [23].

As it said for better tuning of controller parameters a gain phase margin taster has been used that described as follow:

$$C_t(A,\phi)=Ae^{-j\phi} \tag{3}$$

Where A is the gain margin and ϕ is the phase margin. In order to finding controller parameters for a given value of gain margin A of the control system, one need to set $\phi=0$ in "(3)," On the other hand, setting A=1 in "(3)," one can obtain the controller parameters for a given phase margin ϕ. This tester in frequency domain is equal to a lead compensator. Generally a compensator is describing as below [24]:

$$C_t(s)=\frac{K_1(s+z)}{(s+p)} \tag{4}$$

Where K_1 is compensator gain and z, p are compensator zero and pole, respectively. So closed loop system equation is as follow:

$$y=\frac{G(s)C(s)C_t(s)}{1+G(s)C(s)C_t(s)}r \tag{5}$$

And the characteristic equation of closed loop system is as follow:

$$P(s)=1+G(s)C(s)C_t(s) \tag{6}$$

Based on "(4)," the compensator designing is related to choosing the z,p and K_1 in order to obtaining good performance. If in "(4)," $|z|<|p|$ then the compensator is a lead compensator. If the zero being very small (i.e. $|p|>>|z|$) or zero being at the origin then the compensator is equal to a derivative that described as bellow:

$$C_t(s)\approx K_1\frac{s}{p} \tag{7}$$

This derivative has the following frequency response:

$$C_t(j\omega)=jK_1\frac{\omega}{p}=K_1\frac{\omega}{p}e^{+j90^\circ} \tag{8}$$

So for "(4)," the next equation will be arrived:

$$C_t(j\omega)=\frac{K_1(j\omega+z)}{(j\omega+p)}=\frac{K_1(z/(p))[j(\omega/(z))+1]}{[j(\omega/(p))+1]}=\frac{K_2(1+j\omega\alpha\tau)}{(1+j\omega\tau)} \tag{9}$$

Where $p=\alpha z,\tau=1/p$ and $K_2=K_1/\alpha$. So the transfer function for lead compensator is as follow:

$$C_t(s)=\frac{K_1(1+\alpha\tau s)}{\alpha(1+\tau s)}=\frac{K_2(1+\alpha\tau s)}{(1+\tau s)} \tag{10}$$

The phase equation for lead compensator is as follow:

$$\phi(\omega)=\tan^{-1}\alpha\omega\tau-\tan^{-1}\omega\tau \tag{11}$$

The maximum lead occurs on ω_m that it is the through geometric of $p = 1/\tau$ and $z = 1/\alpha\tau$. it means that the maximum lead on logarithm frequency axis is in center of zero and pole frequencies, then

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}} \tag{12}$$

In order to obtaining the maximum angle the "(10)," has been rewritten as follow:

$$\phi = \tan^{-1}\frac{\alpha\omega\tau - \omega\tau}{1+(\omega\tau)^2\alpha} \tag{13}$$

With substituting $\omega_m = 1/\tau\sqrt{\alpha}$ in "(13)," the next equation is obtained:

$$\tan\phi_m = \frac{(\alpha/\sqrt{\alpha}) - (1/\sqrt{\alpha})}{1+1} = \frac{\alpha - 1}{2\sqrt{\alpha}} \tag{14}$$

For a given FOPID controller parameters K_P, K_I, λ and μ the closed-loop system is said to be bounded-input bounded-output (BIBO) stable if the quasi-polynomial $P(s)$ has no roots in the closed right-half of the s-plane (RHP). The stability domain ~~S~~ in the parameter space ~~P~~ with K_P, K_I, λ and μ being coordinates is the region that for $(K_P, K_I, \lambda, \mu) \in S$ the roots of quasi-polynomial $P(s)$ all lie in open left-half of the s-plane (LHP). The boundaries of the stability domain ~~S~~ which are described by real root boundary (RRB), infinite root boundary (IRB) and complex root boundary (CRB) can be determined by the D-decomposition method [26,27]. These boundaries are defined by the equations $P(0, L) = 0$, $P(\infty, L) = 0$ and $P(\pm j\omega, L) = 0$ for $\omega \in (0, \infty)$, respectively, where $P(s, L)$ is the characteristic function of the closed loop system and L is the vector of controller parameters. In applying the descriptions of stability boundaries of the stability domain ~~S~~ to the FOPID in "(6)," the RRB turns out to be simply a straight line given by:

$$KK_2K_P = 0 \rightarrow K_P = 0 \tag{15}$$

To construct the CRB, $j\omega = \infty$ has been substituting into "(6)," and the following equation has been obtained:

$$P(j\omega) = 1 + \frac{KK_2(1 + j\alpha\tau\omega)e^{-j\omega\theta}(K_P + K_I(j\omega)^{-\lambda} - (j\omega)^{\mu})}{(1 + j\tau\omega)(\sum_{i=1}^{n} a_i (j\omega)^i)} \tag{16}$$

For given gain and phase margin, K_P, K_I is obtained as follow:

$$K_P = \frac{-\left(\sum_{i=1}^{n} a_i c_i\right)\left(F(\omega) + \tau\omega D(\omega)\right)}{KK_2\left(A(\omega)F(\omega) - B(\omega)D(\omega)\right)} \frac{-\left(\sum_{i=1}^{n} a_i d_i\right)\left(D(\omega) - \tau\omega F(\omega)\right) + K\left(F(\omega)C(\omega) - D(\omega)E(\omega)\right)}{KK_2\left(A(\omega)F(\omega) - B(\omega)D(\omega)\right)} \tag{17}$$

$$K_I = \frac{\left(\sum_{i=1}^{n} a_i c_i\right)\left(B(\omega) + \tau\omega A(\omega)\right)}{KK_2\left(A(\omega)F(\omega) - B(\omega)D(\omega)\right)} \frac{+\left(\sum_{i=1}^{n} a_i d_i\right)\left(A(\omega) - \tau\omega B(\omega)\right) + K\left(A(\omega)E(\omega) - B(\omega)C(\omega)\right)}{KK_2\left(A(\omega)F(\omega) - B(\omega)D(\omega)\right)} \tag{18}$$

Where

$$c_i = \mathrm{Re}\{(j\omega)^i\}\ ,\quad d_i = \mathrm{Im}\{(j\omega)^i\} \tag{19}$$

$$\begin{cases} A(\omega) = \cos(\omega\theta) + \alpha\tau\omega\sin(\omega\theta) \\ B(\omega) = \sin(\omega\theta) - \alpha\tau\omega\cos(\omega\theta) \end{cases} \tag{20}$$

$$\begin{cases} C(\omega) = \omega^{\mu}(A(\omega)\cos(\frac{\mu\pi}{2}) + B(\omega)\sin(\frac{\mu\pi}{2})) \\ D(\omega) = \omega^{-\lambda}(A(\omega)\cos(\frac{\lambda\pi}{2}) - B(\omega)\sin(\frac{\lambda\pi}{2})) \\ E(\omega) = \omega^{\mu}(B(\omega)\cos(\frac{\mu\pi}{2}) - A(\omega)\sin(\frac{\mu\pi}{2})) \\ F(\omega) = \omega^{-\lambda}(B(\omega)\cos(\frac{\lambda\pi}{2}) + A(\omega)\sin(\frac{\lambda\pi}{2})) \end{cases} \tag{21}$$

In order to plotting stability boundary locus the ω should changing from 0 to ∞. For $A = 1$ and $\phi = 0$, the stability boundaries RRB, IRB and CRB which divide the parameter (K_P, K_I) into stable and unstable regions. The stable region can be found by checking one arbitrary test point within each region. The characteristic equation belonging to the stable region has no RHP roots while the characteristic equation of the unstable region has a certain number of RHP roots. For checking the stability of the fractional-order characteristic equation, an effective numerical algorithm is given in [25]. The region having the stable characteristic equation, which is called the *general stability region*, gives a set of the stabilizing K_P and K_I parameters for the fixed values of μ and λ. It is noted that different choice of μ and λ lead to different general stability regions [16]. In the next example for the given transfer functions general stability region in parameters space (K_P, K_I) is plotted and the good performance of the closed loop system that controlled with FO-PID and lead compensator is obtained.

III. ILLUSTRATIVE EXAMPLE

Example 1

The first order integrating time delay system considered as follow:

$$G(s)=\frac{K}{a_1 s}e^{-\theta s} \tag{22}$$

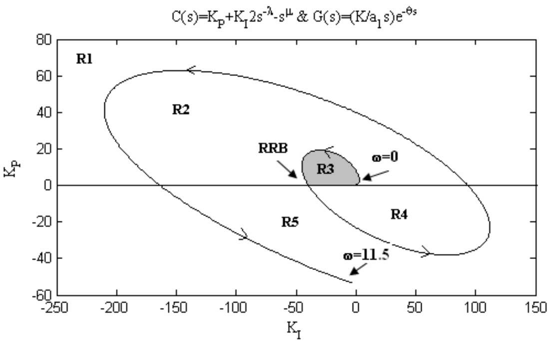

Fig. 1 Stable and unstable regions for first order time delay system without using lead compensator

Where $K=1,\ a_1=5$ and $\theta=1$. At first fractional order PID controller with three tunable parameters by using of frequency domain specification has been designed. Considering μ (and so λ) is 0.5 and ω is changing from 0 to 11.5. So the stable and unstable regions without using lead compensator are as Fig.1. It is seen that from Fig.1 the shaded region (R3) is the stability region for closed loop system. In order to better vision it is shown in Fig.2. Actually it is stability boundary region.

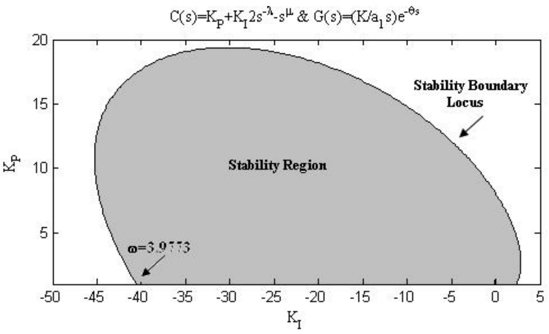

Fig. 2 Stability boundary region for first order time delay system without using lead compensator

Based on Fig.2 it is seen that the maximum of ω in the stability boundary region for closed loop system without using lead compensator is 3.9773. The accuracy of the found stability region can be easily tested using the unit step responses of the closed loop system. In this paper, fractional-order operators have been approximated by continued fraction expansion of the direct discretization by recursive Tustin transformation [10].

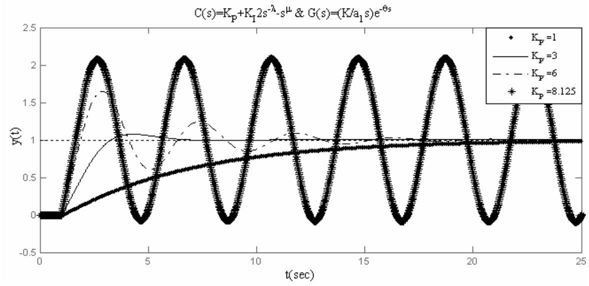

Fig. 3 Step responses for first order time delay system without using lead compensator.

The unit step responses of the $PI^{0.5}D^{0.5}$ control system when K_I is chosen as 0. 5 and various values of K_P is shown in Fig. 3. In this figure, K_P value has been chosen 1, 3, 6 to 8.125. From this figure, it is seen that the control system has more oscillatory response when the value of K_P is increased from 0 to boundary value, $K_P = 8.125$. If the K_P is bigger than the boundary value or smaller than zero, the control system becomes unstable.

Now, for this system the gain margin and phase margin has been considered 11 and 80 respectively. So the gain-phase margin tester has $A=1, \phi=80^{\circ}$. Based on "(17)," and "(18)," K_P and K_I has been computed again. The stability region for the $PI^{0.5}D^{0.5}$ controller with using of lead compensator is shown in Fig. 4.

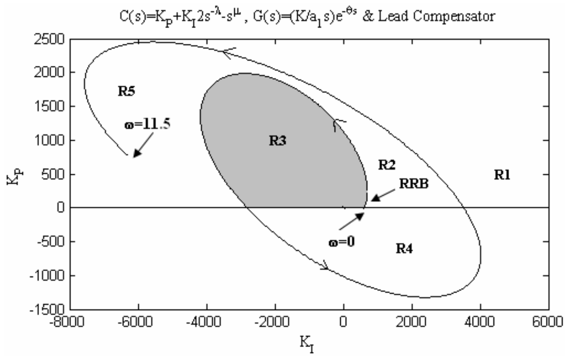

Fig. 4 Stable and unstable regions for first order time delay system with using lead compensator

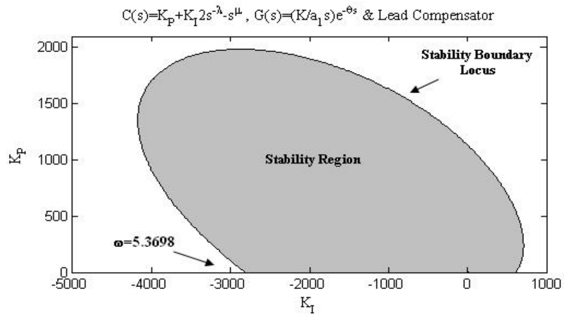

Fig. 5 Stability boundary region for first order time delay system with using lead compensator

The shaded region (R_3) is the stability region or the stability boundary locus. The stability boundary locus for FO-PID has been highlighted in Fig.5 for better clearance. As seen in this figure, the stability region is extended from $\omega = 0$ up to $\omega = 5.3698$ and so this region is bigger than the region that is shown in Fig.2. Therefore, it is true to say that using lead compensator not only improved the gain and phase margin but also increased the stability boundary locus for control system. In order to compensator performance review, the bode diagram for open loop system without and with using of lead compensator are plotted. Fig.6 shows bode diagram for control system by $PI^{0.5}D^{0.5}$ without using lead compensator. According to this figure the gain margin is 10.5 dB and phase margin is 68 degree. To obtaining desired values of gain-phase margin tester it is necessary the system phase margin increased to 80 degree. Doing the procedures provided in section 2 for designing lead compensator, control system was compensated has the bode diagram that shown in Fig.7. It can be seen from this figure that the phase margin in favorite frequency has reached to desired phase margin (i.e. 80 degree) and the system gain has not changed. Step responses of closed loop system by using of lead compensator are shown in Fig.8. From this figure it is seen that the maximum value K_P for compensated system somewhat increased ($K_P = 8.34$). Thus the good performance of closed loop system is obtained.

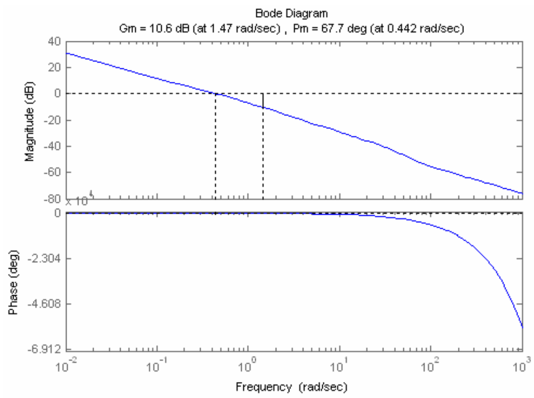

Fig. 6 Bode diagram for control system by $PI^{0.5}D^{0.5}$ without using lead compensator.

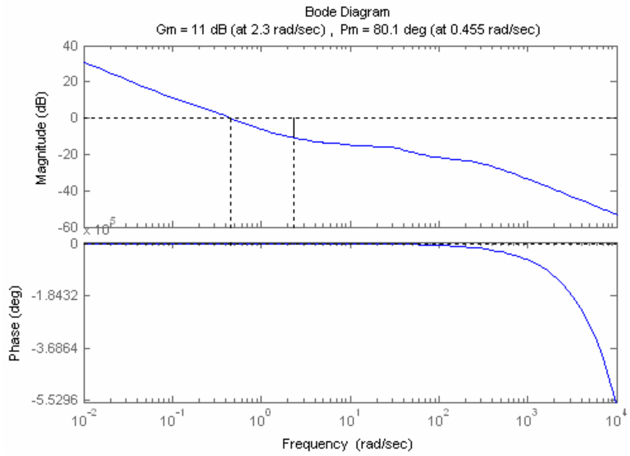

Fig. 7 Bode diagram for control system by $PI^{0.5}D^{0.5}$ without using lead compensator

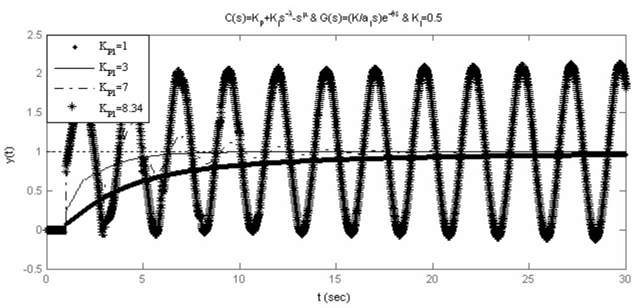

Fig. 8 Step responses for first order time delay system with using lead compensator

IV. CONCLUSION

The stabilization and phase-margin specification are considered, respectively, of this kind of delay systems using FO-PID controller, both based on stability boundary criterion applicable to integer-order time-delay systems. Simulation studies have shown that using the FO-PID with lead compensator can achieve bigger closed-loop stability region compared to the conventional this class of FO-PID controller. On the other hand step response of the closed loop system for bigger value of controller gain has been oscillatory.

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