World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:3, No:12, 2009

Discrete Modified Internal Model Control for a n*th*-order Plant with an Integrator and Dead-time

Manato Ono, Hiromitsu Ogawa, Naohiro Ban, and Yoshihisa Ishida

***Abstract*—** This paper deals with a design method of a discrete modified Internal Model Control (IMC) for a plant with an integrator and dead time. If there is a load disturbance in the input or output side of the plant, the proposed control system can eliminate the steady-state error caused by it. The disturbance compensator in this method is simple and its order is low regardless of that of a plant. The simulation studies show that the proposed method has superior performance for a load disturbance rejection and robustness.

***Keywords*—** Internal Model Control, Smith Predictor, Dead time, Integrator.

I. Introduction

FOR a plant with dead time, a Internal Model Control (IMC) and a Smith Predictor are effective. However, both methods cause a steady state error by an input side load disturbance for a plant with an integrator. To solve the problem, Watanabe et al. [2] have proposed a modified Smith Predictor whose main controller is either a PI or PID controller. Astrom et al. [3] have proposed a new Smith Predictor that provides superior performance. However, Watanabe's method cannot reject a steady state error if the estimated dead time is not accurate and Astrom's method is complicated in terms of setting the parameters in the disturbance compensator. Ishida et al. [4] have proposed a discrete modified IMC method that does not cause a steady-state error by a input side load disturbance for a plant with an integrator and make the parameter setting in the disturbance compensator easier. Furthermore, introducing a predictor as a plant model, they have shown that the robustness of the control system is enhanced. However, this method applies only to a first order plant and the effectiveness for a higher-order plant is not confirmed.

In this paper, a discrete modified Internal Model Control method for a n*th* order plant with and without an integrator and with long dead time is proposed. Moreover, the design method of the disturbance compensator that eliminates a steady-state error caused by a input side load disturbance for a plant with an integrator is given. The effectiveness of the proposed method is showed by comparative simulation studies between the conventional IMC method shown in section 2 and the proposed method shown in section 3 for a nominal system and an uncertain system.

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II. Conventional IMC

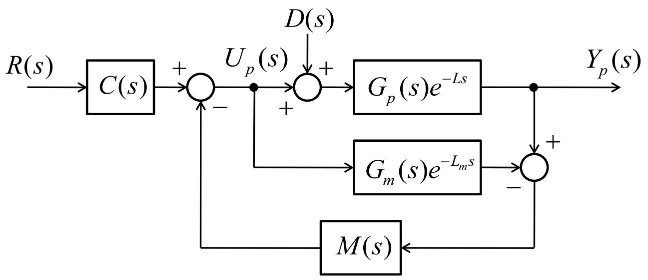


Fig.1 Conventional Internal Model Control

The block diagram of the conventional IMC is shown in Fig.1. $R(s)$ is a reference input, $D(s)$ is a disturbance, $U_p(s)$ is a plant input, $Y_p(s)$ is a plant output, $G_p(s)e^{-Ls}$ is a plant with dead time L, $G_m(s)e^{-L_m s}$ is a plant model with estimated dead time L_m, $C(s)$ is a controller and $M(s)$ is a disturbance compensator. In the conventional IMC, it is assumed that there is no modeling error; that is, $G_p(s)e^{-Ls} = G_m(s)e^{-L_m s}$. The design method of the controller and the disturbance compensator is to satisfy the following equations:

$$\lim_{s\to 0} C(s)G_m(s)e^{-L_m s} = 1 \quad (1)$$

$$\lim_{s\to 0} M(s)G_m(s)e^{-L_m s} = 1 \quad (2)$$

If the plant is a stable and minimum phase system, the controller and the disturbance compensator are given as follows:

$$C(s) = C_m(s) \cdot G_m^{-1}(s), \quad (3)$$

$$M(s) = F(s) \cdot G_m^{-1}(s), \quad (4)$$

where, $C_m(s)$ is a reference model and $F(s)$ is the IMC filter given by

$$F(s) = \frac{1}{(\lambda_1 s + 1)^{\rho}}, \quad (5)$$

where λ_1 is a time-constant that coordinates effect of a disturbance and ρ is selected so that $M(s)$ is proper. Although this design method is very simple, a steady-state error is caused by a load disturbance, when the plant has an integrator. In addition, the order of the IMC filter is involved with that of the plant. That is, the disturbance compensator is not allowed to be of low order compared with the plant.

III. Proposed Method

A. Design method of a predictor

Ishida et al. have proposed the modified discrete IMC using a predictor that improves robustness. However, it is designed only for a first order plant. In this section, the predictor for n*th* order plant is proposed. Consider now the following difference equation of a n*th* order plant model with dead time:

$$y_m(k+1) = -\sum_{i=1}^{n} a_i \cdot y_m(k+1-i) + \sum_{j=0}^{n} b_j \cdot u_m(k+1-j-d_m)$$

$$= \sum_{p=1}^{n} A_p^{[1]} \cdot y_m(k+1-p) + \sum_{q=1}^{n+1} B_q^{[1]} \cdot u_m(k+2-q-d_m), \quad (6)$$

where $y_m(k)$ and $u_m(k)$ are an output and an input of the plant model, respectively, a_i and b_j are the plant model coefficients and d_m is dead time. Multiplying (6) by a forward shift operator, the following equation is given:

$$y_m(k+2) = A_1^{[1]} y_m(k+1) + \sum_{p=2}^{n} A_p^{[1]} \cdot y_m(k+2-p)$$
$$+\sum_{q=1}^{n+1} B_q^{[1]} \cdot u_m(k+3-q-d_m) \quad (7)$$

Substituting (6) into (7), $y_m(k+2)$ is

$$y_m(k+2) = \sum_{p=1}^{n-1}\left(A_1^{[1]} \cdot A_p^{[1]} + A_{p+1}^{[1]}\right) \cdot y_m(k+1-p)$$
$$+A_1^{[1]} \cdot A_n^{[1]} \cdot y_m(k+1-n) + B_1^{[1]} \cdot u_m(k+2-d_m)$$
$$+\sum_{q=2}^{n+1}\left(A_1^{[1]} \cdot B_{q-1}^{[1]} + B_q^{[1]}\right) \cdot u_m(k+3-q-d_m)$$
$$+A_1^{[1]} \cdot B_{n+1}^{[1]} \cdot u_m(k+1-n-d_m)$$
$$= \sum_{p=1}^{n} A_p^{[2]} \cdot y_m(k+1-p) + \sum_{q=1}^{n+2} B_q^{[2]} \cdot u_m(k+3-q-d_m) \quad (8)$$

Iterating the same procedure *t* times,

$$y_m(k+t) = \sum_{p=1}^{n} A_p^{[t]} \cdot y_m(k+1-p) + \sum_{q=1}^{n+t} B_q^{[t]} \cdot u_m(k+t+1-q-d_m)$$
$$= \sum_{p=1}^{n-1}\left(A_1^{[t-1]} \cdot A_p^{[1]} + A_{p+1}^{[t-1]}\right) \cdot y_m(k+1-p)$$
$$+A_1^{[t-1]} \cdot A_n^{[1]} \cdot y_m(k+1-n)$$
$$+\sum_{q=1}^{t-1} B_q^{[t-1]} \cdot u_m(k+t+1-q-d_m)$$
$$+\sum_{q=t}^{n+t-1}\left(A_1^{[t-1]} \cdot B_{q-t+1}^{[1]} + B_q^{[t-1]}\right) \cdot u_m(k+t+1-q-d_m)$$
$$+A_1^{[t-1]} \cdot B_{n+1}^{[1]} \cdot u_m(k+1-n-d_m) \quad (9)$$

is obtained. When $t = d_m$, the equation represents the plant model without dead time as follows:

$$y_m(k+d_m) = \sum_{p=1}^{n} A_p^{[d_m]} \cdot y_m(k+1-p) + \sum_{q=1}^{n+d_m} B_q^{[d_m]} \cdot u_m(k+1-q) \quad (10)$$

B. Design method of a disturbance compensator

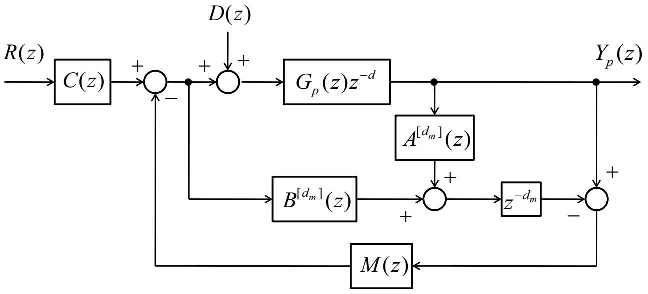


Fig.2 Proposed Internal Model Control

The discrete modified IMC using the predictor (10) is shown in Fig. 2. $A^{[d_m]}(z)$ and $B^{[d_m]}(z)$ are given by

$$A^{[d_m]}(z) = \sum_{p=1}^{n} A_p^{[d_m]} \cdot z^{-p+1} \quad (11)$$

$$B^{[d_m]}(z) = \sum_{q=1}^{n+d_m} B_q^{[d_m]} z^{-q+1} \quad (12)$$

$C(z)$ and $M(z)$ are a controller and a disturbance compensator, respectively. From Fig.2, the disturbance response is obtained as follows:

$$H_d(z) = \frac{Y_p(z)}{D(z)}$$
$$= \frac{\left(1-M(z)B^{[d_m]}(z)z^{-d_m}\right)G_p(z)z^{-d}}{1+M(z)\left\{G_p(z)z^{-d} - \left(B^{[d_m]}(z)+G_p(z)z^{-d}A^{[d_m]}(z)\right)z^{-d_m}\right\}} \quad (13)$$

Note that $B^{[d_m]}(z)+G_p(z)z^{-d}A^{[d_m]}(z)$ represents the predictor stated in the preceding section and if there is no modeling error, the following relation is obtained (Appendix):

$$G_p(z) = B^{[d]}(z) + G_p(z)z^{-d}A^{[d]}(z) \quad (14)$$

Then, the disturbance response is given by

$$H_d(z) = \left(1-M(z)B^{[d]}(z)z^{-d}\right)G_p(z)z^{-d} \quad (15)$$

In this case, to eliminate the influence of a step disturbance, the following equation must be satisfied:

$$\lim_{z\to 1}\left(1-M(z)B^{[d]}(z)z^{-d}\right)G_p(z)z^{-d} = 0 \quad (16)$$

Using (16), the disturbance compensator for plants with and without an integrator can be designed. In the former case, from the discrete time final-value theorem, (16) is rearranged as follows:

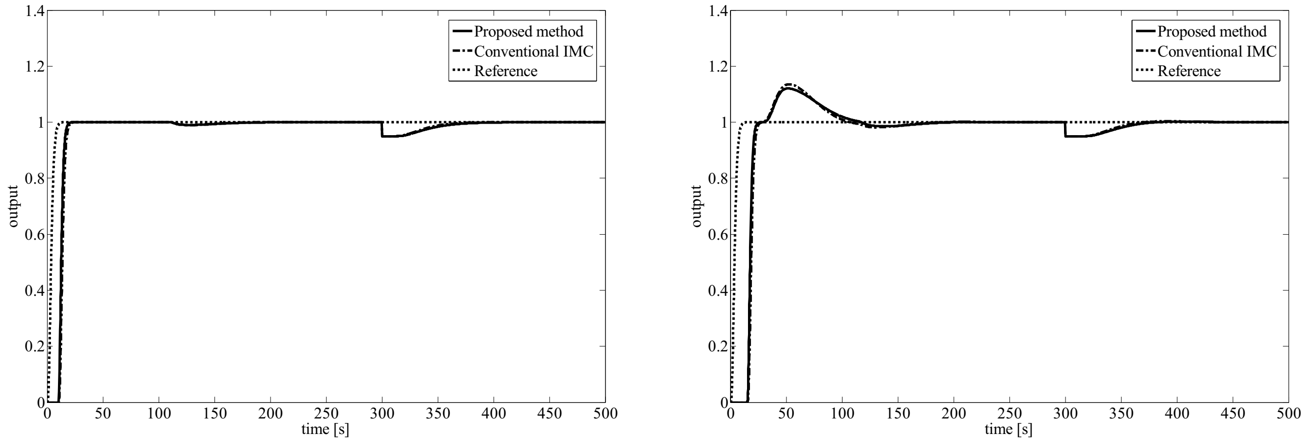
$$\lim_{z\to 1}\left(1-M(z)B^{[d]}(z)z^{-d}\right) = 0 \quad (17)$$

$$\lim_{z\to 1}\frac{d}{dz}\left(1-M(z)B^{[d]}(z)z^{-d}\right) = 0 \quad (18)$$

In addition, the form of the plant transfer function $M(z)$ is

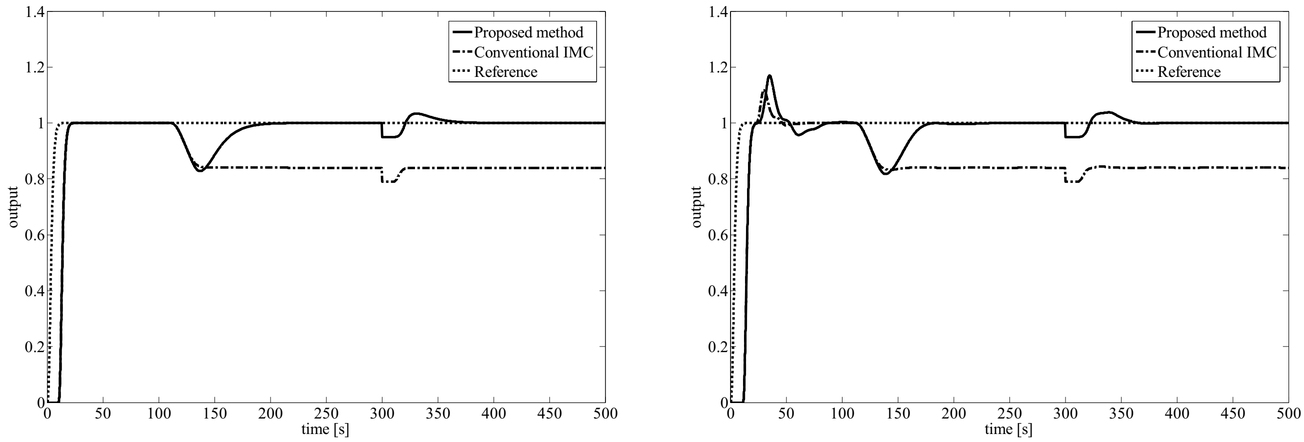
$$M(z) = \frac{\beta_1(z-1)+\beta_0}{\left\{\lambda_2(z-1)+1\right\}^{\rho}}, \quad (19)$$

where λ_2 is the parameter that adjusts the influence of the load disturbance and from (17) and (18), β_0 and β_1 are given by



(a) nominal performance (b) robust performance

Fig.3 The output responses of Case 1



(a) nominal performance (b) robust performance

Fig.4 The output responses of Case 2

$$\beta_0 = \left(\sum_{q=1}^{n+d} B_q^{[d]} \right)^{-1} \tag{20}$$

$$\beta_1 = \beta_0^2 \cdot \sum_{q=1}^{n+d} \left(\left(d+q-1 \right) \cdot B_q^{[d]} \right) + \rho\lambda\beta_0 \tag{21}$$

On the other hand, in the case of the plant without an integrator, the disturbance compensator $M(z)$ is given by

$$M(z) = \frac{\beta_0}{\left\{ \lambda_2 (z-1) + 1 \right\}^{\rho}} \tag{22}$$

$$\beta_0 = \left(\sum_{q=1}^{n+d} B_q^{[d]} \right)^{-1} \tag{23}$$

The parameter β_0 is obtained from (17).

IV. SIMULATION

In this section, the performance of the proposed method is compared with that of the conventional one. A unit step set-point is introduced at time t = 0[s], an input side load disturbance $D_{in}(s) = -0.01/s$ and an output side load disturbance $D_{out}(s) = -0.05/s$ are introduced at time t = 100[s] and t = 300[s], respectively. The Controller $C(z)$ is derived from the following equation:

$$C(z) = C_m(z) \cdot G_m^{-1}(z) \tag{24}$$

A. Case 1

Consider the following plant without an integrator:

$$G_{p1}(s) = \frac{e^{-Ls}}{\left(T_1 s+1\right)\left(T_2 s+1\right)\left(T_3 s+1\right)} \tag{25}$$

where T_1 = 1[s], T_2 = 2[s], T_3 = 3[s], and L = 10[s]. The plant is discretized by zero-order hold at sampling time T_s = 0.1[s]. In this case, the IMC filter is chosen to make the disturbance compensator proper as follows:

$$F_1(s) = 1 \big/ \left(10s+1\right)^3 \tag{26}$$

From (22) and (23), the disturbance compensator for the plant is given by

$$M_1(z) = \frac{1.149}{\left\{ 150(z-1) + 1 \right\}^2} \tag{27}$$

Fig.3 shows the nominal and robust responses of Case 1. In the robust simulation, there is a 50% error in the dead time of the plant; that is, L = 15[s] and L_m = 10[s]. From the results, almost similar stable responses are obtained.

B. Case 2

Consider the following plant with an integrator:

$$G_{p2}(s) = \frac{e^{-Ls}}{s(T_1 s+1)(T_2 s+1)(T_3 s+1)}, \quad (28)$$

where $T_1 = 1$[s], $T_2 = 2$[s], $T_3 = 3$[s], and $L = 10$[s]. The plant is discretized by zero-order hold at sampling time $T_s = 0.1$[s]. In this case, the following IMC filter is chosen:

$$F_2(s) = 1/(2s+1)^4 \quad (29)$$

The disturbance compensator in proposed method is given by

$$M_2(z) = \frac{103.2629z - 103.0435}{\{150(z-1)+1\}^2} \quad (30)$$

Fig.4 shows the response of the nominal performance and robust performance. In the robust simulation, there is a 30% error in the dead time of the plant; that is, $L = 13$[s] and $L_m = 10$[s]. It is evident that the proposed method eliminates the steady-state error caused by both the input and output load disturbance.

V. Conclusion

In this paper, the discrete modified Internal Model Control method for a n*th* order plant is proposed. The proposed method can be applied to the higher-order plant with long dead time. Furthermore, if a plant has an integrator, it can reject a steady-state error caused by a load disturbance. The disturbance compensator in this method can be low order compared to the conventional IMC's compensator and its parameter setting is simpler than the Astrom's method.

Appendix

In this section, we proof $G_p(z) = B^{[d]}(z) + G_p(z) z^{-d} A^{[d]}(z)$.

First, we assume that it is true for $d = l$; that is,

$$G_p(z) = B^{[l]}(z) + G_p(z) z^{-l} A^{[l]}(z)$$

$$= \sum_{q=1}^{n+l} B_q^{[l]} \cdot z^{-q+1} + \frac{\sum_{q=1}^{n+1} B_q^{[1]} \cdot z^{-q+1}}{1 - \sum_{p=1}^{n} A_p^{[1]} \cdot z^{-p}} z^{-l} \cdot \sum_{p=1}^{n} A_p^{[l]} \cdot z^{-p+1} \quad (31)$$

The predictor for $d = l+1$ is described as follows:

$$B^{[l+1]}(z) + G_p(z) z^{-(l+1)} A^{[l+1]}(z)$$

$$= B^{[l+1]}(z) \cdot \frac{\sum_{q=1}^{n+1} B_q^{[1]} \cdot z^{-q+1}}{1 - \sum_{p=1}^{n} A_p^{[1]} \cdot z^{-p}} z^{-(l+1)} \cdot A^{[l+1]}(z) \quad (32)$$

$A^{[l+1]}(z)$ and $B^{[l+1]}(z)$ are rearranged by

$$A^{[l+1]}(z) = A_1^{[l]} \sum_{p=1}^{n} A_p^{[1]} \cdot z^{-p+1} + \sum_{p=1}^{n} A_p^{[l]} \cdot z^{-p+2} - A_1^{[l]} z \quad (33)$$

$$B^{[l+1]}(z) = \sum_{q=1}^{n+t-1} B_q^{[l]} z^{-q+1} + A_1^{[l]} \sum_{q=1}^{n+1} B_q^{[1]} z^{-q-l+1} \quad (34)$$

Substituting these equations to (32),

$$B^{[l+1]}(z) + G_p(z) z^{-(l+1)} A^{[l+1]}(z)$$

$$= \sum_{q=1}^{n+l} B_q^{[l]} \cdot z^{-q+1} + \frac{\sum_{q=1}^{n+1} B_q^{[1]} \cdot z^{-q+1}}{1 - \sum_{p=1}^{n} A_p^{[1]} \cdot z^{-p}} z^{-l} \cdot \sum_{p=1}^{n} A_p^{[l]} \cdot z^{-p+1} \quad (35)$$

is obtained. So, $G_p(z) = B^{[d]}(z) + G_p(z) z^{-d} A^{[d]}(z)$ is true for $d = l+1$. Finally, the formula is true for $d = 1$ is proved as follows:

$$\sum_{q=1}^{n+1} B_q^{[1]} \cdot z^{-q+1} + \frac{\sum_{q=1}^{n+1} B_q^{[1]} \cdot z^{-q+1}}{1 - \sum_{p=1}^{n} A_p^{[1]} \cdot z^{-p}} z^{-1} \cdot \sum_{p=1}^{n} A_p^{[1]} \cdot z^{-p+1}$$

$$= \sum_{q=1}^{n+1} B_q^{[1]} \cdot z^{-q+1} \Bigg/ \left(1 - \sum_{p=1}^{n} A_p^{[1]} \cdot z^{-p}\right) = G_p(z) \quad (36)$$

Therefore, $G_p(z) = B^{[d]}(z) + G_p(z) z^{-d} A^{[d]}(z)$ is true by mathematical induction.

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