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Optimum Performance Measures of Interdependent Queuing System with Controllable Arrival Rates

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***Abstract*—**In this paper, an attempt is made to compute the total optimal cost of interdependent queuing system with controllable arrival rates as an important performance measure of the system. An example of application has also been presented to exhibit the use of the model. Finally, numerical demonstration based on a computing algorithm and variational effects of the model with the help of the graph have also been presented.

***Keywords*—**Computing, Controllable arrival rate, Optimum performance measure.

I. INTRODUCTION

COST analysis of the queuing theory occupies a prominent place in the research of queuing theory, a significant aspect of the optimization theory. Optimization techniques are widely used in the areas of production, manufacturing and planning including the communication systems to effectively assess the performance systems. It has drawn the attention of the researchers seriously engaged in this area of research. In this connection, a box score till date reveals that no avalanche works have been done in this area. Nowadays, a trend has been redirected and shifted to investigate more realistic performance measures of the system as compared to general theoretical approach that embodies hardly a bit of application.

Here, some relevant researches are in sequel. Reference [13] discussed the queuing decision models namely a cost model and an aspiration level model recognizing the higher service levels reducing the waiting time in the system. It was studied the cost analysis of $E_m/C_m/m$ queue emphasizing on superior model on the basis of the revenue functions defined under the models, [5]. Further, the cost analysis of the M/M/R machine repair problem with spares and two modes of failures and cost and profit analysis of Markovian queues with priority classes dealt upon, as in [10], [14]. Moreover, [3] analyzed the cost analysis of the M/M/R machine repair problem with balking, reneging and server breakdowns. Cost analysis of finite M/M/R queuing system with balking, reneging and server breakdown has been discussed, as in [15]. Recently, it was extended the work of [12] by investigating the cost analysis of machine interference model and its dual with balking, reneging and spares, [7], [8]. Further, [9] studied the machine interference model and analyzed the cost of the system by applying an efficient algorithm of computation.

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Queuing models having controllable arrival rates with interdependence are very useful to provide basic framework for efficient design digital communication system, a user at a client workstation is supposed to invoke program images repeatedly. Distributed File System has access for local computation and system service operation. In this system, the amount of data to be transferred and processing to be done at the server for each request depends on the type of request (interdependent arrival-service model). These requests may be control and data access operations. Control operations have high computational component on the other hand data access operations need low computation requirement but have larger data transfers.

In this series of investigation, [1] discussed various queuing models including M/M/1/N with different arrival rates. It is simply easy to consider that the arrival and service processes are independent to only dealing with mathematical tractability leading to theoretical approach of the system. However, in some practical situations, assuming arrival and service processes to be interdependent is more realistic to dealing with such situations, as in [4], [11].

Recently, for the queuing system having controllable arrival rates, [6] have attempted to present a computational approach to the M/M/1/N interdependent queuing model with controllable arrival rates by analyzing and computing the expected number of customers in the system.

A close look at literature related to it reveals that research works on queueing system characterizing arrival and service processes interdependent, no attempt has been made to analyze the total optimal cost of the system as a novel and useful performance measures. For computing this novel performance measure for this particular queueing system a computing algorithm has been developed based on a fast converging N-R method for solving a non-linear equation, which requires least computing time and lesser memory space as compared to other methods (computer program is developed in C++). Numerical demonstration and graphs have also been added to gain a significant insight into the problem

II. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions have been used through out the paper:

- λ_0: initial arrival rate.
- λ_1: reduced arrival rate.

- ε: mean dependence rate (co-variation between the arrival rate and service process)
- μ: mean service rate.
- n: number of customers in the system.
- R, r, arbitrary integers
- N: arbitrary integers (capacity of the system).
- ρ_0 : $(\lambda_0-\varepsilon)/(\mu-\varepsilon)$, traffic load with initial arrival.
- ρ_1 : $(\lambda_1-\varepsilon)/(\mu-\varepsilon)$, traffic load with reduced arrival rate.
- $P_0(0)$: probability of no customers in the system at initial stage.
- C_1: Per unit service cost.
- C_2: Per unit holding cost.

III. BACKGROUND

Here, we have considered a queuing model with M/M/1/N with single server and finite capacity. The arrival and service rates of the system are interdependent and follow a bivariate Poisson process having the joint probability mass function of the form given as;

$$P(X_1=x_1, X_2=x_2; t) = e^{-(\lambda_i+\mu-\varepsilon)t} \sum_{j=0}^{\min(x_1,x_2)} \frac{(\varepsilon t)^j [(\lambda_i-\varepsilon)t]^{x_1-j} (\mu-\varepsilon)^{x_2-j}}{j!\,(x_1-j)!\,(x_2-j)!} \tag{1}$$

x_1, x_2=0,1,2… 0<λi, μ; 0<ε<min (λi, μ), i=0,1 and other symbols have their usual meanings.

The operating strategy of the system is controllable arrival rates with prescribed integers R and r assuming that whenever the queue size reaches a certain prescribed number R, the arrival rate reduces from λ_0 to λ_1 and it continues with rate as long as the content in the queue is greater than some other prescribed integer r (r> 0 and r< R).

In view of [6], obtained the expected number of customers in the system M/M/1/N as under,

$$Ls = \begin{bmatrix} (\lambda_0-\varepsilon)(\mu-\varepsilon)/(\mu-\varepsilon)^2 \;\; -(R-\rho_0 r)\dfrac{(\lambda_0-\varepsilon)^{R+r}}{(\mu-\varepsilon)^{R+r-1}(\mu-\lambda_0)} - \\ (R+r)(R-r-1)\rho_0^{R+r} A \end{bmatrix} P_0(0)$$

$$+\rho_0^{R+r}\,(\mu-\lambda_0)/(\mu-\lambda_1)\,A \left[\frac{1}{2}(R+r)(R-r-1) - \sum_{n=r+1}^{R}\rho_1^{n-r} + (1-\rho_1^{R-r})\sum_{n=R+1}^{N}\rho_1^{n-R}\right] P_0(0) \tag{2}$$

This may be written as

$$Ls = [B-(R-\rho_0 r)\,C-D+EA+F]\,P_0(0), \tag{3}$$

where $P_0(0) = [B_1-(R-r)\,B_2+B_3]^{-1}$,

$A = ([\rho_0^{\,n}-\rho_1^{\,R}]^{-1}$

$B = (\lambda_0-\varepsilon)(\mu-\varepsilon)/(\mu-\varepsilon)^2$, $C = \dfrac{(\lambda_0-\varepsilon)^{R+r}}{(\mu-\varepsilon)^{R+r-1}(\mu-\lambda_0)}$,

$D = (R+r)(R-r-1)C^{R+r}A$, $E = \rho_0^{R+r}\,(\mu-\lambda_0)/(\mu-\lambda_1)$

$F = \left[\dfrac{1}{2}(R+r)(R-r-1) - \sum_{n=r+1}^{R}\rho_1^{\,n-r} + (1-\rho_1^{\,R-r})\sum_{n=R+1}^{N}\rho_1^{\,n-R}\right]$

$B_1 = (\mu-\varepsilon)/(\mu-\lambda_0)$, $B_2 = \rho_0^{R+r}A(\lambda_0-\lambda_1)/(\mu-\lambda_1)$

$B_3 = (\lambda_1-\varepsilon)(\mu-\lambda_0)/(\mu-\lambda_1)^2\;\rho_0^{R+r}\,(\rho_1^{N-r}-\rho_1^{\,n-r})\,A$

And other symbols have their usual significance as given earlier.

IV. ANALYSIS OF OPTIMUM PERFORMANCE MEASURES OF THE SYSTEM

In this section, we construct the total cost function of the system, denoted by TC, in view of [12] and [7] as under:

$$TC = C_1\mu + C_2\,Ls \tag{3}$$

After subjecting the equation (3) to the optimization condition with respect to μ, it yields the nonlinear equation in μ as:

$$C_1 + C_2 P_0(0)\,[(\lambda_0-\varepsilon)D_1 - D_2 - D_3 + (R+r)(R--1\,r)D_4/2 + (R+r)^2(R-r-1)D_5/2 + \rho_0^{R+r}AD_6 + \left(\frac{\mu-\lambda_0}{\mu-\lambda_1}\right)D_7\,D_8] - C_2\,[B-(R-\rho_0 r)C - D - EA + F]\times$$

$$[B_1-(R-r)B_2+B_3]^{-3}[E_1+E_2+E_3+E_4+E_5+E_6]=0$$

where,

$D_1 = -2(\mu-\varepsilon)/(\mu-\lambda_0)^3 + (\mu-\lambda_0)^2$

$D_2 = (R-\rho_0 r)$

$$(\lambda_0-\varepsilon)\left[(\mu-\varepsilon)^{1-r-R}(\mu-\lambda_0)^{-1} - (\mu-\varepsilon)^{1-r-R}(\mu-\lambda_0)^{-2}\right] + (1-r-R)\,(\mu-\varepsilon)^{-r-R}(\mu-\lambda_0)^{-1}$$

$$D_3 = \frac{-r(\lambda_0-\varepsilon)^{R+r+1}}{(\mu-\lambda_0)(\mu-\varepsilon)^{R+r+1}}$$

$$D_4 = \rho_0^{R+r}\,(\rho_0^{\,n}-\rho_0^{\,R})^{-2}\left[-n\frac{(\lambda_0-\varepsilon)^n}{(\mu-\varepsilon)^{n+1}} + R\frac{(\lambda_0-\varepsilon)^R}{(\mu-\varepsilon)}\right]$$

$$D_5 = A\,\frac{(\lambda_0-\varepsilon)^{R+r}}{(\mu-\varepsilon)^{R+r-1}},$$

$$D_6 = -\,[(\mu-\lambda_0)/(\mu-\lambda_1)^2] + \frac{1}{\mu-\lambda_1}$$

$$D_7 = (R+r)A\frac{(\lambda_0-\varepsilon)^{R+r}}{(\mu-\varepsilon)^{R+r-1}} + \rho_0^{R+r}\,(\rho_0^{\,n} - \rho_0^{\,R})^2\left[-n\frac{(\lambda_0-\varepsilon)^n}{(\mu-\varepsilon)^{n+1}} + R\frac{(\lambda_0-\varepsilon)^R}{(\mu-\varepsilon)}\right]$$

$$D_8 = \sum n(r-n)\,\frac{(\lambda_1-\varepsilon)^{n-r}}{(\mu-\varepsilon)^{n-r+1}} + (1-\rho_1^{\,R-r})\sum_{n=R+1}^{N} n(R-r)\,\frac{(\lambda_1-\varepsilon)^{n-R}}{(\mu-\varepsilon)^{n-R+1}} + \sum_{n=R+1}^{N} n\rho_1^{\,n-R}(R-r)\,\frac{(\lambda_1-\varepsilon)^{R-r}}{(\mu-\varepsilon)^{R-r+1}}$$

$$E_1 = -\frac{(\mu-\varepsilon)}{(\mu-\lambda_0)^2} + (R-r)\rho_0^{\,r+R}A\frac{(\lambda_1-\lambda_0)}{(\mu-\lambda_0)^2(\mu-\lambda_1)^2}$$

$E_2 = (R-r)$

$$\frac{(\lambda_0-\lambda_1)}{(\mu-\lambda_1)}\begin{bmatrix} -(r-R)A(\lambda_0-\varepsilon)^{r+R}/(\mu-\varepsilon)^{r+R+1} - \\ \rho_0^{\,r+R}(\rho_0^{\,n}-\rho_0^{\,R})^{-2}\left(\dfrac{-n(\lambda_0-\varepsilon)^n}{(\mu-\varepsilon)^{n+1}} + R(\lambda_0-\varepsilon)^R/(\mu-\varepsilon)\right)\end{bmatrix}$$

$$E_3=\left(\frac{(\lambda_1-\varepsilon)(\mu-\lambda_0)}{(\mu-\lambda_1)^2}\right)\rho_0^{r+R}$$

$$\left[\begin{array}{l}(N-R)(\lambda_1-\varepsilon)^{N-R}\frac{1}{(\mu-\varepsilon)^{nN-R+1}}\\ -(N-R)(\lambda_1-\varepsilon)^{N-R}\frac{1}{(\mu-\varepsilon)^{nN-r+1}}\end{array}\right]$$

$$E_4=-\left(\rho_1^{N-R}-\rho_1^{N-r}\right)\frac{(\lambda_1-\varepsilon)(\mu-\lambda_0)}{(\mu-\lambda_1)^2}\rho_0^{r+R}\left(\rho_0^{n}-\rho_0^{R}\right).$$

$$\left[\frac{-n(\lambda_0-\varepsilon)^n}{(\mu-\varepsilon)^{n+1}}+R\left(\frac{(\lambda_0-\varepsilon)^R}{(\mu-\varepsilon)}\right)\right]$$

$$E_5=A\left[-\frac{(\lambda_1-\varepsilon)(\mu-\lambda_0)(R+r)(\lambda_0-\varepsilon)^{r+R}}{(\mu-\lambda_1)^2(\mu-\varepsilon)^{r+R+1}}+\rho_0^{r+R}\left\{\frac{(\lambda_1-\varepsilon)}{(\mu-\lambda_1)^2}-\frac{2(\lambda_1-\varepsilon)(\mu-\lambda_0)}{(\mu-\lambda_1)^3}\right]\right.$$

$$E_6=-\frac{(\lambda_1-\varepsilon)(\mu-\lambda_0)}{(\mu-\lambda_1)^2}\rho_0^{r+R}A$$

$$\left[\frac{(R-N)(\lambda_1-\varepsilon)^{N-R}}{(\mu-\varepsilon)^{N-R+1}}-(r-N)\frac{(\lambda_1-\varepsilon)^{N-r}}{(\mu-\lambda_1)^{N-r+1}}\right]$$

Now, we compute the optimal service rate and total optimal cost as important performance measures of the system by developing a program in CPP language on the basis of the following algorithm:

- Begin the computation
- Data Input
- Compute row0, row1, A and P_0 (0)
- Compute all Bs and Cs
- Compute D_1- D_8
- Compute E_1- E_6
- Compute equation and its first derivative
- Compute optimum service rate (approximated root value) by using N-R Method
- Compute total optimal cost of the system corresponding to the optimal service rate.

After applying the above algorithm, requisite table (I) is given as under. In this table, for suitable constant values of C_1, C_2, and other parameters (C_1=29, C_2=20, λ_0=15, λ_1 =7, n=5, ε =0.5, R=4, r=2, N= 30), optimal service rate (μ*=8) has been obtained and consequently the total optimal cost of the system (last column) has been presented for different values of various parameters as given in the table. Here, it is worth mentioning that the values of different parameters are chosen hypothetically in order to numerically demonstrate the use of the model.

Furthermore, upon assuming that service times are independently and identically distributed with having parallel arrangement of multi-servers, we can proceed exactly in the same way to obviously obtain similar results but indicating an application in the area of computer networking as we have observed in case of digital communication.

V.VARIATIONAL ANALYSIS

Validity of any model depends on the variation-effect of one parameter on the other parameters involved therein. Here, different types of analyses are being given in fig. 1.as:

- Cost per Unit Service Rate Vs Total Optimal Cost.
- Holding Cost Vs Total Optimal Cost.
- Initial Arrival Rate Vs Total Optimal Cost.
- Reduced Arrival Rate Vs Total Optimal Cost.

VI. CONCLUSION

Finally, we conclude with the remark that present research on optimum performance measures of the interdependent queuing model with controllable arrival rates can pave the way for future progress of research in various fields including technical applications for the digital communication systems and as well as in assessing the performance measures in the form of optimal service and cost of computer networking by applying this queuing approach. Such systems are often encountered in practice, particularly in service-oriented operations, which brings the efficacy of the model closer to a realistic situation. The aim of the numerical demonstration is to study the variability of the model that is, to assess the effect of one parameter on the others especially such parameters which characterize the performance measures of the model. Numerical demonstration carried out with the help of search program is mainly based on the simulations or hypothetical data-input. In this paper, we have preferred the hypothetical data-input to run the search program developed in the paper, which at later stage can also be tested for any real case study. It has also a good deal of potential to the applications in other areas such as inventory management, production management, computer system etc.

TABLE I
COMPUTATION OF TOTAL OPTIMUM COST

C_1	C_2	λ_0	λ_1	n	ε	R	r	N	μ*	TOC
20	13	5	2	5	0.5	4	2	30	8	40
22	13	8	3	5	0.5	4	2	30	8	40
24	15	11	4	5	0.5	4	2	30	8	41
26	17	14	5	5	0.5	4	2	30	8	41
28	19	15	6	5	0.5	4	2	30	8	42
30	21	16	7	5	0.5	4	2	30	8	43
34	25	18	9	5	0.5	4	2	30	8	45
36	27	19	10	5	0.5	4	2	30	8	45
36	29	20	11	5	0.5	4	2	30	8	46

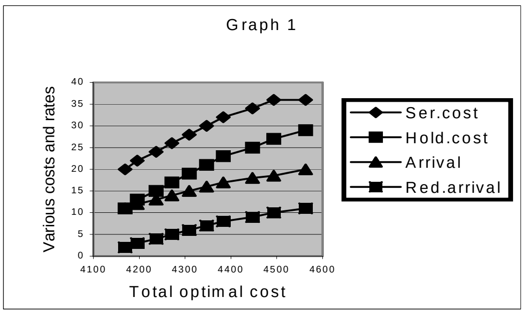

Fig.1. Graphic representation of Total Optimum Cost

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