

# Fuzzy Metric Approach for Fuzzy Time Series Forecasting based on Frequency Density Based Partitioning

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**Abstract**—In the last 15 years, a number of methods have been proposed for forecasting based on fuzzy time series. Most of the fuzzy time series methods are presented for forecasting of enrollments at the University of Alabama. However, the forecasting accuracy rates of the existing methods are not good enough. In this paper, we compared our proposed new method of fuzzy time series forecasting with existing methods. Our method is based on frequency density based partitioning of the historical enrollment data. The proposed method belongs to the  $k$ th order and time-variant methods. The proposed method can get the best forecasting accuracy rate for forecasting enrollments than the existing methods.

**Keywords**—Fuzzy logical groups, fuzzified enrollments, fuzzy sets, fuzzy time series.

## I. INTRODUCTION

IT is obvious that forecasting activities play an important role in our daily life. During last few decades, various approaches have been developed for time series forecasting. Among them ARMA models and Box-Jenkins model building approaches are highly famous. But the classical time series methods can not deal with forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets [11], [23]. Therefore, Song and Chissom [18] presented the theory of fuzzy time series to overcome this drawback of the classical time series methods. Based on the theory of fuzzy time series, Song et al. presented some forecasting methods [16], [18], [19], [20] to forecast the enrollments of the University of Alabama. In [1] Chen and Hsu and in [2], Chen presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series. It has the advantage of reducing the calculation, time and simplifying the calculation process. In [8], Hwang, Chen and Lee used the differences of the enrollments to present a method to forecast the enrollments of the University of Alabama based on fuzzy

time series. In [5] and [6], Huang used simplified calculations with the addition of heuristic rules to forecast the enrollments using [2]. In [4], Chen presented a forecasting method based on high-order fuzzy time series for forecasting the enrollments of the University of Alabama. In [3], Chen and Hwang presented a method based on fuzzy time series to forecast the daily temperature. In [15], Melike and Konstantin presented a new first order time series model for forecasting enrollments of the University of Alabama. In [14], Li and Kozma presented a dynamic neural network method for time series prediction using the KIII model. In [21], Su and Li presented a method for fusing global and local information in predicting time series based on neural networks. In [22], Sullivan and Woodall reviewed the first-order time-variant fuzzy time series model and the first-order time-invariant fuzzy time series model presented by Song and Chissom [18], where their models are compared with each other and with a time-variant Markov model using linguistic labels with probability distributions. In [13], Lee, Wang and Chen presented two factor high order fuzzy time series for forecasting daily temperature in Taipei and TAIEX. In [9], Jilani and Burney and in [10], Jilani, Burney and Ardil presented new fuzzy metrics for high order multivariate fuzzy time series forecasting for car road accident casualties in Belgium.

In this paper, we present a comparison of our proposed method and existing fuzzy time series forecasting methods to forecast the enrollments of the University of Alabama. Our proposed method belongs to the class of  $k$ -step first-order univariate time-variant method. The proposed method gives the best forecasting accuracy rate for forecasting enrollments when compared with existing methods. The rest of this paper is organized as follows. In Section 2, we briefly review some basic concepts of fuzzy time series. In Section 3, we present our method of fuzzy forecasting based on frequency density based partitioning of the enrollment data. In Section 4, we compared the forecasting results of the proposed method with the existing methods. The conclusions are discussed in Section 5.

## II. SOME BASIC CONCEPTS OF FUZZY TIME SERIES

There are number of definitions for fuzzy time series.

Manuscript received June 15, 2007. This work was supported in part by the Higher Education Commission of Pakistan.

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**Definition 1:** Imprecise data at equally spaced discrete time points are modeled as fuzzy variables. The set of this discrete fuzzy data forms a fuzzy time series.

**Definition 2:** Chronological sequences of imprecise data are considered as time series with fuzzy data. A time series with fuzzy data is referred to as fuzzy time series.

**Definition 3:** Let  $Y(t), (t = \dots, 0, 1, 2, \dots)$  be the universe of discourse and  $Y(t) \subseteq R$ . Assume that  $f_i(t), i = 1, 2, \dots$  is defined in the universe of discourse  $Y(t)$  and  $F(t)$  is a collection of  $f(t_i), (i = \dots, 0, 1, 2, \dots)$ , then  $F(t)$  is called a fuzzy time series of  $Y(t), i = 1, 2, \dots$ . Using fuzzy relation, we define  $F(t) = F(t-1) \circ R(t, t-1)$  where  $R(t, t-1)$  is a fuzzy relation and “ $\circ$ ” is the max-min composition operator, then  $F(t)$  is caused by  $F(t-1)$  where  $F(t)$  and  $F(t-1)$  are fuzzy sets.

Let  $F(t)$  be a fuzzy time series and let  $R(t, t-1)$  be a first-order model of  $F(t)$ . If  $R(t, t-1) = R(t-1, t-2)$  for any time  $t$ , then  $F(t)$  is called a time-invariant fuzzy time series. If  $R(t, t-1)$  is dependent on time  $t$ , that is,  $R(t, t-1)$  may be different from  $R(t-1, t-2)$  for any  $t$ , then  $F(t)$  is called a time-variant fuzzy time series. In [19], Song et al. proposed the time-variant fuzzy time-series model and forecasted the enrollments of the University of Alabama based on the model.

### III. A NEW METHOD FOR FORECASTING ENROLLMENTS USING FUZZY TIME SERIES

In this section, we present our method to forecast the enrollments of the University of Alabama based on fuzzy time series based on [9] and [10]. The historical enrollments of the University of Alabama are shown in Table I, [19].

Firstly, based on [1], we defined the partition the universe of discourse into equal length intervals. Then based on frequency density portioning, we redefine the intervals. After this, define some membership function for each interval of the historical enrollment data to obtain fuzzy enrollments to form a fuzzy time series. Then, it establishes fuzzy logical relationships (FLRs) based on the fuzzified enrollments in Table IV. Finally, it uses our proposed method. The proposed method bases on Hsu and Chen approach, [7] of partitioning universe of discourse are as follows:

**Step 1:** Define the universe of discourse  $U$  and partition it into several even and equal length intervals  $u_1, u_2, \dots, u_n$ . For example, assume that the universe of discourse  $U = [13000, 20000]$  is partitioned into seven even and equal length intervals.

TABLE I  
 THE HISTORICAL ENROLLMENTS OF THE UNIVERSITY OF ALABAMA [19]

| YEAR | ENROLLMENTS | YEAR | ENROLLMENTS |
|------|-------------|------|-------------|
| 1972 | 13055       | 1982 | 15433       |
| 1972 | 13563       | 1983 | 15497       |
| 1973 | 13847       | 1984 | 15145       |
| 1974 | 14696       | 1985 | 15163       |
| 1975 | 15460       | 1986 | 15984       |
| 1976 | 15311       | 1987 | 16859       |
| 1977 | 15603       | 1988 | 18150       |
| 1978 | 15861       | 1989 | 18970       |
| 1979 | 16807       | 1990 | 19328       |
| 1980 | 16919       | 1991 | 19337       |
| 1981 | 16388       | 1992 | 18876       |

**Step 2:** Get a weighted aggregation [24] of the fuzzy distribution of the historical enrollments in each interval. Sort the intervals based on the number of historical enrollment data in each interval from the highest to the lowest as given in [1]. Find the interval having the largest number of historical enrollment data and divide it into four sub-intervals of equal length. Find the interval having the second largest number of historical enrollment data and divide it into three sub-intervals of equal length. Find the interval having the third largest number of historical enrollment data and divide it into two sub-intervals of equal length. Find the interval with the fourth largest number of historical enrollment data and let the length of this interval remain unchanged. If there are no data distributed in an interval then discard this interval. For example, the distributions of the historical enrollment data in different intervals are summarized as shown in Table II, [7].

After executing this step, the universe of discourse [13000, 20000] is re-divided into the following intervals [7], see Table III.

**Step 3:** Define each fuzzy set  $A_i$  based on the re-divided intervals and fuzzify the historical enrollments shown in Table I, where fuzzy set  $A_i$  denotes a linguistic value of the enrollments represented by a fuzzy set. We have used triangular membership function to define the fuzzy sets  $A_i$  [10]. The reason for fuzzifying the historical enrollments into fuzzified enrollments is to translate crisp values into fuzzy sets to get a fuzzy time series.

**Step 4:** Establish fuzzy logical relationships based on the fuzzified enrollments where the fuzzy logical relationship “ $A_p, A_q, A_r \rightarrow A_s$ ” denotes that “if the fuzzified enrollments of year  $p, q$  and  $r$  are  $A_p, A_q$  and  $A_r$  respectively, then the fuzzified enrollments of year ( $r$ ) is  $A_r$ ”.

TABLE II  
 THE FREQUENCY DENSITY BASED DISTRIBUTION OF THE HISTORICAL ENROLLMENT DATA [7]

| Intervals      | Number of historical enrollment data |
|----------------|--------------------------------------|
| [13000, 14000] | 3                                    |
| [14000, 15000] | 1                                    |
| [15000, 16000] | 9                                    |
| [16000, 17000] | 4                                    |
| [17000, 18000] | 0                                    |
| [18000, 19000] | 3                                    |
| [19000, 20000] | 2                                    |

TABLE III  
 FUZZY INTERVALS USING FREQUENCY DENSITY BASED PARTITIONING

| Linguistic | Intervals      |
|------------|----------------|
| u1         | [13000, 13500] |
| u2         | [13500, 14000] |
| u3         | [14000, 15000] |
| u4         | [15000, 15250] |
| u5         | [15250, 15500] |
| u6         | [15500, 15750] |
| u7         | [15750, 16000] |
| u8         | [16000, 16333] |
| u9         | [16333, 16667] |
| u10        | [16667, 17000] |
| u11        | [18000, 18500] |
| u12        | [18500, 19000] |
| u13        | [19000, 20000] |

$$t_j = \begin{cases} \frac{1+0.5}{\frac{1}{a_1} + \frac{0.5}{a_2}} & , \text{if } j = 1, \\ \frac{0.5+1+0.5}{\frac{0.5}{a_{j-1}} + \frac{1}{a_j} + \frac{0.5}{a_{j+1}}} & , \text{if } 2 \leq j \leq n-2, \\ \frac{0.5+1}{\frac{0.5}{a_{n-1}} + \frac{1}{a_n}} & , \text{if } j = n. \end{cases}$$

where  $a_{j-1}, a_j, a_{j+1}$  are the mid points of the fuzzy intervals  $A_{j-1}, A_j, A_{j+1}$  respectively. Based on the fuzzify historical enrollments obtained in Step 3, we can get the fuzzy logical relationship group (FLGR) as shown in Table IV.

Divide each interval derived in Step 2 into subintervals of equal length with respect to the corresponding frequency density of the interval. We have assumed thirteen partitions of the universe of discourse of the main factor fuzzy time series. Assuming that  $0 \neq A_i, \forall A_i, i = 1, 2, \dots, 13$ . The proposed method satisfies the following axioms:

TABLE IV  
 THIRD-ORDER FUZZY LOGICAL RELATIONSHIP GROUPS [9]

|          |                                       |           |  |
|----------|---------------------------------------|-----------|--|
| Group 1: | $X_2, X_2, X_3 \rightarrow X_5$       | Group 10: | $X_9, X_5, X_5 \rightarrow X_4$  |
| Group 2: | $X_2, X_3, X_5 \rightarrow X_5$       | Group 11: | $X_5, X_5, X_4 \rightarrow X_4$  |
| Group 3: | $X_3, X_5, X_5 \rightarrow X_6$       | Group 12: | $X_5, X_4, X_4 \rightarrow X_7$  |
| Group 4: | $X_5, X_5, X_6 \rightarrow X_7$       | Group 13: | $X_4, X_4, X_7 \rightarrow X_{10}$   |
| Group 5: | $X_5, X_6, X_7 \rightarrow X_{10}$    | Group 14: | $X_4, X_7, X_{10} \rightarrow X_{11}$  |
| Group 6: | $X_6, X_7, X_{10} \rightarrow X_{10}$ | Group 15: | $X_7, X_{10}, X_{11} \rightarrow X_{12}$<br>$X_{10}, X_{11}, X_{12} \rightarrow$ |
| Group 7: | $X_7, X_{10}, X_{10} \rightarrow X_9$ | Group 16: | $X_{13}$<br>$X_{11}, X_{12}, X_{13} \rightarrow$                                 |
| Group 8: | $X_{10}, X_{10}, X_9 \rightarrow X_5$ | Group 17: | $X_{13}$<br>$X_{12}, X_{13}, X_{13} \rightarrow$                                 |
| Group 9: | $X_{10}, X_9, X_5 \rightarrow X_5$    | Group 18: | $X_{12}$   |

**Axiom 1:**  $t(0) = 0$  and  $t(1) = 1$  (Boundary Condition)

**Axiom 2:**  $t^{\alpha_i}(a_1, a_2, \dots, a_n) \leq t^{\alpha_i}(b_1, b_2, \dots, b_n)$  provided

$$a_i \leq b_i, i = 1, 2, \dots, n$$

(Monotonicity)

**Axiom 3:**  $t^{\alpha}(a_1, a_2, \dots, a_n)$  is continuous

**Axiom 4:**  $(A_k)_{\min} \leq t^{\alpha}(a) \leq (A_k)_{\max}; k = 1, 2, \dots, 8$

(Symmetry)

**Axiom 5:**  $t^{\alpha}(a_1, a_2, \dots, a_n) = a, \forall a \in [0, 1]$

(Idempotency)

In the next section, we have given comparison of different fuzzy time series forecasting methods.

#### IV. A COMPARISON OF DIFFERENT FORECASTING METHODS

In the following, Table V summarizes the forecasting results of the proposed method from 1972 to 1992, where the universe of discourse is divided into thirteen intervals based on frequency density based partitioning. In the following, we use the average forecasting error rate (AFER) and mean square error (MSE) to compare the forecasting results of different forecasting methods, where  $A_i$  denotes the actual enrollment and  $F_i$  denotes the forecasting enrollment of year  $i$ , respectively.

$$AFER = \frac{|A_i - F_i|/A_i}{n} \times 100\%$$

$$MSE = \frac{\sum_{i=1}^n (A_i - F_i)^2}{n}$$

In Table VI, we compare the forecasting results of the proposed method with that of the existing methods. From Table III, we can see that when the number of intervals in the universe of discourse is thirteen and the intervals are sub-partitioned based on frequency density, the proposed method shows smallest values of the MSE and AFER of the forecasting results as compared to other methods of fuzzy time series forecasting. That is, the proposed method can get a

higher forecasting accuracy rate for forecasting enrollments than the existing methods.

TABLE V  
ACTUAL ENROLLMENTS AND FORECASTED ENROLLMENTS OF THE UNIVERSITY OF ALABAMA BASED ON FREQUENCY DENSITY BASED PARTITIONING

| Year                | Enrollments<br>( $A_i$ ) | Fuzzy Rule      | FLRG  | Forecast<br>( $F_i$ ) | $A_i - F_i$ | $(A_i - F_i)^2$     | $\frac{ A_i - F_i }{A_i}$ |
|---------------------|--------------------------|-----------------|---|-----------------------|-------------|---------------------|---------------------------|
| 1971                | 13055                    | A <sub>1</sub>  | A <sub>1</sub> , A <sub>2</sub>                     | 13579                 | -524        | 274778              | 0.040153                  |
| 1972                | 13563                    | A <sub>2</sub>  | A <sub>1</sub> , A <sub>2</sub> , A <sub>3</sub>    | 13798                 | 235         | 55344               | 0.017345                  |
| 1973                | 13847                    | A <sub>2</sub>  | A <sub>1</sub> , A <sub>2</sub> , A <sub>3</sub>    | 13798                 | -49         | 2376                | 0.003520                  |
| 1974                | 14696                    | A <sub>3</sub>  | A <sub>2</sub> , A <sub>3</sub> , A <sub>4</sub>    | 14452                 | -244        | 59427               | 0.016588                  |
| 1975                | 15460                    | A <sub>5</sub>  | A <sub>4</sub> , A <sub>5</sub> , A <sub>6</sub>    | 15373                 | -87         | 7575                | 0.005630                  |
| 1976                | 15311                    | A <sub>5</sub>  | A <sub>4</sub> , A <sub>5</sub> , A <sub>6</sub>    | 15373                 | 62          | 3840                | 0.004047                  |
| 1977                | 15603                    | A <sub>6</sub>  | A <sub>5</sub> , A <sub>6</sub> , A <sub>7</sub>    | 15623                 | 20          | 400                 | 0.001282                  |
| 1978                | 15861                    | A <sub>7</sub>  | A <sub>6</sub> , A <sub>7</sub> , A <sub>8</sub>    | 15883                 | 22          | 487                 | 0.001391                  |
| 1979                | 16807                    | A <sub>10</sub> | A <sub>9</sub> , A <sub>10</sub> , A <sub>11</sub>  | 17079                 | 272         | 73765               | 0.016160                  |
| 1980                | 16919                    | A <sub>10</sub> | A <sub>9</sub> , A <sub>10</sub> , A <sub>11</sub>  | 17079                 | 160         | 25471               | 0.009433                  |
| 1981                | 16388                    | A <sub>9</sub>  | A <sub>8</sub> , A <sub>9</sub> , A <sub>10</sub>   | 16497                 | 109         | 11800               | 0.006629                  |
| 1982                | 15433                    | A <sub>5</sub>  | A <sub>4</sub> , A <sub>5</sub> , A <sub>6</sub>    | 15373                 | -60         | 3604                | 0.003890                  |
| 1983                | 15497                    | A <sub>5</sub>  | A <sub>4</sub> , A <sub>5</sub> , A <sub>6</sub>    | 15373                 | -124        | 15384               | 0.008004                  |
| 1984                | 15145                    | A <sub>4</sub>  | A <sub>3</sub> , A <sub>4</sub> , A <sub>5</sub>    | 15024                 | -121        | 14599               | 0.007978                  |
| 1985                | 15163                    | A <sub>4</sub>  | A <sub>3</sub> , A <sub>4</sub> , A <sub>5</sub>    | 15024                 | -139        | 19272               | 0.009155                  |
| 1986                | 15984                    | A <sub>7</sub>  | A <sub>6</sub> , A <sub>7</sub> , A <sub>8</sub>    | 15883                 | -101        | 10188               | 0.006315                  |
| 1987                | 16859                    | A <sub>10</sub> | A <sub>9</sub> , A <sub>10</sub> , A <sub>11</sub>  | 17079                 | 220         | 48223               | 0.013025                  |
| 1988                | 18150                    | A <sub>11</sub> | A <sub>10</sub> , A <sub>11</sub> , A <sub>12</sub> | 17991                 | -159        | 25136               | 0.008735                  |
| 1989                | 18970                    | A <sub>12</sub> | A <sub>11</sub> , A <sub>12</sub> , A <sub>13</sub> | 18802                 | -168        | 28221               | 0.008856                  |
| 1990                | 19328                    | A <sub>13</sub> | A <sub>12</sub> , A <sub>13</sub>                   | 18994                 | -334        | 111886              | 0.017306                  |
| 1991                | 19337                    | A <sub>13</sub> | A <sub>12</sub> , A <sub>13</sub>                   | 18994                 | -343        | 117988              | 0.017764                  |
| 1992                | 18876                    | A <sub>12</sub> | A <sub>11</sub> , A <sub>12</sub> , A <sub>13</sub> | 18916                 | 40          | 1600                | 0.002119                  |
| <b>MSE=41425.56</b> |                          |                 |   |                       |             | <b>AFER=1.0242%</b> |                           |

TABLE VI  
A COMPARISON OF THE FORECASTING RESULTS OF DIFFERENT FORECASTING METHODS

| Year        | Enrollments | Song Chissom Method [18] | Song Chissom Method [19] | Chen's [2] | Hwang, Chen & Lee's [8] | Huang's [5] | Chen's [4] | Jilani and Burney [9] | Jilani, Burney and Ardil [10] | Proposed Method |
|-------------|-------------|--------------------------|--------------------------|------------|-------------------------|-------------|------------|-----------------------|-------------------------------|-----------------|
| 1971        | 13055       | --                       | --                       | --         | --                      | --          | --         | --                    | 14464                         | 13579           |
| 1972        | 13563       | 14000                    | --                       | 14000      | --                      | 14000       | --         | --                    | 14464                         | 13798           |
| 1973        | 13867       | 14000                    | --                       | 14000      | --                      | 14000       | --         | --                    | 14464                         | 13798           |
| 1974        | 14696       | 14000                    | --                       | 14000      | --                      | 14000       | 14500      | 14730                 | 14710                         | 14452           |
| 1975        | 15460       | 15500                    | 14700                    | 15500      | --                      | 15500       | 15500      | 15615                 | 15606                         | 15373           |
| 1976        | 15311       | 16000                    | 14800                    | 16000      | 16260                   | 15500       | 15500      | 15614                 | 15606                         | 15373           |
| 1977        | 15603       | 16000                    | 15400                    | 16000      | 15511                   | 16000       | 15500      | 15611                 | 15606                         | 15623           |
| 1978        | 15861       | 16000                    | 15500                    | 16000      | 16003                   | 16000       | 15500      | 15611                 | 15606                         | 15883           |
| 1979        | 16807       | 16000                    | 15500                    | 16000      | 16261                   | 16000       | 16500      | 16484                 | 16470                         | 17079           |
| 1980        | 16919       | 16813                    | 16800                    | 16833      | 17407                   | 17500       | 16500      | 16476                 | 16470                         | 17079           |
| 1981        | 16388       | 16813                    | 16200                    | 16833      | 17119                   | 16000       | 16500      | 16469                 | 16470                         | 16497           |
| 1982        | 15433       | 16789                    | 16400                    | 16833      | 16188                   | 16000       | 15500      | 15609                 | 15606                         | 15373           |
| 1983        | 15497       | 16000                    | 16800                    | 16000      | 14833                   | 16000       | 15500      | 15614                 | 15606                         | 15373           |
| 1984        | 15145       | 16000                    | 16400                    | 16000      | 15497                   | 15500       | 15500      | 15612                 | 15606                         | 15024           |
| 1985        | 15163       | 16000                    | 15500                    | 16000      | 14745                   | 16000       | 15500      | 15609                 | 15606                         | 15024           |
| 1986        | 15984       | 16000                    | 15500                    | 16000      | 15163                   | 16000       | 15500      | 15606                 | 15606                         | 15883           |
| 1987        | 16859       | 16000                    | 15500                    | 16000      | 16384                   | 16000       | 16500      | 16477                 | 16470                         | 17079           |
| 1988        | 18150       | 16813                    | 16800                    | 16833      | 17659                   | 17500       | 18500      | 18482                 | 18473                         | 17991           |
| 1989        | 18970       | 19000                    | 19300                    | 19000      | 19150                   | 19000       | 18500      | 18481                 | 18473                         | 18802           |
| 1990        | 19328       | 19000                    | 17800                    | 19000      | 19770                   | 19000       | 19500      | 19158                 | 19155                         | 18994           |
| 1991        | 19337       | 19000                    | 19300                    | 19000      | 19928                   | 19500       | 19500      | 19155                 | 19155                         | 18994           |
| 1992        | 18876       | --                       | 19600                    | 19000      | 19537                   | 19000       | 18500      | 18475                 | 18473                         | 18916           |
| <b>MSE</b>  | 423027      | 775687                   | 407507                   | 321418     | 226611                  | 86694       | 86694      | 82269                 | 227194                        | 41426           |
| <b>AFER</b> | 3.2238%     | 4.3800%                  | 3.1100%                  | 3.1169%    | 2.4452%                 | 1.5294%     | 1.5294%    | 1.4064%               | 2.3865%                       | 1.0242%         |

## V. CONCLUSION

In this paper, we have presented frequency density based partitioning of the historical enrollment data of the University of Alabama and applied improved fuzzy metric for forecasting. The proposed method belongs to the first order and time-variant methods. From Table VI, we can see that the AFER and MSE of the forecasting results of the proposed method are the smallest than that of the existing methods. In the future, we will extend the proposed method to deal with other forecasting problems based on fuzzy time series. We also will develop new methods for forecasting enrollments based on fuzzy parametric and semi-parametric approaches to get a higher forecasting accuracy.

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