

Optimizing Turning Parameters for Cylindrical Parts Using Simulated Annealing Method

Farhad Kolahan, and Mahdi Abachizadeh

Abstract—In this paper, a simulated annealing algorithm has been developed to optimize machining parameters in turning operation on cylindrical workpieces. The turning operation usually includes several passes of rough machining and a final pass of finishing. Seven different constraints are considered in a non-linear model where the goal is to achieve minimum total cost. The weighted total cost consists of machining cost, tool cost and tool replacement cost. The computational results clearly show that the proposed optimization procedure has considerably improved total operation cost by optimally determining machining parameters.

Keywords—Optimization, Simulated Annealing, Machining Parameters, Turning Operation.

I. INTRODUCTION

ECONOMICAL considerations have been the topic of many industrial researches. Regarding machining operations, these considerations usually lead the designers to seek optimum machining parameters in order to minimize machining costs. In turning, cutting speed, feed rate and cutting depth are the most important of these parameters. These are determined based on the constraints imposed by the specifications of the machine, cutting tool and the engaged workpiece. Among these constraints, tool life, cutting forces and the required surface roughness are frequently taken into account.

The number of modeled constraints has a direct clear relevance to the complexity of problem. In several case studies, the number of constraints is limited in order to facilitate finding the optimum [1-5]. In some others, additional considerations such as the chip-tool interface temperature are taken into account [6]. This constraint has been indirectly considered and controlled by the tool life in this paper.

The above mentioned parameters are of discrete nature or imposed to discretization due to practical limitations. Therefore, the discrete or continuous-discrete search space of the problem as well as the complex objective function necessitates the employment of methods other than classic approaches. Heuristic algorithms such as genetic algorithms (GA), simulated annealing (SA), tabu search (TS) and ant colony optimization (ACO) are powerful methods which can

serve in optimization of such problems where non-linear multi-minima functions with numerous variables are engaged. In this paper, simulated annealing is employed as it normally exhibits fast convergence and straightforward implementation.

II. MATHEMATICAL MODEL OF TURNING OPERATION

Defining a proper cost function involves performing several experiments and subsequent statistical analyses. Here, the cost function presented by Onwubolu and Kumalo [6] has been basically used.

This total cost function consists of three cost items including machining cost, tool and tool replacement costs which are initially calculated as time terms and then converted to a cost concept by means of proper coefficients. These items can be briefly explained as follows [6]:

The first term in the cost function, which is normally of higher weight, is the machining cost. It is defined based on the machining time and idle time, as follows:

$$c_1 = k_0 \left\{ \frac{\pi DL}{1000v_r f_r} \left(\frac{d_t - d_s}{d_r} \right) + \frac{\pi DL}{1000v_s f_s} + (h_1 L + h_2) \left(\frac{d_t - d_s}{d_r} + 1 \right) \right\} \quad (1)$$

where D and L are the diameter and length of the workpiece, respectively. The parameter d_t denotes the depth of material to be removed. The parameters d_r and d_s are the depth of cut for each pass of rough and finish machining. Also, v represents the cutting speed, and f the feed rate. The indices r and s represent rough and finish machining, respectively. In addition, k_0 , h_1 and h_2 are some constants.

The second item relates to the process of tool replacement:

$$c_2 = k_0 * t_e \left\{ \frac{1}{T_r} \left[\frac{\pi DL}{1000v_r f_r} \left(\frac{d_t - d_s}{d_r} \right) \right] + \frac{1}{T_s} \left(\frac{\pi DL}{1000v_s f_s} \right) \right\} \quad (2)$$

In the above function, T denotes the tool life and t_e is the tool replacement time.

The third term represents the tool cost which is formulized as the ratio of the cutting process of a certain tool to its defined life, given by:

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$$c_3 = k_t \left\{ \frac{1}{T_r} \left[\frac{\pi DL}{1000 v_r f_r} \left(\frac{d_t - d_s}{d_r} \right) + \frac{1}{T_s} \left[\frac{\pi DL}{1000 v_s f_s} \right] \right] \right\} \quad (3)$$

where k_t is the tool cost for each cutting edge.

As stated before, the total cost is obtained by adding all the above terms:

$$c_{total} = c_1 + c_2 + c_3 \quad (4)$$

At the first look, it seems that the total cost function can be simply minimized in the case of having bigger cutting depth in each pass. However, with the increase of cutting depth, the tool cost rapidly rises. Similar misinterpretation may happen about higher cutting speeds assuming to result in lower cost functions. In fact, higher cutting speeds increases the energy consumption that is directly a lift in total costs. Consequently, some constraints are needed to be defined in order to make the total cost function more realistic. The process constraints considered in this paper are as follows:

1- The available range of cutting speed:

$$v_l \leq v = \frac{\pi DN}{1000} \leq v_u \quad (5)$$

where v_l and v_u are the minimum and maximum allowable cutting speed, respectively. N is also the rotational speed of the spindle.

2- The allowable feed rate:

$$f_l \leq f \leq f_u \quad (6)$$

where f_l and f_u are the minimum and maximum allowable feed rates, respectively. These bounds are determined with respect to the workpiece and tool material.

3- The allowable cutting depth:

$$d_l \leq d \leq d_u \quad (7)$$

where d_l and d_u are the minimum and maximum allowable cutting depth, respectively.

4- The constraint on the tool life prevents the algorithm to select higher values of cutting speed and cutting depth. This is due to the fact that when the tool comes to the end of its life, machining is no longer economical since excessive heat is produced and also surface roughness is affected. Life of tools is predicted with the help of empirical models. Here, the model presented in [6] is adopted:

$$T_l \leq \frac{c_0}{v^p f^q d^r} \leq T_u \quad (8)$$

where c_0 , p , q and r are constants found experimentally. T_l and T_u are also the minimum and maximum predicted life of tool in minute.

5- Cutting forces impose another constraint on the model. The maximum amount of cutting forces F_u should not exceed a certain value as higher forces produce shakes and vibration. This constraint is also modeled as proposed by [6]:

$$F_c = k_f f^\mu d^\nu \leq F_u \quad (9)$$

where k_f , μ and ν are the constants similar to those in (8).

6- Furthermore, the nominal power of the machine P_u limits the cutting process:

$$P_c = \frac{k_f f^\mu d^\nu v}{60000\eta} = \frac{F_c v}{60000\eta} \leq P_u \quad (10)$$

Here, η is the machine efficiency.

In this model demonstrated through (1) to (11), the design variables are the cutting speed v_r , feed rate f_r , and cutting depth d_r , all related to the rough machining. The objective function is also the total cost function c_{total} which is minimized using simulated annealing method.

III. SIMULATED ANNEALING METHOD

Simulated annealing algorithm is a nature-inspired method which is adapted from the process of gradual cooling of metals in nature. In the metallurgical annealing process, a solid is melted at high temperature until all molecules can move about freely, and then a cooling process is performed until thermal mobility is lost. The perfect crystal is the one in which all atoms are arranged in a low level lattice, so the crystal reaches the minimum energy. At the temperature of T , the solid is allowed to reach a certain thermal equilibrium status. The probability of being at the energy level of E is determined by the Boltzmann distribution:

$$pr(E) = \frac{1}{Z(T)} \times \exp\left(-\frac{E}{K_B \cdot T}\right) \quad (11)$$

where $Z(T)$ is a normalization factor and is dependent to the temperature T . The parameter K_B is the Boltzmann constant and the exponential term is the Boltzmann coefficient. With the decrease of temperature, the Boltzmann distribution focuses on a state with lowest energy and finally as the temperature comes close to zero, this becomes the only possible state (see Fig. 1).

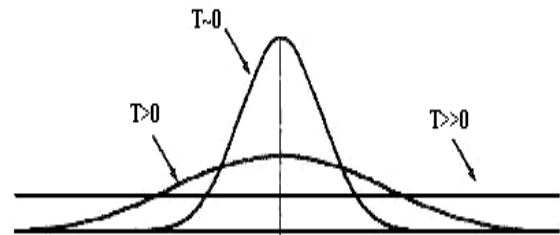


Fig. 1 Distribution of probability for three different temperatures

The SA code employed in this paper operates based on a neighborhood structure where at each step the coded candidate solution is replaced by one of the neighbor combinations. Just like the other optimization methods, the algorithm of simulated annealing consists of operating parameters that should be well set in order to achieve its best performance. These are briefly mentioned in the following.

1- The initial temperature acts as the state of the system at the beginning of the optimization procedure. In general, the initial temperature is set in a way to allow the acceptance of most of the transitions in the neighborhood moves. In the other word, it is desired to satisfy the following condition:

$$\exp\left(-\frac{\Delta c}{T_0}\right) \cong 1 \quad (12)$$

where Δc is the difference between the amount of the objective function of the current and candidate solution.

It has been proved that T_0 is critical in obtaining optimum or in any case, feasible solutions [7]. Bigger amounts of T_0 prevent reaching good solutions in a reasonable time as the search space becomes very large. Smaller T_0 may also reduce the effectiveness of the algorithm as the probability of getting trapped in local minima increases. Here, the initial temperature is set to 1000.

2- Termination criterion determines in what condition the algorithm stops moving ahead. For the SA algorithm, the final temperature of system is a common preference. The maximum number of iterations is another approach which has been selected in this paper. The maximum allowed number of subsequent iterations without any improvement is also another proper criterion.

3- The cooling schedule is an important feature of this algorithm. Different models have been proposed among which the cooling with constant factor is a simple robust strategy first suggested by Kirkpatrick as follows [8]:

$$T_{K+1} = \alpha * T_K \quad K = 0,1,2,\dots \quad (13)$$

In the generalized approach, α may vary with respect to the temperature. Here, it is considered constant and set to 0.98.

The SA algorithm operates based on a biased randomness. In multi-variable problems such as the one investigated here, one of the variables are selected randomly. In the next step, a neighborhood is generated and in the case of being feasible i.e. not violating the constraints, is studied whether it is acceptable for the next move. If this new candidate exhibits lower objective function, the move is accepted. Otherwise, a chance is given to this move as it helps escaping the local minima. This decision depends on the satisfaction of the following inequality:

$$\exp\left(-\frac{\Delta C}{T}\right) > 0.1 \quad (14)$$

It is noted that at each accepted move, the temperature decreases by the cooling schedule.

A range of variation has been assigned for the design variables. As discussed in section I, these parameters which are shown in Table I, are discretized and constrained according to practical limitations.

TABLE I
 THE ALLOWABLE RANGES OF MACHINING PARAMETERS

| Parameters | Cutting Speed (m/min) | Feed Rate (mm/rev) | Cutting Depth (mm) |
|---------------------------|-----------------------|--------------------|--------------------|
| Range of variation | 50 - 250 | 0.1 - 1 | 0.9 - 3 |
| Discretization increments | 5 | 0.05 | 0.05 |

IV. NUMERICAL PROBLEM DESCRIPTION

The problem investigated here is the minimization of manufacturing cost for a cylindrical workpiece which is

machined by a carbide tool. This workpiece is made out of a round bar with the length of 300 and the diameter of 240 mm. The only performed machining operation is turning with the purpose of achieving the desired diameter. The initial parameters are selected randomly and are only verified for being feasible.

The constants and coefficients required to complete the formulation of the optimization should be entirely acquired using statistical and empirical studies. Here, these parameters are used as proposed by Onwubolu and Kumalo [6] and Chen and Tsai [9] which are presented in Table II. It is notable that they are obtained for steels with a middle ranged amount of carbon.

TABLE II
 THE PARAMETERS USED IN (1) – (10)

| | | | |
|--------------------------|--------------------------|------------------------|-----------------------|
| $k_0 = 0.5$ \$/min | $k_t = 2.5$ \$/min | $t_c = 0.75$ min/piece | $t_e = 11.5$ min/edge |
| $h_1 = 7 \times 10^{-4}$ | $h_2 = 0.3$ | $F_u = 5.0$ kg.f | $T_u = 200$ min |
| $\eta = 0.85$ | $c_0 = 6 \times 10^{11}$ | $k_f = 108$ | $T_f = 50$ min |
| $\mu = 0.75$ | $p = 5$ | $q = 1.75$ | $P_u = 200$ kw |
| $v = 0.95$ | $r = 0.75$ | $d_r = 20.1$ mm | $f_s = 0.1$ mm/rev |
| $v_s = 200$ m/min | | | |

V. RESULTS

The optimization is done using a code written in Matlab® on a Pentium IV 2400 GHz CPU. The code was run several times, each for 600 iterations, using different random starting points. Longer execution of the algorithm with more iterations and different start points showed no more improvement. The final result is summarized in Table III and seems to be the global optimum of the problem.

TABLE III
 THE OPTIMIZED TURNING PARAMETERS

| Stage | Total Cost (\$) | Cutting Speed (m/min) | Feed Rate (mm/rev) | Cutting Depth (mm) |
|--------------|-----------------|-----------------------|--------------------|--------------------|
| Final result | 37.58 | 145 | 0.25 | 2.75 |

The cost terms are also presented in Table IV. These include machining cost, tool cost and tool replacement cost.

TABLE IV
 THE OPTIMIZED MACHINING COSTS IN DETAIL

| Cost | Cost Amount (\$) | Cost Percentage (%) |
|-------------------------|------------------|---------------------|
| Machining and idle time | 29.53 | 78.5 |
| Tool | 5.61 | 15 |
| Tool Replacement | 2.44 | 6.5 |

It can be easily understood that the machining cost has the greatest share in the overall cost. In addition, as expected, the costs related to tool and tool replacement, are proportional.

The effect of considering tool cost and tool replacement

cost which helps in modeling the turning process in a more realistic way is also studied. This inclusion can also bring a form of improvement to the model. In the case of considering only the machining cost, as investigated by authors, the lowest cost function obtained is 15.86\$. Adding the tool and tool replacement costs regarding the initial settings, results in a total cost equal to 239.58\$. However, considering these two costs as part of the overall objective function, which is the strategy adopted in this paper, reduces the total cost to 37.58\$ as reported earlier. This reduction is a considerable improvement by just making the model more complex to a reasonable extent.

VI. CONCLUSION

In this paper, the problem of minimizing the machining cost in turning operation of a cylindrical workpiece has been investigated. To model the machining process, several important operational constraints have been considered. These constraints were taken to account in order to make the model more realistic. The multi-criteria objective function is the combination of machining cost, idle time cost, tool cost and tool replacement cost. To optimally determine machining parameters (cutting speed, feed rate and depth of cut), a simulated annealing method was employed. This algorithm is a powerful technique in optimization of problems with discrete search space and multiple local optima. The computational results clearly demonstrated that the proposed solution procedure is quite capable in solving such complicated problems effectively and efficiently.

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