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On the Robust Stability of Homogeneous Perturbed Large-Scale Bilinear Systems with Time Delays and Constrained Inputs

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***Abstract*—**The stability test problem for homogeneous large-scale perturbed bilinear time-delay systems subjected to constrained inputs is considered in this paper. Both nonlinear uncertainties and interval systems are discussed. By utilizing the Lyapunove equation approach associated with linear algebraic techniques, several delay-independent criteria are presented to guarantee the robust stability of the overall systems. The main feature of the presented results is that although the Lyapunov stability theorem is used, they do not involve any Lyapunov equation which may be unsolvable. Furthermore, it is seen the proposed schemes can be applied to solve the stability analysis problem of large-scale time-delay systems.

***Keywords*—**homogeneous bilinear system; constrained input; time-delay, uncertainty; transient response; decay rate.

I. INTRODUCTION

IT is known that not only engineering areas such as nuclear, thermal, and chemical processes but also physical systems such as biology, socio-economics, and ecology, may be modeled as bilinear systems [3, 22]. During the past decades, a number of contributions hence have been devoted to the study of bilinear systems [1, 2, 4-19, 21, 23-27]. Of those works, the stability analysis problem has been studied in [7, 8, 10, 12, 17, 19, 23]. Furthermore, in [1, 2, 4-6, 9, 11, 13-16, 18, 21, 24-26], the stabilizing controller design for bilinear systems have been proposed. In practice, due to the information transmission, natural property of system elements, computation of variables, etc, time delay(s) exist(s) in real-life systems [20]. Besides, when modeling a control system, system perturbations that are occurred as a result of using approximate system model for simplicity, data errors for evaluation, changes in environment conditions, aging, etc, also exist naturally. Therefore, time delay(s) and perturbations ought to be integrated into the model of bilinear systems. Recently, the research of bilinear systems with time-delay and/or perturbations has also been of great interest. In [12, 23, 29], the stability test problem for bilinear time-delay systems has been discussed. In [15] and [21], global stabilization controller design for bilinear systems with time delay has been proposed. Furthermore, the stabilization controller design for perturbed bilinear systems has been discussed in [11, 26]. The stability analysis of bilinear systems with time-delay and perturbations has been addressed in [19]. However, it seems that only few works have deal with the stability analysis for large-scale bilinear systems without time-delay and perturbations [10, 27]. As mentioned in the above descriptions, time-delay and perturbations should be considered in system model. Thus, this paper studies the stability analysis problem for large-scale perturbed bilinear time-delay systems with constrained inputs. Two kinds of perturbation are discussed: nonlinear perturbation and interval matrices. Several criteria that assure the robust stability of the overall systems are derived by using the Lyapunov equation approach associated linear algebraic techniques. It is shown that the obtained criteria do not involve any Lyapunov equation which maybe unsolvable. Furthermore, it is also seen that the presented schemes can be applied to the stability analysis of large-scale perturbed time-delay systems. Finally, we give numerical examples to demonstrate the applicability of the proposed results.

The following symbol conventions are used in this paper. Symbols $\Re$, A^T, $\bar{\lambda}(A)$, $x^T(t)$, $\|x(t)\|$, and $\|A\|$, respectively, means real number field, transpose of matrix *A*, the maximal eigenvalue of a symmetric matrix *A*, transpose of vector $x(t)$, norm of vector $x(t)$ with $\|x(t)\| = \left(x^T(t)x(t)\right)^{1/2}$, and induced norm of matrix *A* with $\|A\| = \bar{\lambda}(A^T A)^{1/2}$. Furthermore, $\mu(A)$ represents the matrix measure of *A* and is defined as $\mu(A) = \bar{\lambda}((A + A^T)/2)$.

II. SYSTEMS WITH NONLINEAR PERTURBATIONS

Consider a homogeneous large-scale perturbed bilinear time-delay system *S* which is described as an interconnection of *N* subsystems $S_1, S_2, \ldots, S_N$ represented by

$$S_i:\ \dot{x}_i(t) = A_i x_i(t) + \sum_{j=1}^{N} f_{ij}(x_j(t-d_{ij}), t) + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij} x_j(t-d_{ij})$$

$$+\sum_{k=1}^{m_i} sat\, u_{ik}(t) B_{ik} x_i(t), \quad i = 1, 2, \ldots, N \qquad (1)$$

where $x_i(\cdot) \in \Re^n$ and $u_{ik}(\cdot) \in \Re$ represent the state vector and the input, respectively, A_i, A_{ij}, and B_{ik} are constant matrices

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with appropriate dimensions, $d_{ij} > 0$ for all i and j with $d_{ii} = 0$ denote the communication delays in the interconnections of the large-scale system *S*, and $f_{ij}(x_j(t-d_{ij}),t)$ represent nonlinear perturbations having the following properties:

$$\left\| f_{ij}(x_j(t-d_{ij}),t) \right\| \le \varepsilon_{ij} \left\| x_j(t-d_{ij}) \right\| \quad (2)$$

where ε_{ij} are positive constants. The constrained inputs $sat\, u_{ik}(t)$ are saturating functions and defined as follows.

$$sat\, u_{ik}(t) = \begin{cases} u_{ik}(t), & if \ \left|u_{ik}(t)\right| \le U_{ik} \\ U_{ik}\,\mathrm{sgn}(u_{ik}(t)), & if \ \left|u_{ik}(t)\right| > U_{ik} > 0 \end{cases} \quad (3)$$

where U_{ik} denote positive constants. From (3), one has

$$\left| sat\, u_{ik}(t) \right| \le U_{ik}, \quad k = 1, 2, \ldots, m_i \quad (4)$$

Define constants β_i, $i = 1, 2, \ldots, N$ as

$$\beta_i \equiv \left\| \sum_{k=1}^{m_i} U_{ik}^2 B_{ik}^T B_{ik} \right\|. \quad (5)$$

Lemma 1[28]: For the Lyapunov equation

$$A^T P + PA = -Q \quad (6)$$

where *A* is a Hurwitz matrix and *Q* is a given positive definite matrix, the unique positive solution *P* has the upper bound

$$\|P\| \le \frac{\|Q\|}{-2\mu(A)} \quad (7)$$

in which $\mu(A) < 0$.

Then the application of Lemma 1 associated with the Lyapunov stability theorem and linear algebraic techniques, we have the following result.

Theorem 1: If the following conditions are met for $i = 1, 2, \ldots, N$

$$\mu(A_i) + \sqrt{2N_i + m_i + 1}\left(\sum_{j=1}^{N} \varepsilon_{ji}^2 + \left\| \sum_{j \ne i}^{N} A_{ji}^T A_{ji} \right\| + \beta_i \right) < 0 \quad (8)$$

where N_i denote the number of $A_{ij} \ne 0$ corresponding to the i-th subsystem with $j = 1, 2, \ldots, N$, then the full order large-scale perturbed bilinear time-delay system (1) with the constraints (3) is robustly stable.

Proof: For convenience, we use x_i, u_{ik}, and f_{ij} to represent $x_i(t)$, $u_{ik}(t)$, and $f_{ij}(x_j(t-d_{ij}),t)$ for all i and j, respectively, in the following and later descriptions. Condition (8) infers that $\mu(A_i) < 0$ which means that matrices A_i are stable. Then, one can conclude that for a given positive constant q_i, the Lyapunov equation

$$A_i^T P_i + P_i A_i = -2q_i I, \ i = 1, 2, \ldots, N \quad (9)$$

has a unique positive definite solution P_i. The Lyapunov function $V(x_i(t),t)$ for large-scale system *S* is chosen as

$$V(x_i(t),t) = \sum_{i=1}^{N} V_i(x_i(t),t)$$

$$= \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + \sum_{\substack{j=1 \\ j \ne i}}^{N} \int_{t-d_{ij}}^{t} x_j^T(s) \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j(s) ds \right\} \quad (10)$$

wherer P_i satisfies the Lyapunov equation (9). Taking the derivative of $V(x_i(t),t)$ along trajectories of (1) gives

$$\dot V(x_i(t),t) = \sum_{i=1}^{N} \left\{ \dot x_i^T P_i x_i + x_i^T P_i \dot x_i + \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j^T \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j \right.$$
$$\left. - \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j^T(t-d_{ij}) \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \right\}$$

$$= \sum_{i=1}^{N} \left\{ \left[A_i x_i + \sum_{j=1}^{N} f_{ij} + \sum_{\substack{j=1 \\ j \ne i}}^{N} A_{ij} x_j(t-d_{ij}) + \sum_{k=1}^{m} sat\, u_{ik} B_{ik} x_i \right]^T P_i x_i \right.$$
$$+ x_i^T P_i \left[A_i x_i + \sum_{j=1}^{N} f_{ij} + \sum_{\substack{j=1 \\ j \ne i}}^{N} A_{ij} x_j(t-d_{ij}) + \sum_{k=1}^{m} sat\, u_{ik} B_{ik} x_i \right]$$
$$+ \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j^T \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j$$
$$\left. - \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j^T(t-d_{ij}) \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \right\}$$

$$= \sum_{i=1}^{N} \left\{ x_i^T (A_i^T P_i + P_i A_i) x_i + \sum_{j=1}^{N} f_{ij}^T P_i x_i + x_i^T P_i \sum_{j=1}^{N} f_{ij} \right.$$
$$+ \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j^T(t-d_{ij}) A_{ij}^T P_i x_i + x_i^T P_i \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j(t-d_{ij}) A_{ij}$$
$$+ x_i^T \left[\sum_{k=1}^{m} sat\, u_{ik} B_{ik}^T P_i + P_i \sum_{k=1}^{m} sat\, u_{ik} B_{ik} \right] x_i$$
$$+ \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j^T \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j$$
$$\left. - \sum_{\substack{j=1 \\ j \ne i}}^{N} x_j^T(t-d_{ij}) \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \right\} \quad (11)$$

We have

$$\sum_{j=1}^{N} f_{ij}^T P_i x_i + x_i^T P_i \sum_{j=1}^{N} f_{ij} \le (N_i + 1) x_i^T P_i^2 x_i + \sum_{j=1}^{N} f_{ij}^T f_{ij} \quad (12)$$

$$\sum_{\substack{j=1\\j\neq i}}^{N} x_j^T(t-d_{ij})A_{ij}^T P_i x_i + x_i^T P_i \sum_{\substack{j=1\\j\neq i}}^{N} x_j(t-d_{ij})A_{ij} \leq N_i x_i^T P_i^2 x_i + \sum_{\substack{j=1\\j\neq i}}^{N} x_j^T(t-d_{ij})A_{ij}^T A_{ij} x_j(t-d_{ij}) \tag{13}$$

$$x_i^T \sum_{k=1}^{m} sat\, u_{ik} B_{ik}^T P_i x_i + x_i^T P_i \sum_{k=1}^{m} sat\, u_{ik} B_{ik} x_i \leq m_i x_i^T P_i^2 x_i + x_i^T \sum_{k=1}^{m} (sat\, u_{ik})^2 B_{ik}^T B_{ik} x_i \tag{14}$$

Substituting the above relations into (10) yields

$$\begin{aligned}
&\dot{V}(x_i(t),t) \\
&\leq \sum_{i=1}^{N} \Big\{ x_i^T(A_i^T P_i + P_i A_i)x_i + (N_i+1)x_i^T P_i^2 x_i \\
&\quad + \sum_{j=1}^{N} f_{ij}^T f_{ij} + N_i x_i^T P_i^2 x_i + \sum_{\substack{j=1\\j\neq i}}^{N} x_j^T(t-d_{ij})A_{ij}^T A_{ij} x_j(t-d_{ij}) \\
&\quad + m_i x_i^T P_i^2 x_i + x_i^T \sum_{k=1}^{m} (sat\, u_{ik})^2 B_{ik}^T B_{ik} x_i \\
&\quad + \sum_{\substack{j=1\\j\neq i}}^{N} x_j^T \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j \\
&\quad - \sum_{\substack{j=1\\j\neq i}}^{N} x_j^T(t-d_{ij}) \left[A_{ij}^T A_{ij} + \varepsilon_{ij}^2 I \right] x_j(t-d_{ij}) \Big\} \\
&\leq \sum_{i=1}^{N} \Big\{ x_i^T(A_i^T P_i + P_i A_i)x_i + (N_i+1)x_i^T P_i^2 x_i + \sum_{j=1}^{N} \left\| f_{ij}^T \right\| \left\| f_{ij} \right\| \\
&\quad + N_i x_i^T P_i^2 x_i + \sum_{\substack{j=1\\j\neq i}}^{N} x_j^T A_{ij\,i}^T A_{ij} x_j + m_i P_i^2 x_i^T x_i + \beta_i x_i^T x_i \\
&\quad + \sum_{\substack{j=1\\j\neq i}}^{N} \varepsilon_{ij}^2 x_j^T x_j - \sum_{\substack{j=1\\j\neq i}}^{N} \varepsilon_{ij}^2 x_j^T(t-d_{ij}) x_j(t-d_{ij}) \Big\} \\
&\leq \sum_{i=1}^{N} \Big\{ x_i^T(A_i^T P_i + P_i A_i)x_i + (N_i+1)\left\| P_i \right\|^2 x_i^T x_i + \sum_{j=1}^{N} \varepsilon_{ij}^2 x_j^T x_j \\
&\quad + N_i x_i^T \left\| P_i \right\|^2 x_i + \sum_{\substack{j=1\\j\neq i}}^{N} x_j^T A_{ij}^T A_{ij} x_j + m_i \left\| P_i \right\|^2 x_i^T x_i + \beta_i x_i^T x_i \\
&\leq \sum_{i=1}^{N} x_i^T \Big\{ -2q_i + \sum_{j=1}^{N} \varepsilon_{ji}^2 + (2N_i + m_i + 1)\left\| P_i \right\|^2 \\
&\quad + \Big\| \sum_{\substack{j=1\\j\neq i}}^{N} A_{ji}^T A_{ji} \Big\| + \beta_i \Big\} x_i
\end{aligned} \tag{15}$$

From Lemma 1 and (9), we hav

$$\left\| P_i \right\| \leq \frac{q_i}{-\mu(A_i)}, \quad i = 1, 2, \ldots, N \tag{16}$$

Substituting this inequality into (15) leads to

$$\begin{aligned}
&\dot{V}(x_i(t),t) \\
&\leq \sum_{i=1}^{N} x_i^T \Big\{ -2q_i + \sum_{j=1}^{N} \varepsilon_{ji}^2 + (2N_i + m_i + 1)\left(\frac{q_i}{\mu(A_i)} \right)^2 \\
&\quad + \Big\| \sum_{\substack{j=1\\j\neq i}}^{N} A_{ji}^T A_{ji} \Big\| + \beta_i \Big\} x_i
\end{aligned} \tag{17}$$

Let q_i be selected as

$$q_i = \frac{\mu^2(A_i)}{2(2N_i + m_i + 1)} \tag{18}$$

Then (17) becomes

$$\dot{V}(x_i(t),t) \leq \sum_{i=1}^{N} x_i^T \left\{ \sum_{j=1}^{N} \varepsilon_{ji}^2 + \Big\| \sum_{\substack{j=1\\j\neq i}}^{N} A_{ji}^T A_{ji} \Big\| + \beta_i - \frac{\mu(A_i)^2}{2N_i + m_i + 1} \right\} x_i \tag{19}$$

Therefore, it is seen that the condition (8) can guarantee $\dot{V}(x_i(t),t)$ is negative and the large-scale system S is robustly stable. Thus, the proof is completed. □

Remark 1: It is seen that if $B_{ik} = 0$ for all i and k, then system (1) become a perturbed large-scale system which is described as an interconnection of N subsystems $S_1, S_2, \ldots, S_N$ represented by

$$S_i: \; \dot{x}_i(t) = A_i x_i(t) + \sum_{j=1}^{N} f_{ij}(x_j(t-d_{ij}), t) + \sum_{\substack{j=1\\j\neq i}}^{N} A_{ij} x_j(t-d_{ij}) \tag{20}$$

where $i = 1, 2, \ldots, N$. Then, according to the proof of Theorem 1, we can obtain the following result without proof.

Corollary 1: The perturbed large-scale system described by (20) is robustly stable if for $i = 1, 2, \ldots, N$,

$$\mu(A_i) + \sqrt{2N_i + 1} \left(\sum_{j=1}^{N} \varepsilon_{ji}^2 + \Big\| \sum_{\substack{j=1\\j\neq i}}^{N} A_{ji}^T A_{ji} \Big\| \right) < 0 \tag{21}$$

III. LARGE-SCALE BILINEAR INTERVAL SYSTEMS

Consider a homogeneous large-scale bilinear interval system $\tilde{S}$ which is described as an interconnection of N subsystems $\tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_N$ represented by

$$\tilde{S}_i: \dot{x}_i(t) = \tilde{A}_i x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij} x_j(t-d_{ij}) + \sum_{k=1}^{m_i} sat\, u_{ik}(t)\tilde{B}_{ik} x_i(t) \quad (22)$$

where $i = 1, 2, \ldots, N$, $x_i(\cdot) \in \Re^n$, $d_{ij} \geq 0$, and $u_{ik}(\cdot) \in \Re$ are the same as those in system (1), $\tilde{A}_i = [\tilde{a}_{ipq}]$, $\tilde{A}_{ij} = [\tilde{a}_{ijpq}]$, and $\tilde{B}_{ik} = [\tilde{b}_{ikpq}]$ are interval matrices with appropriate dimensions and the properties:

$$\tilde{A}_i \in N[U_i, V_i] \text{ with } U_i = [u_{ipq}], V_i = [v_{ipq}]. \quad (23)$$

$$\tilde{A}_{ij} \in N[U_{ij}, V_{ij}] \text{ with } U_{ij} = [u_{ijpq}], V_{ij} = [v_{ijpq}] \quad (24)$$

$$\tilde{B}_{ik} \in N[E_{ik}, F_{ik}] \text{ with } E_{ik} = [e_{ikpq}], F_{ik} = [f_{ikpq}] \quad (25)$$

Functions $N[U_i, V_i]$, $N[U_{ij}, V_{ij}]$, and $N[E_{ik}, F_{ik}]$ present that the set of all matrices $\tilde{A}_i$, $\tilde{A}_{ij}$, and B_{ik} satisfying

$$u_{ipq} \leq \tilde{a}_{ipq} \leq v_{ipq},\ u_{ijpq} \leq \tilde{a}_{ij} \leq v_{ijpq},\ e_{ikpq} \leq b_{ikpq} \leq f_{ikpq} \quad (26)$$

where $p, q = 1, 2, \ldots, n$. Define

$$\hat{A}_i = [\hat{a}_{ipq}] \equiv \frac{U_i + V_i}{2} \text{ and } L_i = [l_{ipq}] \equiv \frac{V_i - U_i}{2} \quad (27)$$

where $p, q = 1, 2, \ldots, n$.

Then the system (22) can be represented as

$$\tilde{S}_i: \dot{x}_i(t) = (\hat{A}_i + \Delta\hat{A}_i) x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij} x_j(t-d_{ij}) + \sum_{k=1}^{m_i} sat\, u_{ik}(t)\tilde{B}_{ik} x_i(t) \quad (28)$$

where $\Delta\hat{A}_i$ denotes parametric uncertainty with the property:

$$\left|\Delta\hat{A}_i\right| \leq L_i \quad (29)$$

which means $\left|\Delta\hat{A}_i\right| = [\left|\Delta\hat{a}_{ipq}\right|]$ and $\left|\Delta\hat{a}_{ipq}\right| \leq l_{ipq}$ for $p, q = 1, 2, \ldots, n$. Define

$$\bar{A}_{ij} = [\bar{a}_{ijpq}] \text{ with } \bar{a}_{ijpq} \equiv \max(\left|u_{ijpq}\right|, \left|v_{ijpq}\right|) \quad (30)$$

$$\bar{B}_{ik} = [\bar{b}_{ikpq}] \text{ with } \tilde{b}_{kij} \equiv \max(\left|e_{ikpq}\right|, \left|f_{ikpq}\right|). \quad (31)$$

This results in

$$\left|\tilde{A}_{ij}\right| \leq \bar{A}_{ij} \text{ and } \left|\tilde{B}_{ik}\right| \leq \bar{B}_{ik}. \quad (32)$$

Then due to the well-known facts that $\|A\| \leq \|\,|A|\,\|$ and $\mu(A) \leq \mu(|A|)$, we have

$$\left\|\tilde{A}_{ij}\right\| \leq \left\|\,\left|\tilde{A}_{ij}\right|\,\right\| \leq \left\|\bar{A}_{ij}\right\|,\ \left\|\tilde{B}_{ik}\right\| \leq \left\|\bar{B}_{ik}\right\|, \text{ and}$$

$$\mu(\Delta\hat{A}_i) \leq \mu(\left|\Delta\hat{A}_i\right|) \leq \mu(L_i) \quad (33)$$

Furthermore, we also define the following constants

$$\bar{\beta}_i \equiv \sum_{k=1}^{m} U_{ik}^2 \left\|\bar{B}_{ik}\right\|^2,\ i = 1, 2, \ldots, N. \quad (34)$$

Let $\tilde{N}_i$ denote the number of $\tilde{A}_{ij} \neq 0$ corresponding to the i-th subsystem with $j = 1, 2, \ldots, N$. Then in light of Theorem 1, (33), and (34), we have the following result.

Theorem 2: If the following conditions are satisfied for $i = 1, 2, \ldots, N$

$$\mu\left(\hat{A}_i\right) + \mu(L_i) + \sqrt{\tilde{N}_i + m_i}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} \left\|\bar{A}_{ji}\right\|^2 + \bar{\beta}_i\right) < 0 \quad (35)$$

then the large-scale bilinear interval system (22) with the constraints (3) is robustly stable.

Proof: By the relation $\mu(A+B) \leq \mu(A) + \mu(B)$, we have

$$\begin{aligned} \mu\left(\hat{A}_i + \Delta\hat{A}_i\right) &\leq \mu\left(\hat{A}_i\right) + \mu(\Delta\hat{A}_i) \leq \mu\left(\hat{A}_i\right) + \mu(\left|\Delta\hat{A}_i\right|) \\ &\leq \mu\left(\hat{A}_i\right) + \mu(L_i) \end{aligned} \quad (36)$$

Then, from (34) and (35), it is obviously that matrices $\hat{A}_i + \Delta\hat{A}_i$ are stable. We choose the Lyapunov function $V(x_i(t), t)$ for the large-scale system (22) as

$$V(x_i(t), t) = \sum_{i=1}^{N} V_i(x_i(t), t)$$

$$= \sum_{i=1}^{N}\left\{x_i^T P_i x_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} \int_{t-d_{ij}}^{t} x_j^T(s)\tilde{A}_{ij}\tilde{A}_{ij} x_j(s) ds\right\} \quad (37)$$

wherer P_i, $i = 1, 2, \ldots, N$, satisfies the following Lyapunov equation

$$(\hat{A}_i + \Delta\hat{A}_i)^T P_i + P_i(\hat{A}_i + \Delta\hat{A}_i) = -2q_i I. \quad (38)$$

Then the solutions P_i have the bounds

$$\|P_i\| \leq \frac{q_i}{-\mu(\hat{A}_i + \Delta\hat{A}_i)} \leq \frac{q_i}{-[\mu(\hat{A}_i) + \mu(L_i)]}. \quad (39)$$

Taking the derivative of $V(x_i(t), t)$ along trajectories of (28) results in

$$\begin{aligned} &\dot{V}(x_i(t), t) \\ &= \sum_{i=1}^{N}\left\{\dot{x}_i^T P_i x_i + x_i^T P_i \dot{x}_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T \tilde{A}_{ij}^T P_i \tilde{A}_{ij} x_j - \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T(t-d_{ij})\tilde{A}_{ij}^T P_i \tilde{A}_{ij} x_j(t-d_{ij})\right\} \\ &= \sum_{i=1}^{N}\left\{x_i^T[(\hat{A}_i + \Delta\hat{A}_i)^T P_i + P_i(\hat{A}_i + \Delta\hat{A}_i)]x_i\right. \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j^T(t-d_{ij})\tilde{A}_{ij}^T P_i x_i + x_i^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} x_j(t-d_{ij})\tilde{A}_{ij} \\ &\quad + x_i^T\left[\sum_{k=1}^{m} sat\, u_{ik}\tilde{B}_{ik}^T P_i + P_i \sum_{k=1}^{m} sat\, u_{ik}\tilde{B}_{ik}\right] x_i \end{aligned}$$

$$\left.+\sum_{\substack{j=1\\ j\neq i}}^{N} x_j{}^T \tilde{A}_{ij}^T \tilde{A}_{ij} x_j - \sum_{\substack{j=1\\ j\neq i}}^{N} x_j{}^T (t-d_{ij}) \tilde{A}_{ij}^T \tilde{A}_{ij} x_j (t-d_{ij}) \right\} \quad (40)$$

By the similar ways as that of the proof of Theorem 1, we obtain

$$\dot{V}(x_i(t),t)$$
$$\leq \sum_{i=1}^{N} \Big\{ x_i^T [(\hat{A}_i + \Delta\hat{A}_i)^T P_i + P_i(\hat{A}_i + \Delta\hat{A}_i)] x_i + \tilde{N}_i x_i^T \|P_i\|^2 x_i$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{N} x_j{}^T \tilde{A}_{ij}^T \tilde{A}_{ij} x_j + m_i \|P_i\|^2 x_i^T x_i + \bar{\beta}_i x_i^T x_i \Big\}$$
$$\leq \sum_{i=1}^{N} x_i^T \left\{ -2q_i + (\tilde{N}_i + m_i)\|P_i\|^2 + \sum_{\substack{j=1\\ j\neq i}}^{N} \|\bar{A}_{ji}\|^2 + \bar{\beta}_i \right\} x_i \,. \quad (41)$$

Using (39) and selecting

$$q_i = \frac{\mu(\hat{A}_i) + \mu(L_i)}{2(\tilde{N}_i + m_i)}, \quad i = 1, 2, \ldots, N \quad (42)$$

inequality (41) becomes

$$\dot{V}(x_i(t),t) \leq \sum_{i=1}^{N} x_i^T \left\{ \sum_{\substack{j=1\\ j\neq i}}^{N} \|\bar{A}_{ji}\|^2 + \bar{\beta}_i - \frac{[\mu(\hat{A}_i) + \mu(L_i)]^2}{\tilde{N}_i + m_i} \right\} x_i \,. \quad (43)$$

From (43), it is seen that the condition (35) can assure the robust stability of the system $\tilde{S}$. Thus, the proof is completed.

Remark 2: Obviously the Lyapunov equation (39) is unsolvable. However, by utilizing the upper bound of the solution P_i, it is seen that an interesting consequence of the proposed schemes is that all obtained robust stability conditions do not involve any Lyapunov equation although the Lyapunov stability theorem is used.

Remark 3: Setting $\tilde{B}_{ik} = 0$ for all i and k in the system $\tilde{S}$, $\tilde{S}$ becomes a large-scale interval time-delay system and its subsystems are described as

$$\tilde{S}_i: \; \dot{x}_i(t) = \tilde{A}_i x_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} \tilde{A}_{ij} x_j(t-d_{ij}), \quad i = 1, 2, \ldots, N \quad (44)$$

In light of Theorem 2, we obtain directly the following result.

Corollary 2: If the following conditions are satisfied

$$\mu(\hat{A}_i) + \mu(L_i) + \sqrt{N_i} \sum_{\substack{j=1\\ j\neq i}}^{N} \|\bar{A}_{ji}\|^2 < 0, \quad i = 1, 2, \ldots, N \quad (45)$$

then the large-scale time-delay system (44) is robustly stable.

IV. ILLUSTRATIVE EXAMPLES

Demonstrations are given as below.

Example 1: Consider a large-scale bilinear perturbed time-delay system (1) as

$$\dot{x}_1(t) = \begin{bmatrix} -5 & 0.5 \\ 0 & -6 \end{bmatrix} x_1(t) + \sum_{j=1}^{3} f_{1j}(x_j(t-d_{1j}), t)$$
$$+\begin{bmatrix} -0.8 & 0.3 \\ 0 & 0.6 \end{bmatrix} x_2(t-d_{12}) + \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.6 \end{bmatrix} x_3(t-d_{13})$$
$$+sat\, u_{11}(t) \begin{bmatrix} 0.3 & -0.1 \\ 0.1 & 0.2 \end{bmatrix} x_1(t)$$
$$\dot{x}_2(t) = \begin{bmatrix} -6 & 0.5 \\ 0.3 & -6 \end{bmatrix} x_2(t) + \sum_{j=1}^{3} f_{2j}(x_j(t-d_{2j}), t)$$
$$+\begin{bmatrix} 0.5 & 0 \\ 0.2 & -0.6 \end{bmatrix} x_1(t-d_{21}) + \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & -0.4 \end{bmatrix} x_3(t-d_{23})$$
$$+sat\, u_{21}(t) \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix} x_2(t)$$
$$\dot{x}_3(t) = \begin{bmatrix} -7 & 1 \\ 0 & -6 \end{bmatrix} x_3(t) + f_{33}(x_3(t), t)$$
$$+\begin{bmatrix} -0.8 & 0.3 \\ 0 & -0.6 \end{bmatrix} x_1(t-d_{31}) + f_{31}(x_1(t-d_{31}), t)$$
$$+sat\, u_{31}(t) \begin{bmatrix} 0.4 & -0.1 \\ 0 & 0.2 \end{bmatrix} x_3(t) \,.$$

Assume $U_{11} = 1.2$, $U_{21} = 1.0$, and $U_{31} = 1.5$. For this case, it is seen that $m_i = 1$, $N_1 = 2$, $N_2 = 2$, and $N_3 = 1$. Now by the stability condition (8), we can estimate the perturbation bounds that can guarantee the asymptotic stability of this large-scale system as □

$$\varepsilon_{11}^2 + \varepsilon_{21}^2 + \varepsilon_{31}^2 < 0.6354$$
$$\varepsilon_{12}^2 + \varepsilon_{22}^2 < 1.3931$$
$$\varepsilon_{13}^2 + \varepsilon_{23}^2 + \varepsilon_{33}^2 < 1.8151$$

Example 2: Consider the following large-scale bilinear interval system:

$$\dot{x}_1(t) = \begin{bmatrix} [-6.5 \;\; -6] & [-0.5 \;\; 0.3] \\ 0 & [-5.5 \;\; -5] \end{bmatrix} x_1(t)$$
$$+\begin{bmatrix} [-0.5 \;\; -0.1] & [-0.2 \;\; 0] \\ [-0.1 \;\; 0.3] & [-0.5 \;\; -0.1] \end{bmatrix} x_2(t-d_{12})$$
$$+\begin{bmatrix} [0.3 \;\; 0.5] & [-0.2 \;\; 0] \\ [0 \;\; 0.2] & [-0.5 \;\; -0.2] \end{bmatrix} x_3(t-d_{13})$$
$$+sat\, u_{11}(t) \begin{bmatrix} [-0.1 \;\; 0.1] & 0 \\ [0.2 \;\; 0.4] & [-0.3 \;\; -0.1] \end{bmatrix} x_1(t)$$
$$\dot{x}_2(t) = \begin{bmatrix} [-6.5 \;\; -6] & [-0.5 \;\; -0.3] \\ [0.1 \;\; 0.2] & [-7.5 \;\; -6.5] \end{bmatrix} x_2(t)$$
$$+\begin{bmatrix} [-0.5 \;\; -0.2] & [-0.2 \;\; 0] \\ [-0.3 \;\; -0.1] & [0.4 \;\; 0.6] \end{bmatrix} x_3(t-d_{23})$$
$$+sat\, u_{21}(t) \begin{bmatrix} [-0.2 \;\; 0.1] & [0 \;\; 0.1] \\ [-0.2 \;\; 0] & [0.3 \;\; 0.4] \end{bmatrix} x_2(t)$$

$$+sat\, u_{22}(t)\begin{bmatrix}\begin{bmatrix}-0.1 & 0.1\end{bmatrix} & 0 \\ \begin{bmatrix}-0.2 & 0.1\end{bmatrix} & \begin{bmatrix}0.3 & 0.6\end{bmatrix}\end{bmatrix}x_2(t)$$

$$\dot{x}_3(t)=\begin{bmatrix}\begin{bmatrix}-5 & -4.5\end{bmatrix} & \begin{bmatrix}0 & 0.3\end{bmatrix} \\ \begin{bmatrix}-0.1 & 0\end{bmatrix} & \begin{bmatrix}-6.5 & -6.2\end{bmatrix}\end{bmatrix}x_3(t)$$

$$+\begin{bmatrix}\begin{bmatrix}0.5 & 0.6\end{bmatrix} & \begin{bmatrix}0.2 & 0.3\end{bmatrix} \\ \begin{bmatrix}-0.1 & 0\end{bmatrix} & \begin{bmatrix}-0.5 & -0.3\end{bmatrix}\end{bmatrix}x_1(t-d_{31})$$

$$+\begin{bmatrix}\begin{bmatrix}-0.4 & -0.2\end{bmatrix} & \begin{bmatrix}-0.3 & -0.2\end{bmatrix} \\ \begin{bmatrix}0.1 & 0.2\end{bmatrix} & \begin{bmatrix}0.2 & 0.4\end{bmatrix}\end{bmatrix}x_2(t-d_{32})$$

$$+sat\, u_{31}(t)\begin{bmatrix}\begin{bmatrix}-0.2 & 0.1\end{bmatrix} & \begin{bmatrix}-0.2 & 0\end{bmatrix} \\ \begin{bmatrix}0.2 & 0.3\end{bmatrix} & \begin{bmatrix}0.3 & 0.4\end{bmatrix}\end{bmatrix}x_3(t)$$

$$+sat\, u_{32}(t)\begin{bmatrix}\begin{bmatrix}-0.1 & 0.1\end{bmatrix} & \begin{bmatrix}-0.2 & -0.1\end{bmatrix} \\ 0 & \begin{bmatrix}-0.5 & -0.3\end{bmatrix}\end{bmatrix}x_3(t).$$

It is seen that $m_1=1, m_2=2, m_3=2, N_1=2, N_2=1$, and $N_3=2$. For this case, we assume $U_{11}=2.5, U_{21}=1.5$, $U_{22}=1.5, U_{31}=1.2$, and $U_{32}=1$. Then, according to the stability condition (35), we have

$$\mu\left(\hat{A}_1\right)+\mu(L_1)+\sqrt{\tilde{N}_1+m_1}\left(\left\|\bar{A}_{31}\right\|^2+\bar{\beta}_1\right)=-0.5568$$

$$\mu\left(\hat{A}_2\right)+\mu(L_2)+\sqrt{\tilde{N}_2+m_2}\left(\left\|\bar{A}_{12}\right\|^2+\left\|\bar{A}_{32}\right\|^2+\bar{\beta}_2\right)=-0.9913$$

$$\mu\left(\hat{A}_3\right)+\mu(L_3)+\sqrt{\tilde{N}_3+m_3}\left(\left\|\bar{A}_{13}\right\|^2+\left\|\bar{A}_{23}\right\|^2+\bar{\beta}_3\right)=-0.3119$$

Therefore, this large-scale bilinear interval system is stable.

V. CONCLUSIONS

The stability analysis problem for homogeneous perturbed bilinear time-delay systems with constrained inputs has been addressed in this paper. By using the Lyapunov equation approach associated with linear algebraic techniques, several delay-independent criteria that guarantee the robust stability of overall systems have been proposed. Although the Lyapunov stability theorem is utilized, it is not necessary to solve any Lyapunov equation for the obtained conditions. It is also shown that these results can be applied to solve the stability analysis for large-scale time-delay systems. Finally, illustrative examples have been given to demonstrate the applicability of the presented schemes. We believe that this work is helpful for controller design of large-scale perturbed time-delay systems.

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