An Erosion-based Modeling of Abrasive Waterjet Turning

I. Zohourkari, and M. Zohoor

***Abstract*—**In this paper, an erosion-based model for abrasive waterjet (AWJ) turning process is presented. By using modified Hashish erosion model, the volume of material removed by impacting of abrasive particles to surface of the rotating cylindrical specimen is estimated and radius reduction at each rotation is calculated. Different to previous works, the proposed model considers the continuous change in local impact angle due to change in workpiece diameter, axial traverse rate of the jet, the abrasive particle roundness and density. The accuracy of the proposed model is examined by experimental tests under various traverse rates. The final diameters estimated by the proposed model are in good accordance with experiments.

***Keywords*—**Abrasive, Erosion, impact, Particle, Waterjet, Turning.

I. Introduction

ABRASIVE waterjet (AWJ) machining is a well-recognized technology for cutting variety of materials such as composites and aerospace alloys [1, 2]. In recent years, AWJ technique was used in milling [3] and specially turning operations [4]. In turning operation, the workpiece is rotated while the AWJ is traversed in axial and radial directions to produce the required geometry. Some authors have reported about the volume removal rate [5], surface finish control [6], flow visualization study [7], and modeling of the turning process [8], using AWJ technique. Unlike conventional turning, AWJ turning is less sensitive to the geometrical workpiece profile. This method is not related to length-to-diameter ratio of the workpiece and therefore enables the machining process to turn long parts with small diameter with close tolerances. This process is ideally suitable for machining materials with low machinability such as ceramics, composites, glass, etc. [9]. Useful works by previous researchers have been done which most of them are based on experimental investigations. From a visualization study Hashish reported that the material removal takes place on the face of the workpiece rather than on the circumference of the workpiece [7]. Ansari and Hashish conducted experimental investigations to study various parameters on the volume of material removed in AWJ turning [10]. The results show that the volume of material removed in AWJ turning is similar to that achieved in AWJ cutting. Zhong and Han [11] studied the influence of variation in process parameters on turning of glass with abrasive waterjet. They reported that lower traverse rate of jet and higher rotational speed of workpiece resulted in lower waviness and surface roughness for turned specimen. Many attempts have been conducted to model AWJ cutting of ductile metallic materials and brittle ceramic materials. However, attempts on modeling of AWJ turning process are very much limited. A semi-empirical model to predict radius reduction in turning using a regression model was presented by Zeng et al. [12]. Based on an empirical approach to model AWJ turning presented by Henning [13], the material removal in AWJ turning process is assumed to be the superposition of volume removed by single particle impacts on the surface of the workpiece.

Empirical models do not explain the mechanics of the process. In addition, To determine the exponents and coefficient of the empirical models, the regression analysis should be undergone. An analytical model was suggested by Ansari and Hashish [5] that relates the volume sweep rate to material removal rate. This model could predict the final diameter of specimen in various set of AWJ turning process parameters. Hashish modified his linear AWJ cutting model for AWJ turning [14]. He considered that material is removed from the face of the rotating workpiece and assumed that the total depth of cut consists of cutting-wear depth and deformation-wear depth in turning. To estimate the cutting-wear depth for shallow impact angle zone, Finnie's theory of erosion was used [15]. To calculate the deformation-wear depth, the Bitter's theory of erosion was used [16, 17]. This analytical model of AWJ, does not consider the continuous change in impact angle, which is the result of the reduction in diameter of the workpiece. A different approach considering the varying local impact angle presented to predict the final diameter by Manu and Babu [18]. They applied Finnie's theory of erosion to model AWJ turning of ductile materials. However, their model is not able to predict accurate final diameter in various traverse rates. Moreover, at angles near to zero (when the impact angle is very low) it predicts higher volume of removed material. Hence the objective of the present work is to develop and experimentally validate a comprehensive process model for AWJ turning of cylindrical specimens subjected to various traverse rates.

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II. Mechanism of AWJ Turning

In abrasive waterjet turning, it is assumed that a jet with a velocity of V, strikes the surface of the rotating workpiece at a speed of N revolutions per minute and an initial diameter of D. The distance between jet centerline and the specimen centerline is termed as the radial position of jet, x. α is the local impact angle that the jet makes with the tangent of surface at point of impact (Fig. 1). Where α can be computed as:

$$\alpha = cos^{-1}\left(\frac{2x}{D}\right) \quad (1)$$

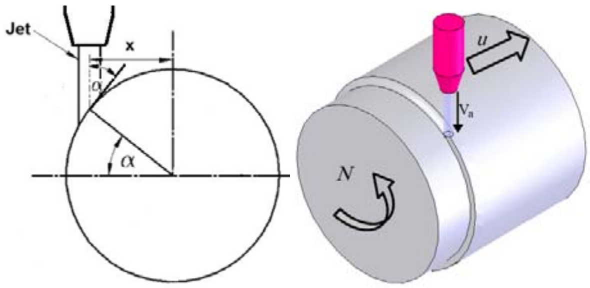


Fig. 1 Schematic diagram of AWJ turning

Turning with AWJ is approximately equivalent to the impact of an inclined jet to a flat surface which moving with a velocity equal to the tangential linear surface velocity of the rotating workpiece. The methodology of AWJ turning involves estimating the volume of material removed by the impacting abrasive particles by employing suitable erosion model. The scope of the presented work is limited to AWJ turning of ductile materials using modified Hashish erosion model. The workpiece material considered is aluminum 6063-T6.

III. Modeling of AWJ Turning

A. Modeling of Abrasive Waterjet Velocity

1. Velocity of Waterjet

The acceleration of the highly pressurized water through the orifice generates high speed waterjets where the hydraulic energy is converted to kinetic energy. According to Bernouli's law [19]:

$$P_{atm} + \frac{\rho_w}{2}V_{th}^2 + \rho_w g h_1 = P + \frac{\rho_w}{2}V_{pipe}^2 + \rho_w g h_2 \quad (2)$$

where P_{atm} is the atmospheric pressure, ρ_w is the water density which is taken as 1000 $^{kg}/_{m^3}$, V_{pipe} is the velocity before the orifice, V_{th} is the theoretical velocity of the water after the orifice, P is the water pressure before the orifice, h_1 and h_2 are the height of two points after and before the orifice respectively.

Assume: $h_1 - h_2 \approx 0$, $P \gg P_{atm}$ and $V_{th} \gg V_{pipe}$

The approximate velocity of the exit-water jet is:

$$V_{th} = \sqrt{\frac{2P}{\rho_w}} \quad (3)$$

Momentum losses occur due to three phenomena which are: (I) wall friction, (II) fluid flow disturbances, and (III) water compressibility. To modify (3), a factor C_v is added, therefore the output water velocity "V_w" becomes:

$$V_w = C_v\sqrt{\frac{2P}{\rho_w}} \quad (4)$$

2. Velocity of Abrasive Particles

The abrasive particle acceleration in an abrasive waterjet is a matter of momentum transfer from the high velocity water to the abrasive particles injected at low velocities which sucks air into the mixing chamber. Using a momentum balance expression:

$$\dot{m}_a V_{a0} + \dot{m}_w V_w + \dot{m}_L V_L = (\dot{m}_a + \dot{m}_L + \dot{m}_w)V_a \quad (5)$$

where $\dot{m}_a, \dot{m}_w$ and $\dot{m}_L$ are the mass flow rates for the abrasives, water and air respectively. V_{a0} and V_L are the input velocities of abrasives and air respectively. V_a is the output velocity of the abrasive waterjet mixture.

Neglecting the amount of air ($\dot{m}_L \approx 0$) and considering $V_{a0} \ll V_a$

$$\dot{m}_w V_w = (\dot{m}_a + \dot{m}_w)V_a \quad (6)$$

A moment transfer efficiency term φ is added for the losses encountered during the process, therefore the velocity of abrasive particles are given by:

$$V_a = \varphi\frac{V_w}{\left[1 + \left(^{\dot{m}_p}/_{\dot{m}_w}\right)\right]} \quad (7)$$

The mass flow rate of water $\dot{m}_w$ is estimated using the expression relating the diameter of waterjet orifice d_0, waterjet velocity V_w, density of water ρ_w and velocity coefficient of orifice C_d as:

$$\dot{m}_w = C_d\frac{\pi}{4}d_0^2 V_w \rho \quad (8)$$

The typical values of C_v, C_d and φ are found to be 0.98, 0.7 and 0.8, respectively. [20]

B. Workpiece Diameter after Each Revolution

The local impact angle of jet "α_k" for k^{th} revolution is given by:

$$\alpha_k = cos^{-1}\left(\frac{2x}{D_k}\right) \quad (9)$$

The volume of material removed during each revolution can be estimated from the rectangular strip of length equal to circumference of the workpiece, width equal to jet diameter and the depth equal to the radial depth of penetration during that revolution. Thus the radius reduction for the k^{th} revolution is given by

$$dr_k = \frac{Q_k}{\pi D_k d_j} \quad (10)$$

Where, Q_k is the volume of material removed at k^{th} revolution, D_k is the workpiece diameter at the beginning of the k^{th} revolution and d_j is the jet diameter. The Workpiece diameter after k^{th} revolution can be obtained as:

$$D_{k+1} = D_k - 2dr_k \quad (11)$$

C. Erosion Models

1. Finnie's Theory of Erosion

Finnie was the first to derive a single-particle erosive cutting model. The model assumes a hard particle with velocity V_a impacting a surface at an angle α. The material of the surface is assumed to be a rigid plastic one. The final expression and boundary conditions for the volume of material removed from the workpiece due to the impact of a single particle can be obtained from (12) [15].

$$Q = \begin{cases} \frac{mV^2}{\psi pk}\left[\sin(2\alpha) - \frac{6}{k}sin^2(\alpha)\right], & tan\alpha \le \frac{k}{6} \\ \frac{mV^2}{\psi pk}\left[\frac{kcos^2(\alpha)}{6}\right], & tan\alpha \le \frac{k}{6} \end{cases} \quad (12)$$

where α is the impact angle, k is the ratio of vertical to horizontal force components, and ψ is the ratio of the depth of contact *l* to the depth of the cut y_t as shown in Fig. 2 , p is the flow stress of the eroded workpiece material and Q is the total volume of target material removed. The total volume removed by multiple particles having a total mass *M* can be obtained from (13) [15].

$$Q = \begin{cases} c\frac{mV_a{}^2}{\psi pk}\left[\sin(2\alpha) - \frac{6}{k}sin^2(\alpha)\right], & tan\alpha \le \frac{k}{6} \\ c\frac{mV_a{}^2}{\psi pk}\left[\frac{kcos^2(\alpha)}{6}\right], & tan\alpha \le \frac{k}{6} \end{cases} \quad (13)$$

The constant *c* is used to compensate for the particles that do not follow the ideal model (some particles impact with each other, or fracture during erosion). Finnie model [15] for erosion is only valid for ductile materials, and does not include any brittle fracture behavior of the material.

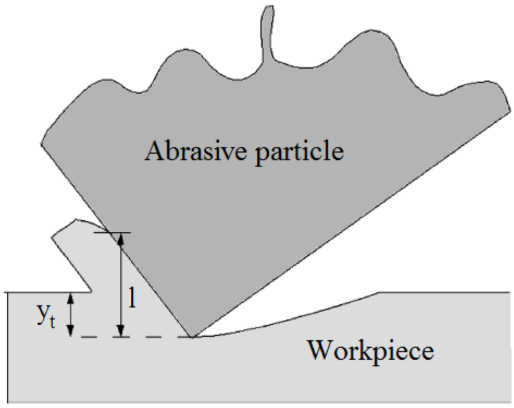


Fig. 2 Depth of cut and length of contact

2. Hashish Modified Model for Erosion

Hashish [14] modified Finnie model for erosion to include the effect of the particle shape as well as modify the velocity exponent predicted by Finnie. The final form of his model, which is more suitable for shallow angles of impact, is given in (13):

$$Q = \frac{7}{\pi}\frac{M}{\rho_p}\left(\frac{V_a}{C_k}\right)^{2.5}\sin(2\alpha)\sqrt{sin\alpha} \quad (13)$$

where C_k can be computed from (14):

$$C_k = \sqrt{\frac{3p \;\; R_f^{3/5}}{\rho_p}} \quad (14)$$

where R_f is the particle roundness factor and ρ_p is the abrasive particle density.

One of the main advantages of this model is that it does not require any experimental constants. In addition, it is a model that accounts for the shape of particles.

D. Number of Revolutions to Achieve Desired Diameter

The jet is moved along the axial direction of the part so as to extend the cutting action along the length of the part. For acceptable turning results, the axial distance moved by the jet during one revolution of the workpiece should be a fraction of the jet diameter. This results in the workpiece surface being subjected to a definite number of cutting passes during the turning operation. Number of cutting passes can be calculated as:

$$n_p = N\frac{d_j}{u} \quad (15)$$

where u is the traverse rate (feed rate) of the jet and *N* is rotational speed of the specimen. Further, due to the interaction between the high velocity abrasive waterjet and the rotating workpiece, material removal takes place, so an appropriate erosion model should apply to estimate material removed at each revolution precisely.

E. *Prediction of Final Diameter*

During each revolution, the workpiece diameter changes and this in turn changes the local impact angle. By applying (2)–(14), the volume of material removed, radial depth and the diameter of work after each revolution can be determined. By repeating the above procedure till the impact angle tends to zero, the final workpiece diameter under any given set of process parameters can be estimated.

IV. Case Study

In order to check the accuracy of the proposed model, an aluminum cylindrical stepped bar (6063-T6) as shown in Fig. 3 was considered. The Process parameters employed for the proposed model are listed in Table I. Waterjet orifice of 0.25 mm diameter and mixing tube (nozzle) of diameter 0.76 mm were assumed for the cutting head. Garnet with a mesh size of 80, roundness factor of 0.4 and particle density of 4000 kg/m^3 was used as the abrasive material. Water pressure is set to 250 Mpa and abrasive mass flow rate is assumed to be $5g/s$.

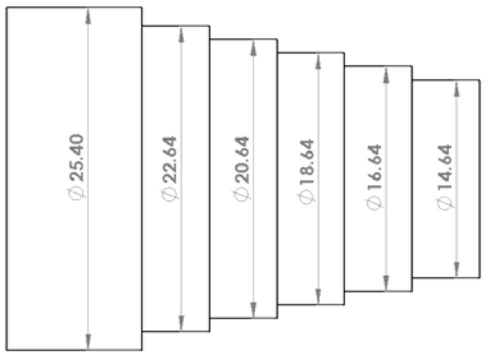


Fig. 3 Geometry of desired specimen

TABLE I
AWJ Turning Parameters

Parameter	Level
Pressure, MPa	250
Nozzle diameter, mm	0.76
Abrasive mass flow rate, g/s	5
Rotational speed, rpm	200
Axial traverse rate, mm/min	2, 2.5, 10, 20

V. Results and Discussion

Since flow stress is an important parameter in Finnie and Hashish erosion models, so this parameter was determined through 12 tests which are listed in Table II. Also material removed predicted by Finnie and Hashish models are shown in Fig. 4.

TABLE II
Determination of Flow Stress

Nozzle Diameter (mm)	Surface speed of workpiece, mm/min	Volume removed, mm3	Jet contact time, s	Finnie's prediction of Flow stress, MPa	Hashish 's prediction of Flow stress, MPa
1.6	1000	587.91	21.38	1136.68	7567.91
	2000	474.04	11.30	744.67	5397.96
	3000	340.76	7.02	643.44	4803.19
	4000	186.64	3.93	657.70	4887.97
1.2	1000	661.49	17.85	843.09	5960.88
	2000	385.39	9.39	761.57	5493.27
	3000	288.08	5.37	582.85	4433.92
	4000	95.67	3.28	1071.42	7219.33
0.76	1000	673.36	27.69	1285.18	8359.51
	2000	454.79	12.40	852.04	6010.45
	3000	360.72	11.54	999.32	6830.42
	4000	290.74	8.28	889.98	6223.56
			Average	874	6099

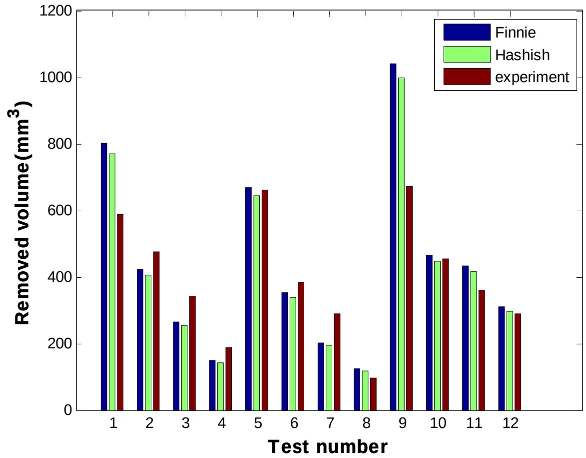


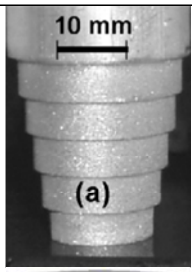
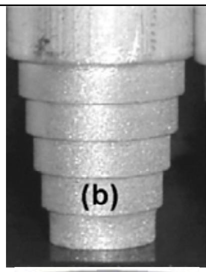
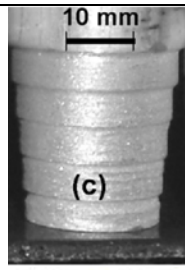
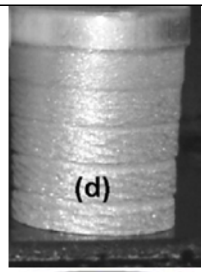
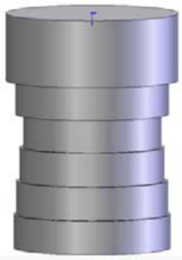
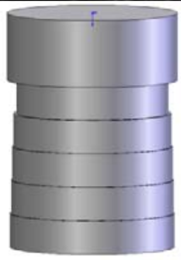
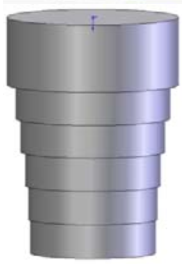
Fig. 4 Material removed predicted by Finnie and Hashish models

In Table III final diameter predicted by the proposed model and Manu model [18] are compared and related errors are described. The results were obtained under traverse rate equal to u=2 mm/min. To investigate the effect of traverse rate and to check the efficiency of the propose model, the predicted diameters obtained by proposed model and Manu model and the comparison with experimental data are inscribed in Table IV.

TABLE III
Prediction of Final Diameter

Initial diameter	Target diameter	Manu model	Error(mm)	Presented model	Error (mm)
25.40	22.640	23.089	0.449	22.640	0
25.40	20.640	20.879	0.239	20.640	0
25.40	18.640	18.711	0.071	18.642	0.002
25.40	16.640	17.110	0.470	16.643	0.003
25.40	14.640	14.943	0.303	14.654	0.014

TABLE IV
COMPARISON OF THE EXPERIMENTAL, PROPOSED AND MANU MODELS

Traverse rate ($^{mm}/_{min}$)	2	2.5	10	20
Experiment [18]				
Manu model				
Presented model				

VI. CONCLUSION

In contrast with reported results obtained by other researchers, the proposed model in this paper, predicts desired geometry of specimen in various traverse rates, successfully. Different flow stress values obtained by Finnie and Hashish erosion models show that the flow stress may even be considered as an empirical constant which accounts for the material property and all other effects which are not accounted in the erosion models. Since the proposed model does not consider the jet divergence, so further attempts should be done to model abrasive waterjet turning more precisely.

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