

# Reliability of Chute-Feeders in Automatic Machines of High Production Capacity

R. Usubamatov, A. Usubamatova, and S. Hussain

**Abstract**—Modern highly automated production systems faces problems of reliability. Machine function reliability results in changes of productivity rate and efficiency use of expensive industrial facilities. Predicting of reliability has become an important research and involves complex mathematical methods and calculation. The reliability of high productivity technological automatic machines that consists of complex mechanical, electrical and electronic components is important. The failure of these units results in major economic losses of production systems. The reliability of transport and feeding systems for automatic technological machines is also important, because failure of transport leads to stops of technological machines. This paper presents reliability engineering on the feeding system and its components for transporting a complex shape parts to automatic machines. It also discusses about the calculation of the reliability parameters of the feeding unit by applying the probability theory. Equations produced for calculating the limits of the geometrical sizes of feeders and the probability of sticking the transported parts into the chute represents the reliability of feeders as a function of its geometrical parameters.

**Keywords**—Chute-feeder, parts, reliability.

## I. INTRODUCTION

Automatic machines of high production capacities have complex transport system for transferring processed parts to the machining area and to storage units. These transport mechanisms fulfill functions of feeding, output, accumulation, linking and separation of the parts workflow [1]. In industrial processes, there are different parts and work pieces like rollers, prisms, plates etc. Simple design parts and work pieces that need to be transported with high reliability some times can cause of jamming into simple design transport systems. Transport systems include plenty of the various trenches, directing channels, slopes and chutes, guide ways and devices, etc. where parts can move by rolling or sliding. The principle function of these mechanisms is based on the mechanically forced displacement of parts, on the action of gravity force and on mixed principle, which combines the gravity force and externally applied force [2]-[4]. The motions of the parts are accomplished by different variants – separately, intermittently or by continuous flow depending on the requirement of productive units.

The displacement of parts by sliding or rolling produced in the guides, chutes and channels have strictly defined rectangular form of its cross section [2]. Most automatic machines have simple construction of transport systems. However, experience of operating transport systems testifies to the low reliability of the simple transport, feeding and storage systems. The functioning brake of such mechanisms is jamming of moving parts into transport systems. This situation reflects the fact that the engineers and scientists do not give attention to the reliability of these simple transport systems.

## II. THE SELECTED DIRECTION OF THE RESEARCH WORK ON RELIABILITY PROBLEMS

For automatic machines of high productive capacity having the duration of working cycle at range of several seconds, questions of the reliability of transport, feeders and storage systems acquire special paramount importance. For example, industrial experience shows that the simplest chute systems remain some of the most unreliable elements of the transport systems in the automatic machines and in production lines for processing complex shape parts [2].

Studies on the reason of stops, brakes and jamming and their comprehensive analysis have made it possible to determine the nature of their appearance and hence to develop design and improve the technological recommendations in order to increase the fitness of the elements of systems [2], [4], [5]. The reason for the stops, fails and brakes of the system is due to the change in the sizes and shapes of parts, design parameter of transport mechanisms, and also in the dynamics of the motion of the parts, where can occur during their collision, which in turn draws either the sticking or releasing of the parts. All these factors obey the laws of the probability theory [7], [8].

Most of the formulas for the calculation of geometrical parameters of transport mechanisms do not consider factors of variations of parameters mentioned above [4]. Existing normative data is presented and analysis show that formulas do not include and experts do not take into account the actual deviation of the sizes of the moving parts and the parameters of transport mechanisms, which naturally affect the reliability of the parts displacement. This circumstance has crucial importance in the calculation of functioning reliability of transport mechanisms.

Simple shape parts like rollers or plates are rolling or slipping freely and chaotically into a chute and comes into contact with boards and under the action of friction forces casually turn. In Fig. 1 is presented a typical design of parts that is possible to find in manufacturing area. The parts are moved into chute can turn due to friction forces between parts

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and chute's boards (Fig. 2). Such situation under the action of dynamic forces can lead to jamming of parts into a chute thus failing the transport system. Parts have chamfers or rounded corners that increase the jamming effect. More often the jamming of parts occurs when the parts stop at their transfer places and accumulate.

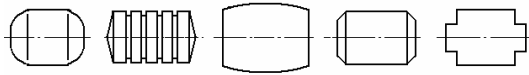


Fig 1. Typical design of parts in manufacturing area.

Among the reasons of parts jamming are factors having casual character such as deviation of the geometrical sizes trays, and chutes, deviation of parts sizes, their uncontrollable movement, turn of parts into a chute etc. All mentioned parameters are subjected to the probability theory and mathematical statistics. It means the probability of parts jamming into a transport system and its reliability can be calculated by mathematical methods i.e., they can be calculated by laws of probability and reliability theory.

Existing normative methods of calculation of the sizes of tray or chute systems represent the formula of a limiting clearance between a tray or chute and parts, which guarantees the reliable moving the parts under condition  $H < W$  (Fig. 2).

$$W = \sqrt{\frac{H^2 + D^2}{1 + f^2}} \quad (1)$$

where  $W$  - the width of the chute's canal;  $H$  - the length of the part;  $D$  - the width or diameter of the part;  $f$  - friction factor between part's and chute's board surfaces.

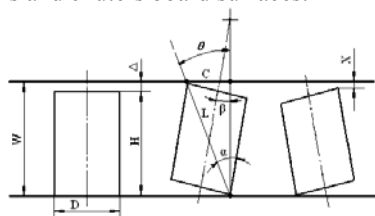


Fig. 2: Jamming of a part into a chute

Studies of the design parameters of the elements for transport systems and parts show that their sizes are random variables, and distributed in a specific range according to the normal law of distribution. The density of probability distribution by the normal law is expressed by the next equation [8]-[10].

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x_i + m_x)^2}{2\sigma_x^2}} \quad (2)$$

where  $f(x)$  - the density of the probability distribution of the specific values if it is a random variable,  $x$  - theoretical frequency;  $x_i$  = specific possible values of random variable;  $m_x$  - mathematical average of the random variable;  $\sigma_x$  - deviation of the random variable;  $e$  - the base of natural logarithms; and  $\pi = 3.14$

Reliability of transporting parts into a chute depends from parameters of a chute and a part (Fig. 2), where  $L$  - the diagonal of the part,  $\Delta$  - the clearance between the part and boarders of the chute,  $x$  - random clearance between the turned part and boarder of the chute. Other parameters are presented above. All the mentioned parameters are described by law of normal distribution and the probability of occurrence of the size of parameters  $H$ ,  $D$ , and  $W$  into some interval  $\pm \delta$ , and calculated by the following expression  $P(B \pm) = F(B +) - F(B -)$  or in the developed form [8]

$$F(B \pm \delta) = \frac{1}{\sigma_B \sqrt{2\pi}} \left[ \int_{-\infty}^{B+\delta} e^{-\frac{(B+\delta-m_B)^2}{2\sigma_B^2}} dB - \int_{-\infty}^{B-\delta} e^{-\frac{(B-\delta-m_B)^2}{2\sigma_B^2}} dB \right] \quad (3)$$

### III. CALCULATION OF RELIABILITY PARAMETERS OF THE MOVING PARTS INTO FEEDERS

During the motion of the parts into the feeding systems, the parts can stop, accumulate and collide, which leads to the failure of the systems due to sticking or jamming of parts into chutes. Fig. 2 shows the case when symmetrical part moving into the chute. The motion of the part into a chute is chaotic, their turn is random, the sizes of the parts and the size of the chute's width are also random and therefore it is possible to calculate the probability dependencies of the parts jamming into a chute by laws of probability theory. The part jamming into a chute depends from a part turn; a definite size of part's height,  $H$ ; a definite size of the part's width,  $D$ ; a definite size of the chute width,  $W$ .

According to the theory of probability all mentioned parameters as events are random, accidental, and independent values, therefore the probability of the parts jamming is calculated as the multiplication of the probability events by next equation [9]

$$P = P_{sw} * P_H * P_D * P_W \quad (4)$$

where  $P$  is probability of the parts jamming into a chute;  $P_{sw}$  is probability of the part turning and coming into contact with the chute board;  $P_H$  is probability of the definitive size of the part height,  $H$  appearing;  $P_D$  is probability of the definitive size of the part width,  $D$ , appearing;  $P_W$  is probability of the definitive size of the chute's width,  $W$  appearing.

### IV. DEFINITION OF THE PROBABILITY VALUE OF THE PART'S TURN INTO THE CHUTE

Naturally, the motion of the part into a chute is chaotic and the part comes into contact, hits chute's boards and collides with other jamming part into a chute. Mathematically it is extremely difficult produce the equation of the probability of location of a moving part into a chute. For simplicity, it is possible to make assumption that any location of a part into a chute has equal probable distribution (Fig. 2). However to get more reliable result it is necessary to add correction factor  $q$ , that should be find empirically.

The part orientation and its contacts with the chute boards is expressed by the probabilistic values  $X$  and  $\beta$ . The limits of their values are shown in Fig. 2.  $X$  is random distance from the part's side to the shut board.  $\beta$  is an angle created by the

part axis and a line perpendicular to the chute board, when the part turns and comes into contact with the chute boards. The values of  $X$  and  $\beta$  have a continuous uniform distribution as accepted above.  $X$  is distributed on the interval from 0 to  $W-H$  and in two directions to the left and right boards [8]. The turning of a part is defined by an angle  $\beta$  and its value changes along the interval from  $\theta$  to  $\alpha$ , and in two directions as  $-\beta$  to  $+\beta$  in clockwise and counterclockwise directions correspondingly (Fig. 2). The angle  $\alpha$  is the angle between the diagonal  $L$  of a part and its axis, and the angle,  $\theta$  is the angle between  $L$  and a line perpendicular to the chute board when the part contacts with one. So the density of the uniform distribution random variable,  $X$  is expressed by the equation

$$f(x) = \left( \frac{1}{2(W-H)} \right) \quad (5)$$

where  $f(x)$  - the density of the uniform distribution random variable, and the number 2 represents the two directional distributions as  $X \in (0, W-H)$  and  $\beta \in (\theta, \alpha)$ . Hence  $f(x) = 0$  by  $X \notin (0, W-H)$  or  $f(\beta) \notin (\theta, \alpha)$ , and other parameters are presented above.

With reference to Fig. 2, it is possible to consider a distance  $X$  having the magnitudes of possible values of random variable.  $X$  and  $\beta$  position and orientation of a part into a chute are possible if the next condition is feasible as  $X = W - H \cos \beta - D \sin \beta$ . Hence, the probability distribution of any part's position into the chute is expressed by next equation [8] - [10].

$$f(X) = \frac{1}{2(W - H \cos \beta - D \sin \beta)}, \quad (6)$$

The probability of any part's location into the chute is expressed by next equation

$$P_{sw} = \int_0^\beta f(x) d\beta = \int_0^\beta \frac{1}{2(W - H \cos \beta - D \sin \beta)} d\beta \quad (7)$$

The integral of trigonometric function is defined by substitution  $\text{tg}(\beta/2) = t$ , and further transformation gives  $\beta = 2 \arctg t$ ,  $d\beta = \frac{2dt}{1+t^2}$ . Trigonometric functions are presented by the next identity formulas

$$\cos \beta = \frac{1 - \text{tg}^2 \frac{\beta}{2}}{1 + \text{tg}^2 \frac{\beta}{2}}, \quad \sin \beta = \frac{2 \text{tg} \frac{\beta}{2}}{1 + \text{tg}^2 \frac{\beta}{2}} \quad (8)$$

After substitutions of the defined formulas (8) into Eq. (7) and transformations

$$P_{sw} = \frac{1}{2} \int_0^{\beta_{\max}} \frac{2dt}{(1+t^2)[W - H\left(\frac{1-t^2}{1+t^2}\right) - D\left(\frac{2t}{1+t^2}\right)]} = \int_0^{\beta_{\max}} \frac{dt}{(W+H)t^2 - 2Dt + W - H} \quad (9)$$

The integral (9) with a square trinomial in a denominator is transformed in the form of a difference of squares

$$(W+H)t^2 - 2Dt + W - H = (W+H) \left[ t^2 - \frac{2Dt}{W+H} + \left(\frac{D}{W+H}\right)^2 - \left(\frac{D}{W+H}\right)^2 + \frac{W-H}{W+H} \right] = (W+H) \left[ \left( t - \frac{D}{W+H} \right)^2 \pm k^2 \right] \quad (10)$$

where  $\pm k^2 = -\left(\frac{D}{W-H}\right)^2 + \frac{W-H}{W+H}$ , and the sign ( $\pm$ ) depends from the sign of an expression that locates at the right, positive or negative, i.e. will be roots of a trinomial expression complex or valid.

Integral  $P_{sw}$  becomes

$$P_{sw} = \frac{1}{W+H} \int_0^{\beta_{\max}} \frac{dt}{\left( t - \frac{D}{W+H} \right)^2 \pm k^2} \quad (11)$$

Next step is necessary to do in which the last integral (11) is substituted by the variable

$$t - \frac{D}{W+H} = y, \quad dt = dy. \quad (12)$$

Then the integral (11) becomes

$$P_{sw} = \frac{1}{W+H} \int_0^{\beta_{\max}} \frac{dy}{y^2 \pm k^2}. \quad (13)$$

The expression (13) is standard integral, where depending on signs ( $\pm$ ), gives following solutions.

1. For a sign (+k)

$P_{sw} = \frac{1}{(W+H)k} \arctg \frac{y}{k} \Big|_0^{\beta_{\max}}$ , and after substitutions the expressions  $y$  and  $k$ , that presented above

$$P_{sw} = \frac{1}{(W+H)k} \left[ \frac{\arctg \frac{\text{tg}(\beta_{\max}/2) - D/(W+H)}{k}}{\arctg \frac{-D/(W+H)}{k}} \right] \quad (14)$$

where  $\beta_{max}$  is defined from expression  $\beta_{max} = \alpha - \theta$ . (Fig. 2), where trigonometric identities

$$tg \frac{\alpha - \theta}{2} = \frac{\sin \theta (1 - \cos \alpha) - \sin \alpha (1 - \cos \theta)}{\sin \alpha \sin \theta + ((1 - \cos \alpha)(1 - \cos \theta))}$$

where from Fig. 2,  $\sin \alpha = D/L$ ,  $\cos \alpha = H/L$ ,  $\sin \theta = C/L$ ,  $\cos \theta = W/L$ ,  $C = \sqrt{L^2 - W^2}$  and  $L = \sqrt{H^2 + D^2}$ .

After substitutions and transformations

$$c = tg \frac{\alpha - \theta}{2} = \frac{\sqrt{H^2 + D^2 - W^2}(\sqrt{H^2 + D^2} - H) - D(\sqrt{H^2 + D^2} - W)}{D\sqrt{H^2 + D^2 - W^2} + (\sqrt{H^2 + D^2} - H)(\sqrt{H^2 + D^2} - W)}$$

However, trigonometric identities  $\arctg a - \arctg b = \arctg \frac{a-b}{1+ab}$ , after substitutions to Eq. (14) and transformations

$$P_{sw} = \frac{1}{ek} \left[ \arctg \frac{c}{1 - (c-D/e)D/e} \right] \quad (15)$$

where  $e = W + H$ ;  $c$  and  $k$  are presented above.

2. For a sign (-k)

$P_{sw} = \frac{1}{2(W+H)k} \ln \left| \frac{y-k}{y+k} \right|_{\beta_{max}}$ , and after substitutions of the expressions  $y$  and  $k$ , that presented above

$$P_{sw} = \frac{1}{2(W+H)k} \left[ \frac{\ln \left| \frac{tg(\beta_{max}/2) - D/(W+H) - k}{tg(\beta_{max}/2) - D/(W+H) + k} \right|}{\ln \left| \frac{-D/(W+H) - k}{-D/(W+H) + k} \right|} \right]$$

where the expression  $tg(\beta_{max}/2)$  is presented above. After substitutions and transformations

$$P_{sw} = \frac{1}{2ek} \ln \left| \frac{(D/e)^2 - k^2 - c[(D/e) - k]}{(D/e)^2 - k^2 - c[(D/e) + k]} \right| \quad (16)$$

where expressions  $e$ ,  $c$  and  $k$  are presented above.

After substitutions Eq. (3) and Eq. (15) or Eq. (16) into Eq. (4), the probability of parts jamming in the integrated form is presented by the following equation

$$P = qP_{sw} \int_{-\infty}^{H+\delta} \int_{-\infty}^{D+\delta} \int_{-\infty}^{W+\delta} f(H)dH * f(D)dD * f(W)dW, \quad (17)$$

where  $q$  - correction factor that considers the turn of a part into a chute does not have equal probable distribution and defined empirically;  $P_{sw}$  - the probability of any location part into the chute is presented by Eq. (15) or Eq. (16), and sub integral expressions are presented by Eq. (3). Other parameters are presented above.

The probabilities of appearance of the critical sizes of the part and the chute that lead to jamming are defined by Eq. (3) on a basis of distribution laws of their sizes. In practice, the distribution of the chute's width size is many times more than of the distribution of sizes of transported parts. Hence, in the calculation of probability of parts jamming into a chute, it is possible to neglect distribution of the part sizes due to their insignificance. In such case, calculation of probability of parts jamming into a chute is fulfilled by following simple equation.

$$P = qP_{sw} \int_{-\infty}^{W+\delta} f(W)dW \quad (18)$$

where all parameters are presented above.

In real industrial conditions, the feeding systems of automatic machines can contain, in one chute,  $N$  parts that move separately, independently and simultaneously. Therefore the probability of parts jamming can increase in  $N$  times. In this case the probability of a feeding system brake pays off according to laws of the probability theory by the following formula [8]

$$P_N = 1 - (1 - P)^N, \quad (19)$$

where  $N$  is quantity of parts moving into one chute.  $P$  is probability of parts jamming into one chute of the feeding system, counted by Eq. (18).

The presented dependencies for the calculation of the probability of parts jamming into a chute can be used as a first approximation at design stage. Calculation of reliability of feeding systems in automatic machines of high productive capacity allows raising their quality and efficiency of use.

## V. CASE STUDY

For the part and chute there are next data (Fig. 2): shut width  $W = 33.0 \pm 1.0$  mm has normal distribution of sizes; part height  $H = 30.0$ mm; a part width  $D = 15.0$ mm;  $f = 0.1$  - friction factor. Clearance  $\Delta = 3 - 5$  mm, due to deviation of the width of part. In the feeding system there are  $N = 20$  parts that move separately. An industrial machine has work cycle time  $T = 25$  seconds. Assume that corrections factor  $q = 0.7$ . Find the average time when feeding system of the machine will work without brakes.

Solution: The jamming of parts takes place at some critical sizes of a chute's width and geometrical parameters of a part. Calculation by Eq. (1) gives the critical size of the chute's width.

$$W = \sqrt{\frac{H^2 + D^2}{1 + f^2}} = \sqrt{\frac{30.0^2 + 15.0^2}{1 + 0.1^2}} = 33.374 \text{mm}$$

Probability of appearance the critical sizes of the chute's width, which lead to jamming, calculated by Eq. (3) and gives next solutions [9].

$$F(W \pm \delta) = \frac{1}{\sigma_W \sqrt{2\pi}} \left[ \int_{-\infty}^{W+\delta} e^{-\frac{(W+\delta-m_B)^2}{2\sigma_B^2}} dW - \int_{-\infty}^{W-\delta} e^{-\frac{(W-\delta-m_B)^2}{2\sigma_B^2}} dW \right]$$

where  $m_W = 33,0$  mm – mean size;  $\sigma_W = 2.0/6 = 0.3333$ mm – standard deviation

For the chute width

$$P(34.0 - 33.374) = \frac{1}{0.3333\sqrt{2\pi}} \left[ \int_{-\infty}^{34.0} e^{-\frac{(34-33.0)^2}{2*0.3333^2}} dW - \int_{-\infty}^{33.374} e^{-\frac{(33.374-33.0)^2}{2*0.3333^2}} dW \right] = 0.13$$

The probability of the part turning into a chute is defined by formula  $P_{sw}$  and after substitution of all the initial data yields

$$e = W + H = 33.0 + 30.0 = 63.0 \text{ mm}$$

$$\pm k^2 = -\left(\frac{D}{W-H}\right)^2 + \frac{W-H}{W+H} \cdot \pm k^2 = -(15/3)^2 + 3/63,$$

$$-k^2 = -24.952, k = 4.995$$

$$c = \frac{\sqrt{H^2 + D^2 - W^2}(\sqrt{H^2 + D^2} - H) - D(\sqrt{H^2 + D^2} - W)}{D\sqrt{H^2 + D^2 - W^2} + (\sqrt{H^2 + D^2} - H)(\sqrt{H^2 + D^2} - W)} = 0.1428$$

$$P_{sw} = \frac{1}{ek} \left[ \arctg \frac{c}{1 - (c - D/e)D/e} \right] =$$

$$\frac{1}{63 * 4.995} \left[ \arctg \frac{0.1428}{1 - (0.1428 - 15/63)(15/62)} \right] = 0.025$$

The probability of the part sticking is defined by formula (18),

$$P = 0.7 * 0.025 * 0.13 = 0.0023.$$

The probability that one out of the 20 parts will stick into feeding system is defined by Eq. (19),

$$P_{20} = 1 - (1 - 0.0023)^{20} = 0.045 \approx 0.05$$

## VI. RESULTS AND DISCUSSION

The probability of parts sticking into the feeding system of the automatic machine will have 5 cases per 100 cycles or per  $100 * 25/60 = 41.7$ min. In average the feeding system of a machine will brake after 8.3 min. It shows low reliability of the feeding system of the automatic machine. The reliability of the feeding systems of machines depends on the geometry of the transport unit. Such approach provides the chances and possibilities for engineers to calculate the reliability of the machines via geometrical dimensions of their units. Presented Eq. (17) for calculating the probability of parts jamming into a feeding system of a complex automatic machine allows developing methodology for calculating the reliability of feeding system as a function of its geometrical dimension and sizes.

## VII. CONCLUSION

Intensification of production processes leads to the necessity of enhancing the reliability of all types of facilities, which involve fabrication of machine parts. This research

presents solution in the area of the calculation of reliability of transport system, particularly reliability of functioning of chutes that represents the number of jamming of parts with shoulders into a chute per some time. Practically, there is an answer on how long a transport system work without stops. It is shown that jamming of the parts depends from deviation of the part's sizes and from probability of part's turning into a chute. A methodology for the calculation of the reliability of the functioning of the chute is developed, which contains the following steps:

- Defined the deviations of the part and chute sizes to obtain the dependencies of their distributions.
- Define the limit sizes of the part and the chute width when the parts are jamming.
- Calculate the probability of a part turn into chute.
- Calculate the probability of a part and a shut limit sizes that can create the condition for parts jamming into chutes.
- Calculate the reliability of feeding systems.

The presented Eq. (17) is concerned with the simple shape of the parts. The methodology of calculation of reliability for transport systems allows deciding for more complex problems of reliability for industrial machines of high productive capacity.

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