Recursive Path-finding in a Dynamic Maze with Modified Tremaux's Algorithm

Nien-Zheng, Yew, Kung-Ming, Tiong and Su-Ting, Yong

Abstract—Number Link is a Japanese logic puzzle where pairs of same numbers are connected using lines. Number Link can be regarded as a dynamic multiple travelers, multiple entries and exits maze, where the walls and passages are dynamically changing as the travelers move. In this paper, we apply the Tremaux's algorithm to solve Number Link puzzles of size 8×8, 10×10 and 15×20. The algorithm works well and produces a solution for puzzles of size 8×8 and 10×10. However, solving a puzzle of size 15×20 requires high computer processing power and is time consuming.

I. INTRODUCTION

A maze is a tour puzzle with complex branching passages where the traveler must choose a route. The mission in a maze is to find an exit from a certain entry point by avoiding obstacles [1] [2] [3]. Maze solving algorithms are generally classed into two types: (i) traveler-based and (ii) maze-based [4].

Number Link is a Japanese logic puzzle that involves connecting pairs of numbers, each with its own continuous line. According to the publishers of Number Link, solving a Number Link puzzle requires inspired guessing and puzzle sense, and there is no perfect logical method to solve this puzzle [5].

Taking a closer look at Number Link however, it can be seen that a Number Link puzzle can actually be perceived as a dynamic multiple travelers, multiple entries and exits maze. In this research, a traveler-based algorithm known as Tremaux's algorithm is used to find the solution for Number Link puzzles of size 8×8, 10×10 and 15×20.

II. NUMBER LINK

A. Background

Number Link was developed by Issei Nodi and it was first published in 1987 by Nikoli Co., Ltd. This company publishes puzzle magazines and develops their own puzzle games. The popular Sudoku and Kakuro were produced by Nikoli as well [5].

B. What is Number Link?

Number Link puzzles come in sizes of $m \times n$, where $m = n$ or $m \neq n$ (m representing rows and n representing columns). An example of Number Link 7×7 is shown in Fig.

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1. The objective of the puzzle is to connect the number pairs (e.g. 1 pairs up with 1, 2 pairs up with 2, and so on), each with its own continuous line.
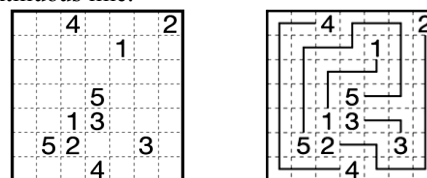

Fig. 1 Example of Number Link and its solution [5]

There are three ground rules governing the solution of Number Link. Firstly, pairs of the same numbers are connected with a single continuous line. Secondly, lines go through the center of the cells, horizontally, vertically, or changing direction, and never twice through the same cell. Thirdly, lines cannot cross, branch off or go through the cells with numbers [5]. Additionally, all cells must be marked by a line.

III. TREMAUX'S ALGORITHM

Tremaux's algorithm is an efficient depth-first search algorithm that is applicable by humans (i.e. it is a traveler-based algorithm) to find the way out of a maze. The algorithm may not provide the shortest route solution but will find a solution for all mazes [4]. Tremaux's algorithm is presented as a flowchart in Fig. 2. In this algorithm, all visited paths are marked so that the way to return to the source is recorded and all visited pathways would not be visited again.   When arriving at a junction, a pathway is chosen and marked. If arrive at a marked junction, an unmarked pathway is chosen. If that is not possible, the path where you come from is chosen. This algorithm will never pick the same marked pathway twice. The process is repeated until the exit is found, or in the case of no exit, it will return to the original position with all paths visited.
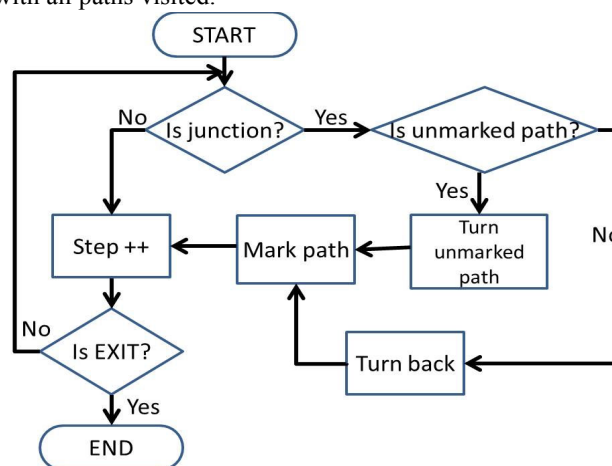

Fig. 2 Tremaux's algorithm [4]

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IV. SOLVING NUMBER LINK WITH MODIFIED TREMAUX'S ALGORITHM

A. Modified Tremaux's Algorithm

Before applying the Tremaux's algorithm to the Number Link puzzle, a slight modification has to be made to the original algorithm. This is due to the structure of the Number Link puzzle. The modified Tremaux's algorithm is shown as a flowchart in Fig. 3.
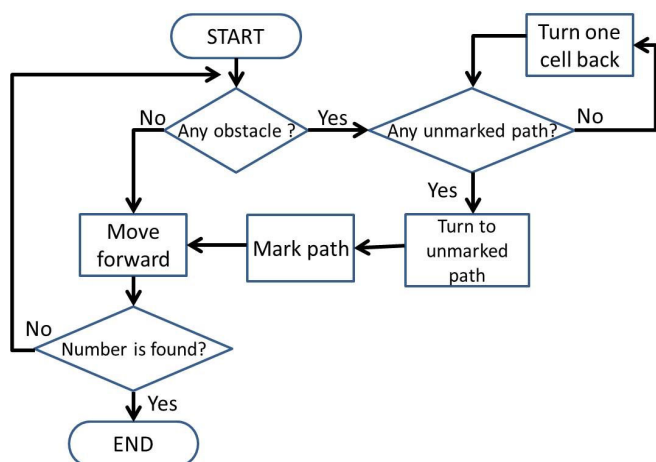

Fig. 3 The flowchart of modified Tremaux's algorithm applied in solving Number Link

In Number Link, each small square is practically a 'junction' where pathways can be chosen from the current position. The traveler starts from a number and continues to move in its current straight line direction until an obstacle is reached. Obstacles can be any numbers other than itself, marked paths or the boundaries of the puzzle. When reaching an obstacle, the traveler will choose a direction to turn, always starting with north, followed by east, south and west directions and will choose an unmarked path. It will then continue to move in a straight line direction until another obstacle is reached where it will again make a decision where to turn to. If there is no unmarked path that the traveler can turn to, it will turn back one cell and finds an unmarked path, if any. If there is none, it will turn back one more cell and again finds an unmarked path, if any. This process is repeated until all the pairs of numbers are connected.

B. An Example of the Modified Tremaux's Algorithm Applied to Number Link

This section explains the use of modified Tremaux's algorithm, coded into the Number Link Solver, to solve a Number Link puzzle example (Fig. 4).
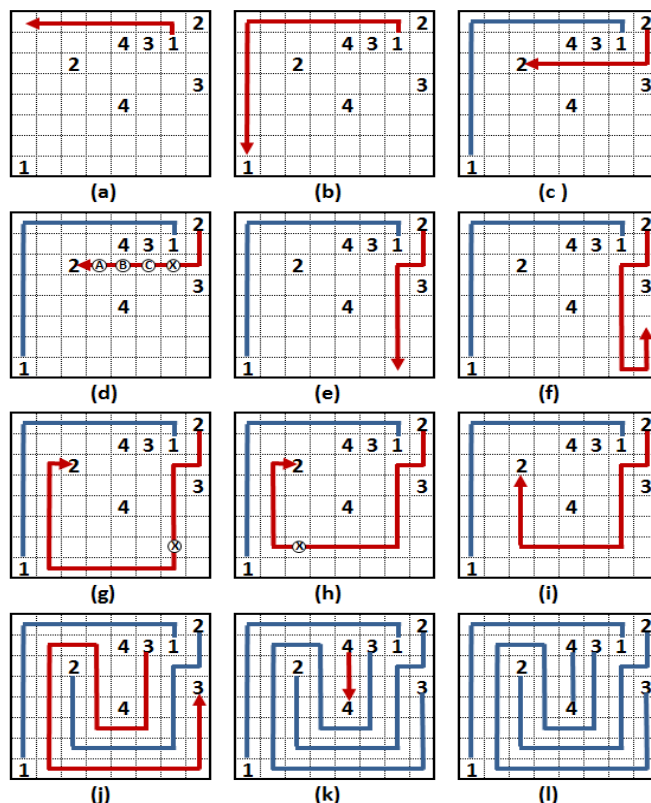

Fig. 4 Solving Number Link with modified Tremaux's Algorithm

The Number Link Solver chooses starting numbers from first to last row, left to right. The link starts with Number 1 (Fig. 4a). When the link reaches the obstacle (boundary), it changes its direction to the south, the only available path (Fig. 4b) and the pair of Number 1 is connected. Next, the algorithm chooses Number 2 (Fig. 4c) and the pair of Number 2 is connected. However, this blocks Number 3. The link has to move backward until point X (Fig. 4d) because if the link moves back to point A, B or C, there are still no available paths for Number 3. Starting from point X, the link changes its direction to the south (Fig. 4e) and moves on to the east (Fig. 4f) but ends up nowhere. Therefore, the link turns to the west (Fig. 4g) and the pair of Number 2 is connected. However, the pair of Number 3 is separated by the link. Therefore, the link has to move backward until point X (Fig. 4g) to avoid this. Then, starting from that point X, the link moves to the west, then north, and the pair of Number 2 is connected (Fig 4h), but again the pair of Number 3 is separated by the link. Therefore, the link has to move backward until point X (Fig. 4h) and starting from that point, the link moves to the north and the pair of Number 2 is connected (Fig. 4i). The same procedure applies in finding the link to connect the pairs of Number 3 and 4 to complete the solution (Fig. 4l).

C. Number Link Solver

The Number Link Solver is developed using Visual Basic with a simple interface design. At the beginning of the program, the user has to choose the required dimension of the

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puzzle (Fig. 5). Three options are given i.e. 8×8, 10×10 and 15×20 (these sizes are actual puzzles from a Nikoli Number Link puzzle book [6]).
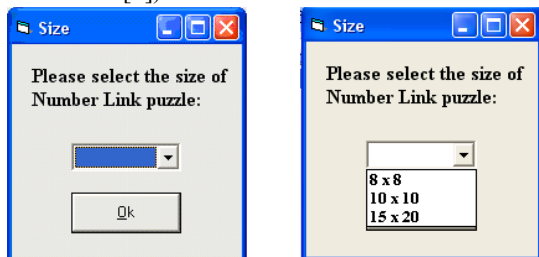

Fig. 5 User Interface to choose the dimension of Number Link

After the dimension is selected, an empty Number Link template is displayed based on the selected dimension. An example of 8×8 Number Link template is shown in Fig. 6.
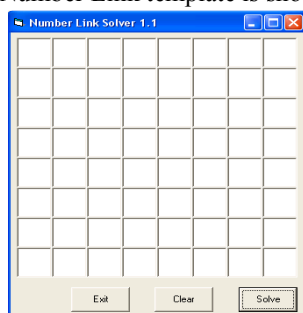

Fig. 6 Number Link template for 8×8

The template allows a user to key in the pairs of numbers. Then, the user can presses the solve button to view the solution. In this application, the solution is displayed using numbers instead of lines (Fig. 7). There are two extra buttons on the template: clear and exit. Pressing the clear button deletes all the numbers in the template, whereas exit is meant for quitting the application.

D. Limitations of the Number Link Solver

The Number Link Solver cannot check for empty cells, if any. If a proper Number Link puzzle is entered into the template, the solver always successfully finds the answer (Fig. 7). However, if an improper Number Link puzzle is entered into the template, the solver will still give a 'solution', albeit with one or more cells left unfilled or filled with zeros (Fig. 8).
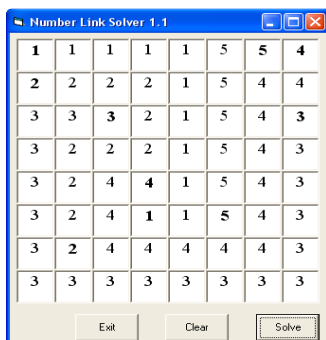

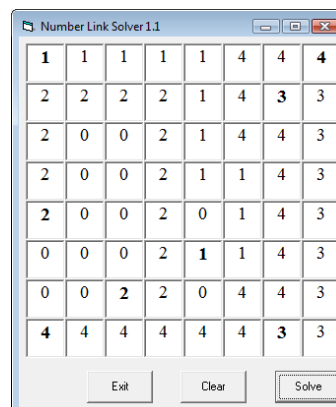Fig. 7 A proper Number Link puzzle is solved

Fig. 8 An improper Number Link puzzle with empty cells

When running the Number Link Solver with Intel® Core™2 Duo with 2GB of RAM, 8×8 and 10×10 Number Link puzzles can be solved within a few seconds. However, when solving a 15×20 Number Link puzzle, it takes a long time to produce a solution and sometimes the computer even runs out of memory and hangs. This indicates that when dealing with an increasingly huge number of cells, the use of modified Tremaux's algorithm to solve Number Link puzzles is not efficient anymore. This is because too much memory is needed to store all the various marked and unmarked pathways while the program is running to find the solution. At present, only a numerical output can be produced instead of lines, which may be a bit difficult to see. Also, only one solution will be provided, if any. Sometimes a puzzle may have more than one solution but the Number Link Solver only provides one solution and does not look for alternative solutions.

V. CONCLUSION

The Number Link puzzle is an interesting and challenging puzzle to study as it is a dynamic multiple travelers, multiple entries and exits maze. The use of modified Tremaux's algorithm in Number Link Solver is able to find the solution without problems for decent puzzle sizes. It will be interesting to see how other maze solving algorithms manage the Number Link puzzle in terms of performance.

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