Calculation of Masses and Magnetic Moment of the Nucleon using the MIT Bag Model

Mahvash Zandy Navgaran, Maryam Momeni Feili

Abstract**—**The bag radius of the nucleon can be determined by MIT bag model based on electric and magnetic form factors of the nucleon. Also we determined the masses and magnetic moment of the nucleon with MIT bag model, using bag radius and compared with other results, suggests a suitable compatibility.

Keywords**—**MIT bag model, masses and magnetic moment of the nucleon, bag radius of the nucleon.

I. INTRODUCTION

THE study of the electric and magnetic form factors the nucleons is of fundamental importance in understanding their electromagnetic structure. The electromagnetic form factors of the nucleon have been a longstanding subject of interest in nuclear and particle physics, and have been the subject of sustained experimental and theoretical investigations for almost 50 years [1].The original MIT bag model was presented about three decades ago [2, 3]. This model is defined by the equation of motion and boundary condition for each field degree of freedom inside the bag and homogeneous boundary condition at the surface of the bag. Hadrons are considered as static extended objects in space. The internal structure of these objects includes quarks and varying gluon field.In MIT bag model, it is supposed that a region of space called "bag" including hadrons fields are fixed. The pressure of hadron constituents in the surface is constant and the vacuum around the bag imposes an external pressure on the surface of the bag. As this external pressure increases more than the internal one, the bag shrinks [4]. B is the only parameter of this theory. Hadrons constituent fields in the bag can carry any spin or quantum numbers. In this paper we generally supposed that the fields in the bag are massless, that is, free Lagrangian is considered for the fields without any interaction in Lagrangian.Although the fields in the bag are free in first approximation, at the next level, it is suggested that the fields couple weakly. Weak coupling is considered for the quantum numbers in the hadron. The hadron fields in the bag are colored quarks and gluons [3]. Models such as MIT bag model describe two features of QCD in the quark model [5], asymptotic in short distances and Confinement in large distances.The goal of this paper is to perform the numerical analysis the masses and magnetic moment of the nucleon with MIT bag model based on the electric and magnetic form factors. Finally the obtained results are compared to the experimental and previous calculated values.

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II. FORM FACTORS OF THE NUCLEON

Elastic electron scattering off the lightest nuclei, hydrogen and deuterium, yields information about the nuclear building blocks, the proton and the neutron [5].

The proton form factors G_E and G_M, , were determined by first converting the experimental elastic e-p cross section $\sigma = (E,\theta)$to the reduced cross section σ_R[6], as:

$$\sigma_R(Q^2,\varepsilon) = \varepsilon(1+\tau)\frac{E}{E'}\frac{\sigma(E,\theta)}{\sigma_{Mott}} = \tau G_{Mp}^2(Q^2) + \varepsilon G_{Ep}^2(Q^2) \quad (1)$$

Where $\sigma_{Mott} = \frac{\alpha^2 cos^2\left(\frac{\theta}{2}\right)}{4E^2 sin^4\left(\frac{\theta}{2}\right)}$ an$\varepsilon = \left(1 + 2(1+\tau)tan^2\left(\frac{\theta}{2}\right)\right)^{-1}$ is the is the transverse polarized of virtual photon. By measuring the reduced cross section σ_R at several ε points for a fixed Q^2, and by making a linear fit to ε, we obtain $\tau G_{Mp}^2(Q^2)$ from the intercept and $G_{Ep}^2(Q^2)$ from the slope.

Quasielastic e-d spectra at each kinematic point were obtained as function of missing mass squared, $W^2 = M^2 - 2M(E-E') - Q^2$.

In this portion $\varepsilon = \left(1 + 2(1+\tau')tan^2\left(\frac{\theta}{2}\right)\right)^{-1}$ is the longitudinal polarization of the virtual photon, with $\tau' = \frac{V^2}{Q^2}$ and $V = E - E'$.

The measured e-d cross section per nucleon, $\sigma(E - E',\theta)$,were converted to reduce cross sections, defined as:

$$\sigma_R = \varepsilon(1+\tau')\frac{\sigma(E,E',\theta)}{\sigma_{Mott}G_D^2} = \frac{R_T}{G_D^2} + \varepsilon\frac{R_L}{G_D^2} \quad (2)$$

To extract the neutron form factors, R_L and R_T were fitted with the model shapes for both the quasielastic and inelastic contributions. The quasielastic component was modeled with a nonrelativistic plane wave impulse approximation (PWIA) calculation [7] using paris deuteron wave function [8].

In the PWL quasielastic potion of R_L is proportional to$\left(G_E^p\right)^2 + (G_E^n)^2$, and that R_T is proportional [9] to $\left(G_M^p\right)^2 + (G_M^n)^2$. The neutron form factors were determined by subtracting the proton form factors measured from the coefficients of the quasielastic fits [10].

III. NUECLEON BAG REDIUS

In the other hand, the nucleon form factors can be calculated in other theory models.There are many calculations of the nucleon electromagnetic form factors within different hadronic models. Indeed the understanding of these form factors is extremely important in any effective theory or models of strong interaction [11]. The electromagnetic form factors of proton and neutron are calculated using MIT bag model wave function and parameters [12, 13]. The MIT bag model is a conceptually very simple phenomenological model developed in 1974 at the Massachusetts Institute of Technology in Cambridge (USA) shortly after the formulation of QCD. It soon became a major tool for hadrons theorists. According to the model quarks are forced by a fixed external pressure to move only inside a given spatial region. Within this region (bag) quarks occupy single-particle orbitals similar to nucleons in the nuclear shell model [4]. The corresponding wave function can be obtain by solving the Dirac equation for free fermions and appropriate boundary condition at the bag surface guarantee that no quark can leave the bag. In the following, we shall investigate spherical bags; this is for sure the most obvious assumption for hadrons in the ground state, and it has the additional advantage that the solutions can be found analytically. Since the surface of the bag isspherical and all quarks are in the lowest eigenmode, the electric and magnetic form factors for the neutron can be written as [11]:

$$G_E(Q^2) = \int_0^R 4\pi r^2 \; dr j_0(Qr)[g^2(r) + f^2(r)] \tag{3}$$

$$G_M(Q^2) = 2m_N \int_0^R 4\pi r^2 \; dr \frac{j_1(Qr)}{Q}[2g(r)f(r)] \tag{4}$$

Where m_N is the nucleon mass. The function $g(r)$ and $f(r)$Are defined by:

$$g(r) = Nj_0\left(\frac{\omega r}{R}\right) \tag{5}$$

$$f(r) = Nj_1\left(\frac{\omega r}{R}\right) \tag{6}$$

Where $\omega = 2.04$ in the lowest mode and $N^2 = {}^{\omega}\!/_{8\pi R^3 j_0^2(\omega)(\omega-1)}$. According to the nucleon electric and magnetic form factors calculated, the radius of the bag can be obtained based on Eq.3 and Eq.4 at each Q^2. The results are shown in Tables 1 and 2 . It seen that Rvalue is inverses proportion to the increase of theQ^2. Since Q^2 increases cause an external pressure, B, imposed on the surface of the bag and makes it contract, hence its radius decreases[12].In the calculation, to obtain the static radius of the bag, the limit value of the nucleons bag radius can be calculated in limit$Q^2 \to 0$ based on the fits of the graphs in Figure 3 and 4 which

Our results shown in Table 3.

IV. Nucleon Mass

In the MIT bag model , nucleons masses determined based on some terms. We summarize them briefly [14]:

(a)The quantum fluctuations contribute two terms

Which depend only on the radius of the nucleon. The Volume term is:

$$E_V \equiv \frac{4}{3}\pi B R^3 \tag{7}$$

The remainder of the zero-point energy is:

$$E_0 \equiv -\frac{Z_0}{R} \tag{8}$$

we can expect Z_0, to be positive and of order unity.

(b)The quarks contribute their rest and kineticenergies to the nucleon's mass . If$N_0, N_s, m_0, and\; m_s$the respective numbers and masses of thenonstrange and strange quarks, and if ω is the frequency defined by:

$$\omega(m,R) \equiv \frac{1}{R}[x^2 + (mR)^2]^{\frac{1}{2}} \tag{9}$$

Where $x = x(mR)$ and obtained by:

$$\tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{\frac{1}{2}}} \tag{10}$$

Then this term is:

$$E_Q \equiv N_0\omega(m_0, R) + N_s\omega(m_s, R) \tag{11}$$

(c) The gluon interaction has color magnetic exchange and color electric parts. The color magnetic exchange term will beWritten in the form:

$$E_M = a_{00}M_{00} + a_{0s}M_{0s} + a_{ss}M_{ss} \tag{12}$$

In Eq.(12) M_{00} is the color magnetic interaction between two nonstrange quark, M_{0s} is that between a nonstrange and strange quark and M_{ss} , the interaction energy between two strange quarks. The values of M_{00} , M_{os} , and M_{ss} can be read off of Fig. 1. The value ofa_{00} for nucleon is (-3) . the color electric energy is given by:

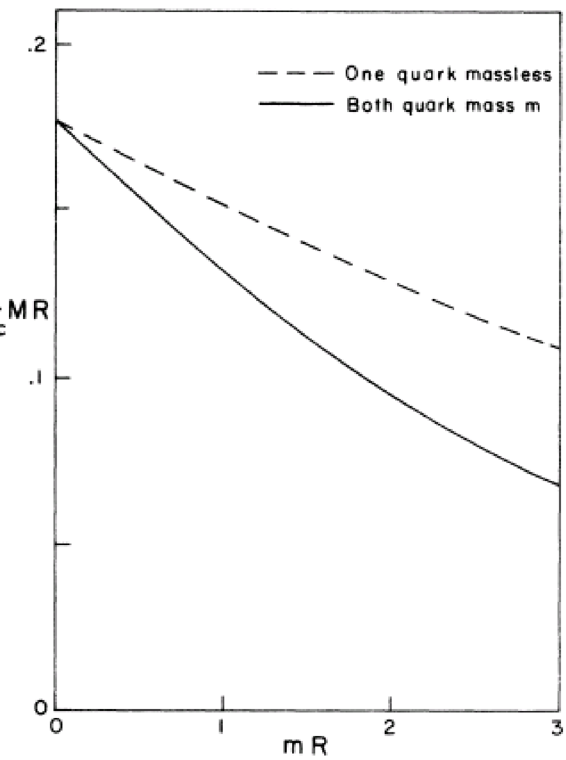

Fig. 1 Magnetic gluon exchange energy of two quarks as a function of mR . M is the quantity referred to in Eq.(12). Ref [14]

$$E_E = b\epsilon \tag{13}$$

Where ϵ is the color electric interaction energy of a strange and a nonstrange quark including bothself-interaction and exchange graphs. The coefficient b is one or zero depending upon whether thequark content of the hadron is mixed or not. Where for nucleon is zero.

The mass of hadron of radius R is then given by:

$$M(R) = E_V + E_0 + E_Q + E_M + E_E \tag{14}$$

Where the individual terms are given by:

$$E_V = \frac{4}{3}\pi B R^3$$

$$E_0 = \frac{-Z_0}{R} \tag{15}$$

$$E_Q = N_0 \omega(m_0, R)$$

Without considering the mass of quarks m_0 in above equation is zero. Then :

$$E_Q = N_0 \frac{1}{R}[x^2 + 0]^{\frac{1}{2}} = \frac{N_0 x}{R} = \frac{3(2.04)}{R} = \frac{6.12}{R} \tag{16}$$

Where due to nucleons do not have strange quark, then Eq. (11) as follow:

$$E_M = a_{00} M_{00} = -3M_{00} \tag{17}$$

The value ofM_{00}can be read off of Fig.2. then:

$$\frac{3}{8 \propto_c} M_{00} R = 0.175$$

$$\propto_c = 0.55 \tag{18}$$

$$M_{00} R = 0 \cdot 256$$

Then :

$$M_N = \frac{4}{3}\pi B R^3 - \frac{z_0}{R} + \frac{6 \cdot 12}{R} - \frac{0 \cdot 768}{R} \tag{19}$$

Values of Z_0 and B in Eq.(19) according to Ref.[15] are $Z_0 = 1 \cdot 84$ and $B^{\frac{1}{4}} = 0 \cdot 145\ (Gev)$. Finally, mass of the nucleon is:

$$M_N = \frac{4}{3}\pi(0 \cdot 145)^4 R^3 - \frac{1 \cdot 84}{R} + \frac{6 \cdot 12}{R} - \frac{0 \cdot 768}{R} \tag{20}$$

According to the calculated static radius, the numerical value of masses of the nucleons can be obtained by others. We shown our results in Table 3.

V. MAGNETIC MOMENT

The magnetic moment of the proton in MIT bag model is defined as [4]:

$$\mu_p = \int d^3 r_1 \int d^3 r_2 \int d^3 r_3\, \psi_p^\dagger \sum_i \left(\frac{\hat{Q}_i}{2} \hat{r}_i \times \hat{\alpha}_i\right) \Psi_p \tag{21}$$

here Ψ_p denotes the proton wave function and $\hat{Q}_i$ is the charge operator. Since the SU(6) wave function of the proton is symmetric under permutations of the indices 1, 2, and 3, we have:

$$\int \Psi_p^\dagger \left(\frac{\hat{Q}_1}{2} \hat{r}_1 \times \hat{\alpha}_1\right) \Psi_p d^3 r_1 d^3 r_2 d^3 r_3 = \int \Psi_p^\dagger \left(\frac{\hat{Q}_2}{2} \hat{r}_2 \times \hat{\alpha}_2\right) \Psi_p d^3 r_1 d^3 r_2 d^3 r_3 = \int \Psi_p^\dagger \left(\frac{\hat{Q}_1}{2} \hat{r}_3 \times \hat{\alpha}_3\right) \Psi_p d^3 r_1 d^3 r_2 d^3 r_3 \tag{22}$$

We insert Ψ_p in Eq. (21) .then:

$$\mu_p = \frac{1}{6} \int d^3 r_1 \times \left[10 u^\uparrow(1)^\dagger \left(\frac{\hat{Q}_1}{2} \hat{r}_1 \times \hat{\alpha}_1\right) u^\uparrow(1) + 2u^\downarrow(1)^\dagger \left(\frac{\hat{Q}_1}{2} \hat{r}_1 \times \hat{\alpha}_1\right) u^\downarrow(1) + 4d^\downarrow(1)^\dagger \left(\frac{\hat{Q}_1}{2} \hat{r}_1 \times \hat{\alpha}_1\right) d^\downarrow(1) + 2d^\uparrow(1)^\dagger \left(\frac{\hat{Q}_1}{2} \hat{r}_1 \times \hat{\alpha}_1\right) d^\uparrow(1)\right] \tag{23}$$

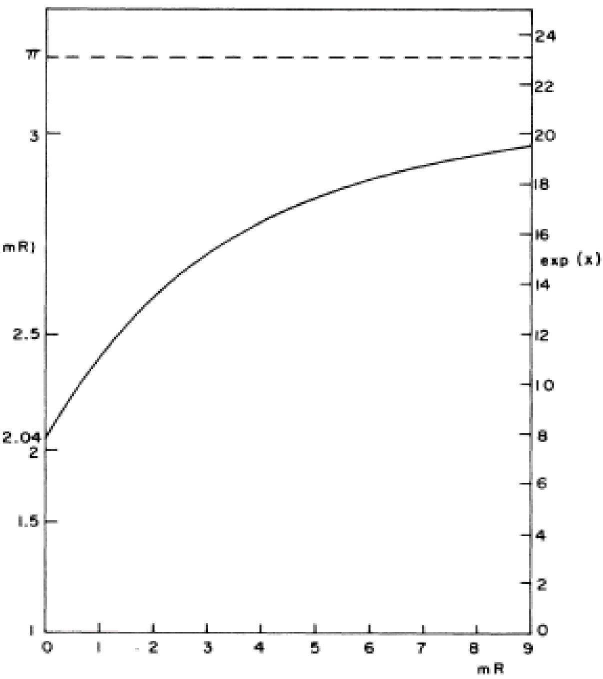

Fig. 2 Eigen frequency x(mR) of the lowest quarkMode with massin a spherical cavity of radius R.Ref [14]

Next we insert the quark charges and wave function of quarks obtain:

$$\mu_p = \frac{e}{2} N^2 \int_0^R dr\ r^2 \int d\Omega \left(j_0(Er) x_{-1}^{\frac{1}{2}\dagger}, -i j_1(Er) x_{-1}^{\frac{1}{2}\dagger} \hat{\sigma}_r\right) \times \begin{pmatrix} 0 & \hat{r} \times \hat{\alpha} \\ \hat{r} \times \hat{\alpha} & 0 \end{pmatrix} \begin{pmatrix} j_0(Er) x_{-1}^{\frac{1}{2}} \\ i j_1(Er) \hat{\sigma}_r x_{-1}^{\frac{1}{2}} \end{pmatrix} = \frac{e}{2} N^2 \int_0^R dr\, r^2 \int d\Omega\ \mathrm{i} j_0(Er) j_1(Er) x_{-1}^{\frac{1}{2}\dagger} [(\hat{r} \times \hat{\alpha}) \hat{\sigma}_r - \hat{\sigma}_r (\hat{r} \times \hat{\alpha})] x_{-1}^{\frac{1}{2}} \tag{24}$$

Using commutation relations of pauli matrices ,σ, therefor :

$$\mu_p = \frac{e}{2} N^2 \int_0^R dr\, r^2 \int d\Omega\ \mathrm{i} j_0(Er) j_1(Er) \frac{1}{4\pi} (1 \quad 0) [2ir\sigma_r - 2ir\sigma] \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{eN^2}{4\pi} \int_0^R dr\, r^3 j_0(Er) j_1(Er) \int d\Omega \left[e_3 - \cos\theta \begin{pmatrix} \sin\theta \cos\varphi \\ \sin\theta \sin\varphi \\ \cos\theta \end{pmatrix}\right] \tag{25}$$

Now we can easily perform the φ, θ , and r integrations:

$$\mu_p = e_3 \frac{Re}{ER(ER-1)} \frac{4ER-3}{12} \tag{26}$$

According to the calculated static radius, the numerical value of magnetic moment of the proton can be obtained by others.. With similar calculation obtained the magnetic moment of neutron. We shown our results in Table 3.

VI. CONCLUSION

The electric and magnetic form factors of the proton are calculated and through using them in the MIT bag model, the bag radius can be calculated . In the limit of$Q^2 \to 0$, the static radius of the bag can be obtained and based on this, masses and magnetic moment of the nucleon can be calculated and compared with the results of others. There should be no difference between the proton and the neutron mass , because the proton and the neutron have the same quark structure.

Finally,as we have remarked, the magnetic moment of the proton is too small.This is a consequence of the fact that the magnetic moment of a quark is associated with the overlap of the small and large components of theDirac wave function and this is rather small.

TABLE I
COMPARISON OF CALCULATED MAGNETIC AND ELECTRIC FORM FACTORS WITHREF[15]AND RATIO OF THESE FORM FACTORS TO DIPOLE FIT .ADDITIONALLY, THE RADIUS OF THE BAG IS SHOWN

Q		G			R_p(fm)
0.65	0.2688	0.2713	0.986	0.995	0.941
0.9	0.1941	0.1971	0.998	1.013	0.873
2.2	0.0557	0.0627	0.936	1.053	0.671
2.75	0.0382	0.0442	0.907	1.050	0.619
3.75	0.0236	0.0262	0.931	1.034	0.546
4.25	0.0202	0.0210	0.986	0.986	0.518
5.25	0.0155	0.0142	1.09	1.000	0.472

TABLE II
RESULTS FOR NEUTRON FORM FACTOR, AND THE RADIUS OF THE BAG IS SHOWN REF.[16]

Q			μ	R
1.75	0.00377	0.1644	1.0329	0.7629
2.5	-0.0514	0.1014	1.0844	0.6713
3.25	0.402	0.06422	1.045	0.6131
4	0.303	0.0480	1.105	0.5629

TABLE III
THE RESULTS OF OUR CALCULATIONS AND COMPARISON WITH THOSE OBTAINED BY OTHERS

	R	R	M (Gev)	M (Gev)		
Our results	1.011	1.044	0.934	0.938	1.919	-1.34
EXP	-	-	0.938	0.93955	2.793	-1.911
Ref [4]	1.005	-	-	-	1.9	-1.27
Ref[14]	0.986	-	0.938	-	-	$-2/3$
Ref [17]	0.95	-	0.940	-	2.71	-

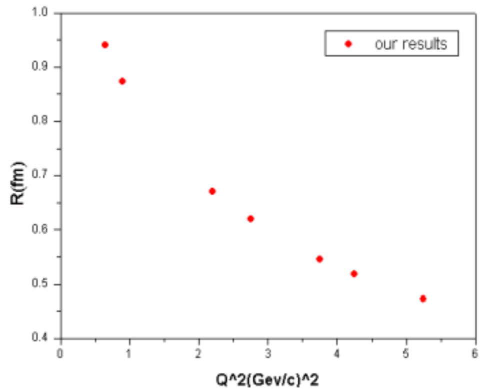

Fig. 3 The bag radius according to values of Q^2 ref[15]

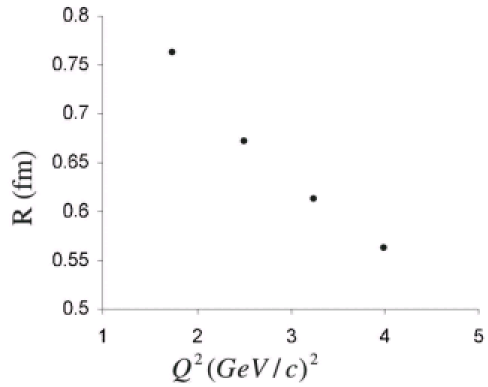

Fig. 4 the bag radius of a neutron ref [17]

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