Using Genetic Algorithms in Closed Loop Identification of the systems with Variable Structure Controller

O.M. Mohamed vall, and M. Radhi

***Abstract*—**This work presents a recursive identification algorithm. This algorithm relates to the identification of closed loop system with Variable Structure Controller. The approach suggested includes two stages. In the first stage a genetic algorithm is used to obtain the parameters of switching function which gives a control signal rich in commutations (i.e. a control signal whose spectral characteristics are closest possible to those of a white noise signal). The second stage consists in the identification of the system parameters by the instrumental variable method and using the optimal switching function parameters obtained with the genetic algorithm. In order to test the validity of this algorithm a simulation example is presented.

***Keywords*—**Closed loop identification, Variable structure controller, Pseud-random Binary Sequence, Genetic algorithms.

I. Introduction

THE purpose of closed loop identification is to identify a process model while the process is still under feedback control [5],[8]. There are serval reasons for performing the identification in closed loop: the system might be unstable in open loop or the system contains inherent feedback mechanisms [5]. Safety and/or economic reasons are also often strong reasons for performing identification experiments in closed loop. The main problem with closed loop identification is the correlation between the unmeasurable noise and the input, induced by the loop. Servals classical closed loop identifications approaches are available to cope with this problem, broadly categorized into three main groups: the direct approach, the indirect approach, and the joint input-output approach.

The direct approach: apply a prediction error method and identify the open-loop system using measurement of the input and the output, ignoring possible feedback. This approach gives consistency and optimal accuracy, given that the true noise characteristics are correctly modeled. A drawback of the direct approach is that we need good noise model. In practice this means that we must include a sufficiently flexible, parameterized noise model (which out-rules output error models). In case a fixed, or too ''small'', noise model is used the results will be biased. The reason for this bias error is that there is correlation between the output noise and the input. This is also why other methods, like instrumental variables, spectral analysis and subspace methods, fail when applied directly to closed loop data.

The indirect approach: identify the closed-loop system using measurements of the reference signal and the output and use this estimate to solve the open-loop system parameters using the knowledge of the controller. For this approach the feedback structure must be know (and linear), and it is also required that an external reference signal is used and that this measurable [7].

The joint Input-output approach: identify the transfer function from the reference signal and the output and from the reference signal and the input and use them to compute an estimate of the open-loop system.

In this work we use the first approach such as the direct approach and it is supposed that the noise can be modeled. As it is mentioned previously the results of identification by this method can be biased. This due to the correlation between the output noise and the input. To solve this problem we apply one of the identification methods based on the decorrelation of the observations vector and of the prediction error such as the instrumental variable with observations delayed method.

Here the system to identify is a closed loop system with Variable Structure Controller (VSC). It should be noted that the signal of variable structure controller has the form of a Pseud-random Binary Sequence PRSB (often used as input signal for the identification). This was the original idea of this work.

In this work a genetic algorithm is applied to determine the parameters of the switching function which give a control signal whose spectral characteristics are nearest possible to those of a white noise signal.

II. Problem Formulation

Consider a linear SISO closed-loop system depicted in Fig. 1.

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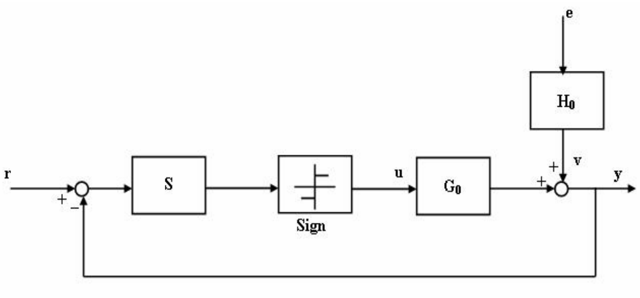

Fig. 1 Closed loop system with VSC

Where G_0 represents the true process to be identified, u(t) describes the process input signal (the variable structure controller signal), y(t) the process output signal, {e(t)} is white noise with variance λ_0 and r(t) is the reference signal.

With this notation, the true system is given by :

$$\begin{cases} y(t) = G_0(q)u(t) + v(t) \\ v(t) = H_0(q)e(t) \end{cases} \quad (1)$$

the input *u(t)* is given by:

$$u(t) = -K.Sgn(S) \quad (2)$$

K is a constant and it is the maximal value of the controller output. *S* is called switching function. *S* is defined as:

$$S = e + \lambda_1 \varepsilon^{(1)}(t) + ... + \lambda_{n-1}\varepsilon^{(n-1)}(t) \quad (3)$$

where $\varepsilon(t) = r(t) - y(t)$, λ_i is a constant and $\varepsilon^{(i)}(t)$ is the *ith* derivative of ε *(t)* for *i* =1..*n*-1. *n* is the true system order. *Sgn(S)* is a sign function, which is defined as:

$$Sgn(S) = \begin{cases} +1 & if\ S \le 0 \\ -1 & if\ S > 0 \end{cases} \quad (4)$$

Let us consider that in the closed-loop this system can be described by the following model:

$$y(k) = -a_1 y(k-1) - ... - a_n y(k-n) + b_1 u(k-1) + ... + b_n u(k-n) + v(k) \quad (5)$$

This mathematical model can be written in the following compact form:

$$y(k) = \theta^T \psi(k) + e(k) \quad (6)$$

Where

$\theta^T = [a_1 ... a_2\, b_1 .. b_n]$ and

$\psi^T = [-y(k-1) - .... - y(k-n)u(k-1)...u(k-n)]$

We can estimate the model by the straightforward fit:

$$\hat{\theta} = arg\ \min_{\theta} V_N(\theta) \quad (7)$$

with

$$V_N(\theta) = \sum_{k=1}^{N} \varepsilon^2(k,\theta) \quad (8)$$

and

$$\varepsilon(k,\theta) = y(k) - \hat{y}(k/\theta) = y(k) - \hat{\theta}^T \psi(k) \quad (9)$$

$\varepsilon(k,\theta)$ is the prediction error.

The problem arising here consists in developing a recursive algorithm able to identify θ. It should be noted that the recursive algorithm of identification RLS cannot solve this problem, and this, because of the noise *v*. Indeed, this noise *v* is strongly correlated with the observations, and thereafter the use of the identification algorithm RLS give results biased. To solve this problem we thus propose to use the Recursive Instrumental Variable (RIV) method. In more we exploit the fact that the control signal, here, has the characteristics of a white noise.

The general idea of the instrumental variable method consists in creating a new observations vector which is not correlated with the noise to be able to obtain $E\left\{\psi(k)\varepsilon(k+1)\right\} = 0$. The new vector thus created is called variable instrumental. There are many possible ways to construct the instrumental variable. For instance, in closed loop it may be built from delayed inputs and outputs. The new observations vector will be:

$$\begin{aligned} Z^T = [&-y(k-1-d) - ... - y(k-n-d)... \\ &u(k-1-d)...u(k-n-d)] \end{aligned} \quad (10)$$

If the noise *v* is assumed to be a moving average of order n_V, *d* must satisfy the condition: $d \ge n_V$.

In this work, we use the following Recursive Instrumental Variable (RIV) algorithm:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)Z(k)\varepsilon(k) \quad (11)$$

$$P(k) = P(k-1) - \frac{P(k-1)Z(k)\psi^T(k)P(k-1)}{1 + \psi^T(k)P(k-1)Z(k)} \quad (12)$$

$$\varepsilon(k) = y(k) - \hat{\theta}^T(k-1)\psi(k) \quad (13)$$

where

$$\begin{aligned} Z^T = [&-y(k-1-d) - ... - y(k-n-d)... \\ &u(k-1)...u(k-n)] \end{aligned} \quad (14)$$

It should be noted that in our case, one can take the inputs not delayed because the input signal *u* has the characteristics of a white noise and he is thus not correlated with the noise *v*. this represents an originality of this work.

III. Genetic Algorithms Applied to Determine the Switching Function Parameters

A. Genetic Algorithms

Genetic Algorithms (GA)[10] are search algorithms that use operations found in natural genetics to guide the trek through a search space. GA uses a direct analogy of natural behavior. They work with a population of individuals, each representing a possible solution to a given problem. Each individual has assigned a fitness score according to how good solution to the problem it is.

Any GA starts with a population of randomly generated solutions, chromosomes, and advances toward better solutions by applying genetics operators, modeled on the genetic processes occurring in nature. The most usual operators are as follows:

- Selection: The main goal is selecting the chromosomes with the best qualities for integration in the next generation (these would depend on the cost function for each individual).
- Crossover: By combining the chromosomes of two individuals. New chromosomes are generated and integrated into the population.
- Mutation: Random variations of parts of the chromosome of an individual in the population generate new individuals.

The fig. 2 shows the structure of a simple GA.

```
Begin (1)
  t=1
  Initialize Population(t)
  Evaluate fitness Population(t)
  While (Generations < Total Number) do
  Begin (2)
     Select Population(t+1) out of Population(t)
Apply Crossover on Population(t+1)
     Apply Mutation on Population(t+1)
     Evaluate fitness Population(t+1)
     t = t + 1
  End (2)
End (1)
```

Fig. 2 Structure of standard genetic algorithm

The variations of the GA can be distinguished by the kind of condition used for chromosomes and the genetic operators used.

GA has demonstrated very good performances as global optimizers in many types of applications.

B. Determination of the Switching Function Parameters by the Genetic Algorithms

Here, it is a question of applying the genetic algorithms to determine the switching function parameters which give a control signal whose spectral characteristics are nearest possible to those of a white noise signal. The autocorrelation function of a white noise signal verifies:

$$R(\tau) = \begin{cases} \sigma^2 & if \quad \tau = 0 \\ 0 & if \quad \tau \neq 0 \end{cases} \tag{15}$$

and We note also that the autocorrelation function of the VSC signal highly depends on the switching function parameters. In order the get a VSC signal with an autocorrelation function like that of that of a white noise signal, we propose to determine the Switching function parameters that minimize the criterion :

$$J = \sum_{i=2}^{N} (R_{uu}(i))^2 \tag{16}$$

R_{uu} is the autocorrelation function of the VCS signal.

We then propose to deal with this problem of minimization with AG. The reason of this is that the function of autocorelation of the signal of CSV cannot be expressed analytically.

The application of the GA to determine the switching function parameters can be reformulated as follows:

1. Starting with an initial population randomly generated (N vectors $(\lambda_1 \ldots\ldots \lambda_{n-1})^T$. The λ_i are the switching function parameters.
2. Calculation of the fitness function (in our case this function is *J*) value for each individual (vector).
3. Selection of the best individuals (we chose a probability of selection equal to *0.75*).
4. Creation on a new population (from the old one) by the application of the operators :
   - Crossover (with a Crossover probability *PC = 0.95*)
   - Mutation (with a Mutation probability *Pm= 0.01*)
5. While the termination condition is not met, return at step 2.

IV. Simulation Example

To illustrate the performances of the proposed algorithm, we consider the following numerical example. The process to identify is described by (1), where

$$G_0 = \frac{0.51650q^{-1} + 0.78900q^{-2}}{1 - 1.98320q^{-1} + 0.96320q^{-2}}, \; H_0 = \frac{0.00996q^{-1}}{1 - 0.992q^{-1}}$$

The parameter vector to be estimated is therefore given by $\theta = (-1.983, 0.9632, 0.51650, 0.78900)$. $e(t)$ is a mean zero Gaussian white noise with variance 0.1, $u(t) = -10.Sgn(S)$, $S = \varepsilon(t) + \lambda\varepsilon^{(t)}(t)$, $\varepsilon(t) = 2 - y(t)$.

Initially GA is used in order to determine the λ value which minimizes (16).

The Fig. 3 and Fig. 4 show, respectively the evolution of the λ and of the fitness function during the optimization.

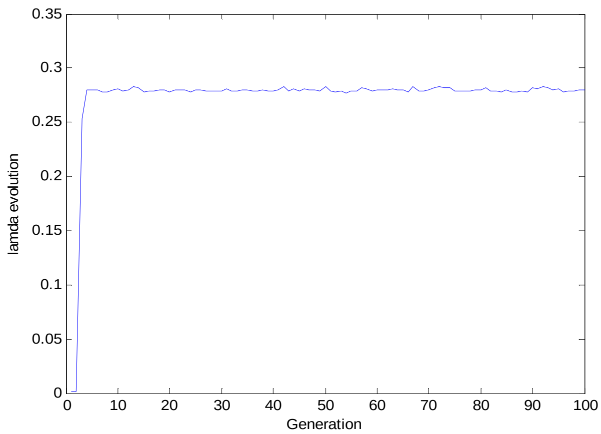

Fig. 3 Evolution of the Switching function parameter during the optimization

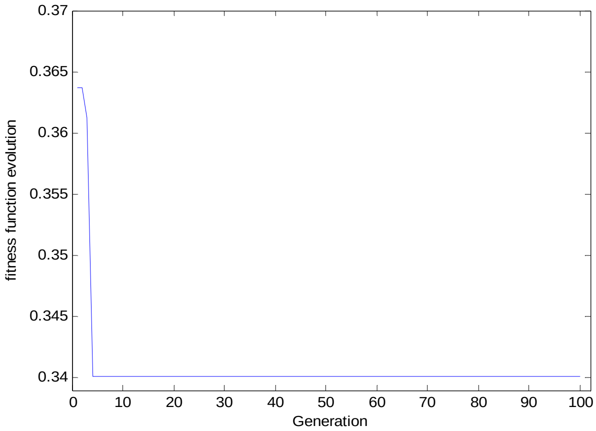

Fig. 4 Evolution of the fitness function during the optimization

In the second time the parameters of the system are estimated by the RIV method exposed previously and this from the output signal and control signal using the optimal value of λ obtained with the GA. The comparison between the actual and the estimated values of these parameters is presented in the Fig. 5.

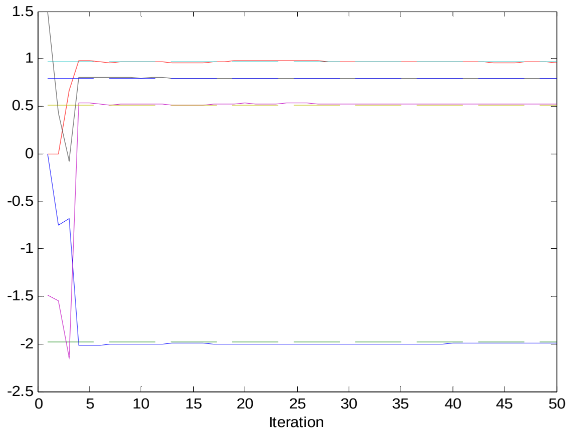

Fig. 5 Comparison between the actual (dashed) and estimated (solid) values of the system parameters to identify

This last figure shows the validity of the identification approach suggested.

V. Conclusion

In this paper, a recursive identification algorithm for the systems in sliding mode has been presented. This algorithm includes two stages. The first consists to the use of a genetic algorithm to determinate of the switching function parameters which gives a control signal whose spectral characteristics are closest possible to those of a white noise signal. The second stage consists to the estimate of parameters of system (by a RIV method). Finally, the effectiveness of the proposed approach has been demonstrated by simulation example.

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