Maximizer of the Posterior Marginal Estimate for Noise Reduction of JPEG-compressed Image

Yohei Saika and Yuji Haraguchi

Abstract—We constructed a method of noise reduction for JPEG-compressed image based on Bayesian inference using the maximizer of the posterior marginal (MPM) estimate. In this method, we tried the MPM estimate using two kinds of likelihood, both of which enhance grayscale images converted into the JPEG-compressed image through the lossy JPEG image compression. One is the deterministic model of the likelihood and the other is the probabilistic one expressed by the Gaussian distribution. Then, using the Monte Carlo simulation for grayscale images, such as the 256-grayscale standard image "Lena" with 256×256 pixels, we examined the performance of the MPM estimate based on the performance measure using the mean square error. We clarified that the MPM estimate via the Gaussian probabilistic model of the likelihood is effective for reducing noises, such as the blocking artifacts and the mosquito noise, if we set parameters appropriately. On the other hand, we found that the MPM estimate via the deterministic model of the likelihood is not effective for noise reduction due to the low acceptance ratio of the Metropolis algorithm.

Keywords—Noise reduction, JPEG-compressed image, Bayesian inference, the maximizer of the posterior marginal estimate

I. INTRODUCTION

RESEARCHERS have investigated image compression, such as the JPEG (Joint Photographic Expert Group) compressed image [1] in various areas of image processing technology. Now, lossless JPEG image compression is now established. However, even now, lossy JPEG-compressed images have been used in various areas of information processing technology, because high quality compressed image has been constructed by the lossy JPEG image compression utilizing the DCT (discrete cosine transformation) and the quantization in spatial momentum space. However, in the patterns of such JPEG compressed images, there appear various types of noises, such as the block-type and the mosquito noises due to the conventional procedure of the JPEG image compression. Therefore, in order to obtain original image with high image quality, it is necessary to reduce such noises from the JPEG-compressed image. For this purpose, a lot of methods [2]-[7] have been attempted. On the other hand, based on analogy between statistical mechanics and Bayesian inference via the maximizer of the posterior marginal (MPM) estimate, theoretical physicists have applied statistical mechanics to information science [8]-[10].

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In the early stage of this field, statistical mechanical techniques have been applied to image restoration and error-correcting codes [11],[12], such as the cluster variation method and the replica theory established in the theory of spin glasses. Then, the statistical mechanics has been applied to various problems in information technology, such as the low-density parity checking codes [13]. Statistical mechanics to information has become an established field to investigate theoretical aspect of various problems in information, such as image processing technology [14] and quantum information [15].

In this study, we constructed a method of noise reduction for JPEG-compressed image converted by the conventional JPEG image compression utilizing the DCT and quantization on the basis of the Bayesian inference using the MPM estimate. In this method, we tried two kinds of the likelihood which enhances grayscale images converted into the JPEG-compressed image by the conventional JPEG-compressed images. One is the deterministic model giving a constraint which stabilizes grayscale images converted into the JPEG-compressed image by the lossy JPEG image compression scheme. The other is the probabilistic model of the likelihood enhancing grayscale images converted into the JPEG-compressed image by the lossy JPEG image compression. Then, in order to reduce noises, such as the blocking artifacts and the mosquito noise, we use the model of the true prior enhancing smooth structures by suppressing difference of gray-levels between neighboring pixels. Using the Monte Carlo simulation for grayscale images, such as the 256-grayscale standard image "Lenna" with 256×256 pixels, we examine the efficiency of the MPM estimate for noise reduction of the JPEG-compressed image converted by the lossy JPEG image compression scheme. First we tried the MPM estimate using the likelihood expressed by the deterministic model stabilizes grayscale images converted into the JPEG-compressed image. The Monte Carlo simulation clarified that the MPM estimate using the likelihood expressed in the deterministic form does not work for this problem and therefore that this fact suggests that the Monte Carlo simulation is difficult to construct the Bayes-optimal solution for noise reduction of the JPEG-compressed image converted by the lossy JPEG image compression scheme. So, we then tried the MPM estimate using the likelihood expressed in the Gaussian probabilistic distribution. The simulation clarified that the MPM estimate using the probabilistic version of the likelihood is successful in reducing noises introduced into the JPEG compressed image by the lossy JPEG image compression scheme, if we set parameters appropriately.
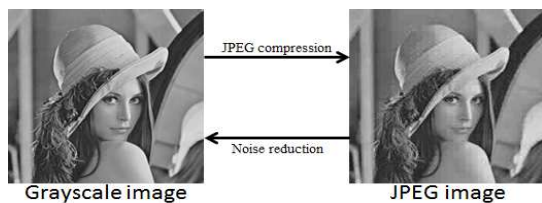
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Fig. 1 JPEG compression for grayscale images and noise reduction of the JPEG image

Fig. 2 (a) the 256-grayscale standard image "Lenna" with 256×256 pixels, (b) the JPEG-compressed image converted by the lossy JPEG image compression (q=2, MSE=83.26), (c) the grayscale image obtained by the MPM estimate when q=2, J=0.23, T_m=1 and h=1.0 (MSE=91.87), (d) the grayscale image obtained by the MPM estimate when q=2, J=0.31, T_m=1 and h=1.0 (MSE=77.85)
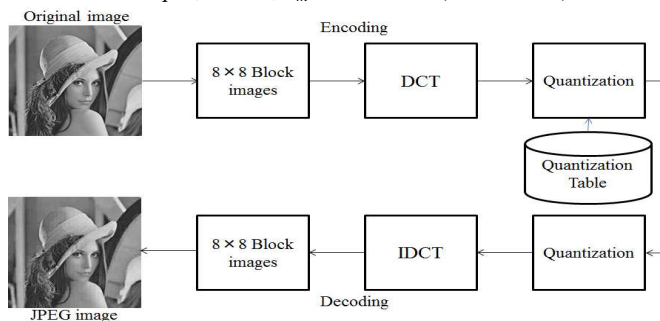

Fig. 3 Lossy JPEG image compression using the blocking and quantization due to quantization table
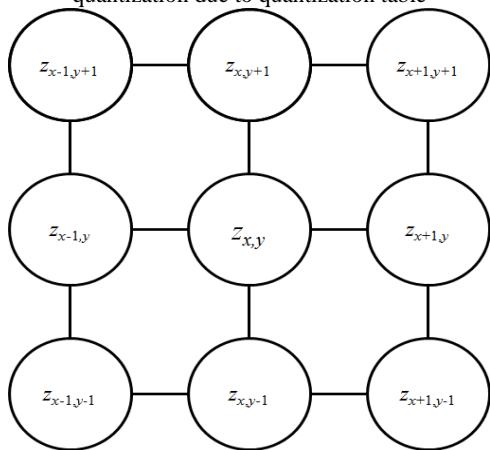

Fig. 4 the Q-Ising model on the square lattice

Then, as seen from the reconstructed grayscale images, we found that the MPM estimate using the likelihood expressed in the Gaussian probabilistic distribution.

The content of this paper is organized as follows. Next, we overviewed general formulation for the problem of noise reduction for the JPEG-compressed image converted by the lossy JPEG image compression scheme. Then, we discussed this method from the viewpoint of statistical mechanics of the Q-Ising model on the square lattice. Next, we showed the performance of the Bayesian inference using the MPM estimate to the problem of noise reduction of the JPEG-compressed image using the Monte Carlo simulation for the 256-grayscale standard images. The last part was devoted to summary and discussion.

II. GENERAL FORMULATION

In this section, we overviewed the general formulation for the problem of noise reduction of JPEG-compressed image converted by the lossy JPEG image compression scheme for the 256-grayscale images.

As shown in Fig. 1, the present formulation is composed of two parts. One is the forward process representing the lossy JPEG compression for 256-grayscale images. Then, the other is then the inverse process representing the noise reduction for the JPEG-compressed image based on the Bayesian inference using the MPM estimate. In this method, we tried two kinds of the likelihood representing the model of the JPEG image compression scheme. One is the deterministic model giving the constraint that we permit grayscale images which are converted into the JPEG-compressed images. Then, the other is the likelihood expressed by the Gaussian probability distribution stabilizing grayscale images which are converted into the JPEG-compressed images through the lossy JPEG image compression shown as below.

In the forward process, we first consider an original grayscale image $\{\xi_{x,y}\}$, where $\xi_{x,y}$=0,…,Q-1, x, y=1,…,L. If we discuss the statistical performance of the present method, we use a set of original image $\{\xi_{x,y}\}$ generated with the probability distribution expressed by Pr($\{\xi_{x,y}\}$). On the other hand, we discuss the performance for realistic images, we consider a typical 256-grayscale image, such as the 256-grayscale standard image "Lenna" with 256×256 pixels (Fig. 2(a)). Then, as shown in Fig. 3, we convert the original image $\{\xi_{x,y}\}$ through the procedure of the conventional lossy JPEG image compression. In this procedure, we first split the original image $\{\xi_{x,y}\}$ into a set of 8×8 images $\{\eta_{i,j}(m, n)\}$, where $\eta_{i,j}(m, n)$=0,…,Q-1, i=x%8, j=y%8, m=1,…,L/8, n=1,…,L/8. Then, we transform each 8×8 image $\{\eta_{i,j}(m, n)\}$ into the spatial-frequency representation $\{\zeta_{u,v}(m, n)\}$ by using the DCT (discrete cosine transformation) in two dimensions as

$$\zeta_{u,v}(m,n) =$$

$$\frac{1}{4} C(u)C(v) \sum_{i=1}^{8} \sum_{j=1}^{8} \cos\frac{(2i+1)ui}{16} \cos\frac{(2j+1)vj}{16} \eta_{i,j}(m,n) \quad (1)$$

where

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$$C(u) = \begin{cases} 1/\sqrt{2} & (u=0) \\ 1 & (u \neq 0) \end{cases}. \qquad (2)$$

Next, in order to reduce high-spatial frequency components $\{\zeta_{u,v}(m,n)\}$, we carry out quantization by

$$\tau_{u,v}(m,n) = \left\lfloor \frac{\zeta_{u,v}(m,n)}{qQ(u,v)} + \frac{1}{2} \right\rfloor, \qquad (3)$$

using the quantization table $Q(u,v)$:

$$Q(u,v) = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 66 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 57 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 36 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix} \qquad (4)$$

Then, if we would like to observe the pattern of the JPEG image, we transform $\{\zeta_{u,v}(m,n)\}$ by the block combination of 8 $\times 8$ images after the inverse DCT transformation. The JPEG image obtained from the original image in Fig. 2(a) is shown in Fig. 2(b). As shown in Fig. 2(b), there appear the blocking artifacts and the mosquito noises both of which are typical in the JPEG-compressed image.

In the inverse process, we construct the Bayesian inference using the MPM estimate to reduce noises appearing in the JPEG-compressed image. In order to reduce noises of the JPEG-compressed image $\{\tau_{x,y}\}$, we use the model system $\{z_{x,y}\}$($z_{x,y}=0,\ldots,Q-1$, x, $y=1,\ldots,L$) in Fig. 4. Then, we reduce noises appearing in the JPEG-compressed image $\{\tau_{x,y}\}$ so as to maximize the posterior marginal probability as

$$\hat{z}_{x,y} = \arg \max_{z_{x,y}=0,\ldots,Q-1} \sum_{\{z\} \neq z_{x,y}} \Pr(\{z\} \mid \{\tau\}). \qquad (5)$$

where the posterior probability is estimated based on the Bayes formula:

$$\Pr(\{z\} \mid \{\tau\}) = \frac{\Pr(\{z\}) \Pr(\{\tau\} \mid \{z\})}{\sum_{\{\tau\}} \Pr(\{z\}) \Pr(\{\tau\} \mid \{z\})} \qquad (6)$$

using the model of the true prior $\Pr(\{\tau_{x,y}\})$ and the likelihood $\Pr(\{\tau_{x,y}\} \mid \{z_{x,y}\})$. In this study, we use the model prior which enhances smooth structures as

$$\Pr(\{z\}) = \frac{1}{Z_m} \exp\left[-\beta J \sum_{n.n.} (z_{x,y} - z_{x',y'})^2 \right]. \qquad (7)$$

As shown in this equation, we enhance smooth structures by suppressing difference of the gray-levels between neighboring pixels. Then, we try two kinds of likelihood enhancing grayscale images converted into the JPEG-compressed image by the above lossy JPEG image compression scheme. One is the deterministic model which gives a constraint permitting grayscale images converted into the JPEG-compressed image

through the above lossy JPEG image compression. That is, the explicit form of the likelihood is expressed as

$$\Pr(\{\tau\} \mid \{z\}) = \prod_{x=1}^{L} \prod_{y=1}^{L} \delta(f_{x,y}(\{z\}) - \tau_{x,y}), \qquad (8)$$

where $f_{x,y}(\{z\})$ is the pixel value of the model system $\{z_{x,y}\}$ at the (x,y)-th site of the JPEG-compressed image converted from $\{z_{x,y}\}$ by the lossy JPEG image compression scheme. Then, the other is the likelihood expressed by the Gaussian probability distribution as

$$\Pr(\{\tau\} \mid \{z\}) \propto \exp\left[-\beta J \sum_{x=1}^{L} \sum_{y=1}^{L} \left(f_{x,y}(\{z\}) - \tau_{x,y} \right)^2 \right], \qquad (9)$$

which enhances grayscale images $\{z_{x,y}\}$ converted into the JPEG-compressed image $\{\tau_{x,y}\}$ by the above lossy JPEG compression scheme. Though researchers have often used the likelihood expressed by

$$\Pr(\{\tau\} \mid \{z\}) \propto \exp\left[-\beta h \sum_{x=1}^{L} \sum_{y=1}^{L} \left(z_{x,y} - \tau_{x,y} \right)^2 \right] \qquad (10)$$

for convenience, however, as we here treat the above models given in eqs. (8) and (9).

In order to clarify the performance of the Bayesian inference using the MPM estimate for noise reduction of the JPEG-compressed image, we use the performance measure based on the mean square error between original and reconstructed images as

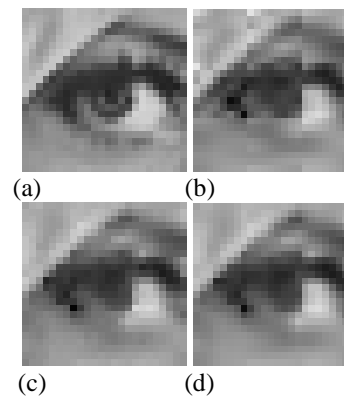$$\sigma = \sum_{x=1}^{L} \sum_{y=1}^{L} (z_{x,y}^R - \xi_{x,y})^2, \qquad (11)$$

(a) (b)

(c) (d)

Fig. 5 (a) the 256-grayscale standard image "Lenna" with 256×256 pixels (part; eye), (b) the JPEG-compressed image converted from (a) by the lossy JPEG image compression (part; eye), (c) the reconstructed image due to the MPM estimate using the Gaussian probabilistic likelihood, (part; eye), (d) the reconstructed image due to the MPM estimate using the Gaussian probabilistic likelihood (part; eye)

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Fig. 6 (a) the 256-grayscale standard image "Lenna" with 256×256 pixels (part; cheek), (b) the JPEG-compressed image converted from (a) by the lossy JPEG image compression (part; cheek), (c) the reconstructed image due to the MPM estimate using the Gaussian probabilistic likelihood, (part; cheek), (d) the reconstructed image due to the MPM estimate using the Gaussian probabilistic likelihood(part; cheek)
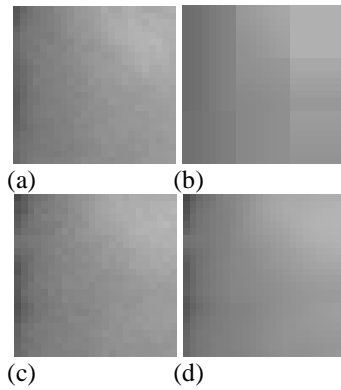
Here $\{z^R_{x,y}\}$ is the reconstructed image by the present method. This variable takes zero, if the image reconstruction is carried out completely.

III. STATISTICAL MECHANICAL VIEWPOINT

In this section, based on statistical mechanics, we discussed the present methods for noise reduction of the JPEG-compressed image. From the statistical mechanical point of view, the MPM estimate is regarded as constructing the equilibrium state of the Q-Ising model whose Hamiltonian is given in the following part of this section. In the case of the MPM estimate via the deterministic version of the likelihood, the system is, from the viewpoint of statistical mechanics, regarded as the Q-Ising model whose Hamiltonian is expressed as

$$H(\{z\}) = J \sum_{x=1}^{L} \sum_{y=1}^{L} \left\{ (z_{x,y} - z_{x+\delta_x,y})^2 + (z_{x,y} - z_{x,y+\delta_y})^2 \right\}, \tag{12}$$

under the constraint:

$$f_{x,y}(\{z\}) = \tau_{x,y} \tag{13}$$

for every pixel $(x, y=1,\ldots,L)$. In this case, it is considered to be difficult to construct thermal equilibrium state of this system based on the Metropolis algorithm, as the transition probability becomes small extremely under the constraint in eq. (11). On the other hand, in the case of the MPM estimate via the probabilistic version of the likelihood, the system is, from the viewpoint of statistical mechanics, regarded as the Q-Ising model whose Hamiltonian is expressed as

$$H(\{z\})$$
$$= J \sum_{x=1}^{L} \sum_{y=1}^{L} \left\{ (z_{x,y} - z_{x+\delta_x,y})^2 + (z_{x,y} - z_{x,y+\delta_y})^2 \right\} \tag{14}$$
$$+ h \sum_{x=1}^{L} \sum_{y=1}^{L} (f_{x,y}(\{z\}) - \tau_{x,y})^2$$

In this case, it is not so difficult to construct the equilibrium state of this system in the region $J \sim h$, as there is no constraint, such as eq. (11). However, if $J << h$, the system is approximately same as the previous case, so it is expected that constructing thermal equilibrium state is difficult using the conventional Metropolis algorithm.

IV. PERFORMANCE

Here, we investigated the performance of the Bayesian inference via the MPM estimate for noise reduction of the JPEG-compressed image converted through the lossy JPEG image compression.

When we numerically estimated the performance of this method, we carried out the Monte Carlo simulation in the following conditions. We first considered grayscale images, such as the 256-grayscale standard image "Lenna" with 256×256 pixels. Then, each image was converted into the JPEG-compressed image by the above lossy JPEG image compression. Next, when we reduced noises of the JPEG-compressed image, we carried out the Monte Carlo simulation with 20000 MCS for each image based on the Metropolis algorithm. When we estimated the performance of this method for noise reduction of the JPEG-compressed image, we numerically estimated the mean square error between original and reconstructed images.

First, we examined the efficiency of the MPM estimate whose Hamiltonian is given in eq. (11) under the constraint in eq. (12). In this case, as the Metropolis algorithm does not work under this constraint, this method was impossible to reduce noises appearing in the JPEG-compressed image. So, we tried the MPM estimate using the likelihood expressed in the Gaussian probability distribution. As shown in Figs. 2 (c) and (d), we found that the MPM estimate is useful for noise reduction of the JPEG-compressed image, if we set the parameters appropriately. Further, as shown in Figs. 5(c), (d) and Figs. 6(c) and (d), it was clearly seen that the blocking artifacts are reduced by this method, if we appropriately set parameters.

V. SUMMARY AND DISCUSSION

In previous sections, on the basis of the analogy between statistical mechanics of the Q-Ising model and the Bayesian inference using the MPM estimate, we have constructed the methods for noise reduction of the JPEG-compressed image converted by the lossy JPEG image compression scheme. Then, we discussed the properties of the MPM estimate for noise reduction of the JPEG-compressed image from the viewpoint of statistical mechanics. From this viewpoint, we clarified that the thermal equilibrium state is expected to be constructed easier, if we applied the likelihood using the Gaussian probabilistic distribution to the MPM estimate under the appropriate condition. Then, in order to clarify the efficiency of this method for noise reduction of the JPEG-compressed image, we carried out the Monte Carlo simulation for the 256-grayscale image "Lenna" with 256×256 pixels. The simulations clarified that the MPM estimate using the likelihood expressed in the

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Gaussian distribution is successful in reconstructing the original image from the JPEG-compressed image, if we appropriately set parameters. Also, we found that the blocking artifacts are reduced in the pattern of the reconstructed image.

As a future problem, in order to improve the performance of noise reduction of the lossy JPEG-compressed image, we will construct a realistic method for noise reduction of the JPEG-compressed image by introducing prior information on original realistic images.

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