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Nonlinear Controller for Fuzzy Model of Double Inverted Pendulums

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***Abstract*—**In this paper a method for designing of nonlinear controller for a fuzzy model of **D**ouble **I**nverted **P**endulum is proposed. This system can be considered as a fuzzy large-scale system that includes offset terms and disturbance in each subsystem. Offset terms are deterministic and disturbances are satisfied a matching condition that is mentioned in the paper. Based on Lyapunov theorem, a nonlinear controller is designed for this fuzzy system (as a model reference base) which is simple in computation and guarantees stability. This idea can be used for other fuzzy large-scale systems that include more subsystems. Finally, the results are shown.

***Keywords*—**Controller, Fuzzy Double Inverted Pendulums, Fuzzy Large-Scale Systems, Lyapunov Stability.

I. INTRODUCTION

INDUSTRIAL systems are usually complicated and stochastic; in other hand these systems are large-scale. So, analysis of stability, performance and reliability is important aspect of designing in these systems. Also, it is necessary that the mathematical models of systems are available; these models are nonlinear model usually. Advent of fuzzy systems [1] and extension of these systems, established many methods for modeling, designing and analyzing. It has proven that all nonlinear systems can be modeled by a fuzzy system [2]. Nowadays, ability in designing and flexibility are important advantage of the mentioned systems. [3] Presented a new fuzzy system which was linear state space in consequents that has named as Takagi-Sugeno fuzzy systems ($T-S$). So, analysis of these models (as a model reference base) is an important issue. Advent of theses methods and models needed new methods for stability analysis and controlling. [4][5][6][7] Present methods and conditions for stability in continuous and discrete fuzzy systems.

These fuzzy models (namely $T-S$) may include nonlinear terms in consequent as offset terms, interconnections between states, time delay, disturbance and etc. Fuzzy model of Double Inverted Pendulums can be considered as fuzzy large-scale system that includes two subsystems and each subsystem consists of offset terms and disturbance. Since, these types of systems (similar to Inverted Pendulum, Mass-Spring-Damper and etc) can be proper primary models for analyzing of industrial systems and military, they are significant models.

Recently, some paper and literatures are presented for analyzing of fuzzy Large-scale systems. Some attempts have focused on stability and designing of fuzzy large-scale systems. [8],[9] presented criterions for stability problem of fuzzy-large scale systems and [10] studied the decentralized PDC for fuzzy large-scale systems. [11] Presented an approach to stability analysis and H_∞ controller based on LMIs method for fuzzy large-scale systems.

In this paper, main attempt focused on presenting a method and an appropriate idea for fuzzy large-scale system as a nonlinear system which includes nonlinear terms as offset terms, interconnections between states and disturbance. Also, it is focused on an idea that is applicable for other complicated fuzzy systems which include more subsystems.

The contributions of this paper are threefold. First, we introduce Fuzzy Double Inverted Pendulums, second, we introduce a nonlinear controller for this system and analyze stability and the third is results.

Nomenclature: Throughout this paper, the superscript "T" denotes matrix transposition and the notation. $X \geq Y$ $(respectively, X > Y)$ where X and Y are matrix, means that $X-Y$ is positive semi-definite (respectively, positive definite). $\|A\|_p$ means $p-norm$ of matrix A. "min" and "max" are abbreviator of minimum and maximum respectively.

$(\lambda_{min}(A), \lambda_{max}(A), \lambda_i)$ denote to $(minimum\ Eigen\ value\ of\ A, maximum\ Eigen\ value\ of\ A, i-th\ Eigen\ value\ of\ A)$ consequently. Also, $|a|$ means absolute value of a.

II. FUZZY MODEL OF DOUBLE INVERTED PENDULUMS

We consider the problem of balancing double inverted pendulums connected by a torsional spring as Figure. 1. This system is extract from [11]. The equations of motion of the pendulums are defined by:

$$\dot{x}_{11}(t) = x_{12}(t)$$

$$\dot{x}_{12}(t) = \frac{m_1 gr}{J_1} sin\big(x_{11}(t)\big) - \frac{k}{J_1} x_{11}(t) + \frac{u_1}{J_1} + \frac{k}{J_1} x_{21}(t) + \frac{v_1(t)}{J_1}$$

$$\dot{x}_{21}(t) = x_{22}(t)$$

$$\dot{x}_{22}(t) = \frac{m_2 gr}{J_2} sin\big(x_{21}(t)\big) - \frac{k}{J_2} x_{21}(t) + \frac{u_2}{J_2} + \frac{k}{J_2} x_{11}(t) + \frac{v_2(t)}{J_2} \quad (1)$$

Where$x_1(t) = [x_{11}(t)\,, x_{12}(t)]^T$, $x_2(t) = [x_{21}(t)\,, x_{22}(t)]^T$ are state vectors of subsystem1 and subsystem2. It is assumed that both θ_i *and* $\dot{\theta}_i$(angular position and rate)are available, where $\theta_1 = x_{11}(t)$and $\theta_2 = x_{21}(t)$ are the angular displacements of the pendulums from the vertical reference, the end masses of pendulum are $m_1 = 2\ kg$and $m_2 = 2.5\ kg$, the moment of inertia are $J_1 = 2\ kg$ and $J_2 = 2.5\ kg$, the constant of the connecting torsional spring is $k = 2\ N.m/rad$, the pendulum height is $r = 1\ m$, the gravitational acceleration is $g = 9.81\ m/s^2$, the torsional spring is relaxed when the pedulums are all in the upright position. Therefore the origin($x_{11} = x_{12} = x_{21} = x_{22} = 0$) is the equilibrium point of this system.

Two pendulums are linearized around the origin and $x_i = [\pm 88^{\circ}\,, 0]^T$ and then obtain the following fuzzy large-scale system model:

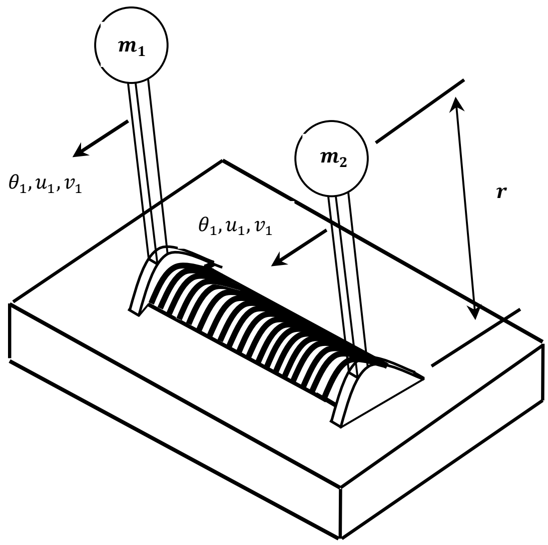

Fig. 1 Double Inverted Pendulums

Subsystem S_1:

$Rule1$: $if\ x_{11}(t)\ is\ about - 88^{\circ}$

$$then\ \ \dot{x}_1(t) = A_1^1 x_1(t) + \alpha_1^1 + B_1^1 u_1 + D_1^1 v_1(t) + C_{12} x_2(t)$$

$Rule2$: $if\ x_{11}(t)\ is\ about\ 0$

$$then\ \ \dot{x}_1(t) = A_1^2 x_1(t) + \alpha_1^2 + B_1^2 u_1 + D_1^2 v_1(t) + C_{12} x_2(t)$$

$Rule3$: $if\ x_{11}(t)\ is\ about + 88^{\circ}$

$$then\ \ \dot{x}_1(t) = A_1^3 x_1(t) + \alpha_1^3 + B_1^3 u_1 + D_1^3 v_1(t) + C_{12} x_2(t)$$

$Where$

$$A_1^1 = \begin{bmatrix} 0 & 1 \\ -0.6576 & 0 \end{bmatrix}, A_1^2 = \begin{bmatrix} 0 & 1 \\ 8.81000 & 0 \end{bmatrix}$$

$$A_1^3 = \begin{bmatrix} 0 & 1 \\ -0.6576 & 0 \end{bmatrix}$$

$$B_1^1 = B_1^2 = B_1^3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$$

$$C_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\alpha_1^1 = \begin{bmatrix} 0 \\ -9.2783 \end{bmatrix}, \alpha_1^2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \alpha_1^3 = \begin{bmatrix} 0 \\ 9.2783 \end{bmatrix}$$

$$D_1^1 = D_1^2 = D_1^3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$$

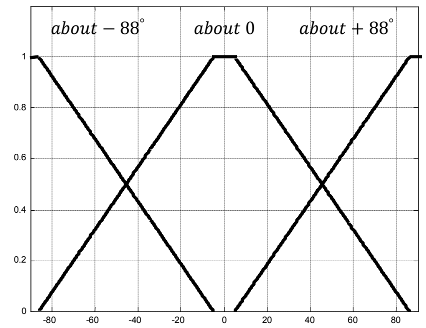

Fig. 2 Membership functions of first subsystem

- ***Subsystem S_2:***

$Rule1$: $if\ x_{21}(t)\ is\ about - 88^{\circ}$

$$then\ \ \dot{x}_2(t) = A_2^1 x_2(t) + \alpha_2^1 + B_2^1 u_2 + D_2^1 v_2(t) + C_{21} x_1(t)$$

$Rule2$: $if\ x_{21}(t)\ is\ about\ 0$

$$then\ \ \dot{x}_2(t) = A_2^2 x_1(t) + \alpha_2^2 + B_2^2 u_1 + D_2^2 v_2(t) + C_{21} x_1(t)$$

$Rule3$: $if\ x_{21}(t)\ is\ about + 88^{\circ}$

$$then\ \ \dot{x}_2(t) = A_2^3 x_2(t) + \alpha_2^3 + B_2^3 u_2 + D_2^3 v_2(t) + C_{21} x_1(t)$$

$Where$

$$A_2^1 = \begin{bmatrix} 0 & 1 \\ -0.4576 & 0 \end{bmatrix}, A_2^2 = \begin{bmatrix} 0 & 1 \\ 9.0100 & 0 \end{bmatrix}$$

$$A_2^3 = \begin{bmatrix} 0 & 1 \\ -0.4576 & 0 \end{bmatrix}$$

$$B_2^1 = B_2^2 = B_2^3 = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}$$

$$C_{21} = \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \end{bmatrix}$$

$$\alpha_2^1 = \begin{bmatrix} 0 \\ -9.2783 \end{bmatrix}, \alpha_2^2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \alpha_2^3 = \begin{bmatrix} 0 \\ 9.2783 \end{bmatrix}$$

$$D_2^1 = D_2^2 = D_2^3 = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}$$

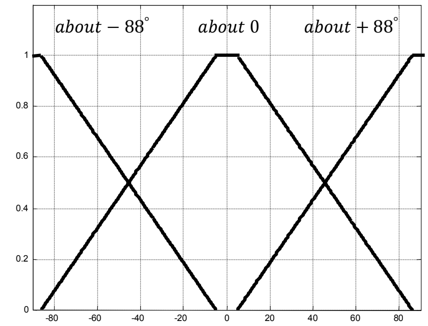

Fig. 3 Membership functions of second subsystem

III. Nonlinear Controller and Stability Analysis

In the first it is necessary to introduce following lemma.

Lemma1 [12]

a. For any real vectors x, y and a real matrix $P > 0$ of appropriate dimensions,

$$2x^T y \leq x^T P^{-1} x + y^T P y$$

b. Let A, D, E and $F(t)$ be real matrices of appropriate dimensions with $\|F(t)\| \leq 1$. Then for any scalar $\epsilon > 0$, the following inequality holds

$$DF(t)E + E^T F^T(t) D^T \leq \epsilon^{-1} D D^T + \epsilon E^T E$$

For stabilization of fuzzy model of double inverted pendulum as a fuzzy large-scale system, a nonlinear controller is considered as follow:

$$u_i(t) = -\sum_{k=1}^{2} m_i^k(t)\left(K_i^k x_i(t) + \gamma_i\right) \quad (i = 1,2) \tag{2}$$

$$subject\ to: \quad \sum_{k=1}^{2} m_i^k(t) = \sum_{l=1}^{3} \mu_i^l(x_i(t)) = 1 \ \ \& \ \ 0 \leq m_i^k(t) \leq 1 \tag{3}$$

Where K_i^k is state feedback gain with appropriate dimension and $m_i^k(t)$ is a nonlinear function that is defined as $(18.a)$ and $(18.b)$. After using above controller, the closed-loop fuzzy subsystem becomes

$$\dot{x}_i = \sum_{l=1}^{3} \mu_i^l(x_i(t)) \left(A_i^l x_i(t) - B_i^l \sum_{k=1}^{2} m_i^k(t)\left(K_i^k x_i(t) + \gamma_i\right) + C_{ij} x_j(t) + D_i^l v_i(t) + \alpha_i^l \right)$$

$$(i = 1,2 \ \ \& \ \ j = 1,2 \ \ \& \ \ for\ C_{ij} \ \ j \neq i) \tag{4}$$

Defining $Y_i^{lk} = A_i^l - B_i^l K_i^k$ yields

$$\dot{x}_i(t) = \sum_{l=1}^{3}\sum_{k=1}^{2} \left(\mu_i^l(x_i(t)) m_i^k(t)\left(Y_i^{lk} x_i(t) + D_i^l v_i(t) + \left(\alpha_i^l - B_i^l \gamma_i\right) + C_{ij} x_j(t)\right)\right) \tag{5}$$

Let the Lyapunov function candidate be

$$V(t) = \sum_{i=1}^{2} V_i(t) \quad such\ that \quad V_i(t) = x_i^T(t) P_i x_i(t) \tag{6}$$

$for\ i = 1$, taking the derivative of $V_1(t)$ then

$$\dot{V}_1(t) = \dot{x}_1^T(t) P_1 x_1(t) + x_1^T(t) P_1 \dot{x}_1(t) \tag{7}$$

Then

$$\begin{aligned}\dot{V}_1(t) &= \sum_{l=1}^{3}\sum_{k=1}^{2} \Big(\mu_1^l(x_1(t)) m_1^k(t) \Big(x_1^T(t) {Y_1^{lk}}^T P_1 x_1(t) \\ &+ {D_1^l}^T v_1(t) P_1 x_1(t) + (\alpha_1^l - B_1^l \gamma_1)^T P_1 x_1(t) \\ &+ x_2^T(t) C_{12}^T P_1 x_1(t) \Big)\Big) \\ &+ \sum_{l=1}^{3}\sum_{k=1}^{2} \Big(\mu_1^l(x_1(t)) m_1^k(t) (x_1^T(t) P_1 Y_1^{lk} x_1(t) \\ &+ x_1^T(t) P_1 v_1(t) D_1^l + x_1^T(t) P_1 (\alpha_1^l - B_1^l \gamma_1) \\ &+ x_1^T(t) P_1 C_{12} x_2(t)) \Big)\end{aligned} \tag{8}$$

Using ***lemma1.a*** yields:

$$\begin{aligned}&\left(\left(\alpha_1^l - B_1^l \gamma_1\right)^T P_1 x_1(t)\right) \\ &\qquad \leq \left(0.5 \left(\alpha_1^l - B_1^l \gamma_1\right)^T P^{-1} \left(\alpha_1^l - B_1^l \gamma_1\right) + x_1^T(t) P_1 P P_1 x_1(t)\right)\end{aligned} \tag{9}$$

Let $P = P_1^{-1}$ then

$$\begin{aligned}&\left(\left(\alpha_1^l - B_1^l \gamma_1\right)^T P_1 x_1(t)\right) \\ &\qquad \leq \left(0.5 \left(\left(\alpha_1^l - B_1^l \gamma_1\right)^T P_1 \left(\alpha_1^l - B_1^l \gamma_1\right) + x_1^T(t) P_1 x_1(t)\right)\right)\end{aligned} \tag{10}$$

Also, similar to above $(P = P_1)$

$$\begin{aligned}&\left(x_1^T(t) P_1 \left(\alpha_1^l - B_1^l \gamma_1\right)\right) \\ &\qquad \leq \left(0.5 \left(\left(\alpha_1^l - B_1^l \gamma_1\right)^T P_1 \left(\alpha_1^l - B_1^l \gamma_1\right) + x_1^T(t) P_1 x_1(t)\right)\right)\end{aligned} \tag{11}$$

And using of ***lemma1.b***:

(It is assumed $v_1(t) = \beta_1 f(t)$, where $\|f(t)\| \leq 1$)

$$\left(x_1^T(t)P_1(v_1(t)I)D_1^l + {D_1^l}^T(v_1(t)I)P_1x_1(t)\right) \leq \left(\tau_1^{-1}{\beta_1}^2 {D_{1l}}^T D_1^l + \tau_1 x_1^T(t){P_1}^2 x_1(t)\right) \quad (12)$$

So,

$$\dot{V}_1(t) \leq \sum_{l=1}^{3}\sum_{k=1}^{2}\left(\mu_1^l(x_1(t))m_1^k(t)\left(x_1^T(t)\left({Y_1^{lk}}^T P_1 + P_1 Y_1^{lk} + \tau_1{P_1}^2 + P_1\right)x_1(t) + x_2^T(t)C_{12}^T P_1 x_1(t) + x_1^T(t)P_1C_{12}x_2(t)\right)\right) + \sum_{l=1}^{3}\left(\mu_1^l(x_1(t))\left((\alpha_1^l - B_1^l\gamma_1)^T P_1(\alpha_1^l - B_1^l\gamma_1) + \tau_1^{-1}{\beta_1}^2 {D_1^l}^T D_1^l\right)\right) \quad (13)$$

It is assumed that

$$\left((\alpha_1^l - B_1^l\gamma_1)^T P_1(\alpha_1^l - B_1^l\gamma_1) + \tau_1^{-1}{\beta_1}^2 {D_1^l}^T D_1^l\right) < 0$$

Then with Schur's Complement and $P_1 = q_1^{-1}$

$$-(\alpha_1^l - B_1^l\gamma_1)^T(-q_1^{-1})(\alpha_1^l - B_1^l\gamma_1) + \tau_1^{-1}{\beta_1}^2 {D_1^l}^T D_1^l < 0 \quad (15)$$

And $if\ \tau_1 \geq 1\ then$

$$\left(-(\alpha_1^l - B_1^l\gamma_1)^T(-q_1^{-1})(\alpha_1^l - B_1^l\gamma_1) + \tau_1^{-1}{\beta_1}^2 {D_1^l}^T D_1^l\right) \leq \left(-(\alpha_1^l - B_1^l\gamma_1)^T(-q_1^{-1})(\alpha_1^l - B_1^l\gamma_1) + \tau_1{\beta_1}^2 {D_1^l}^T D_1^l\right) \quad (16)$$

Thus, if $\begin{bmatrix} -q_1 & (\alpha_1^l - B_1^l\gamma_1) \\ (\alpha_1^l - B_1^l\gamma_1)^T & \tau_1{\beta_1}^2 {D_1^l}^T D_1^l \end{bmatrix} < 0$, then the right-hand side of (16) is less than zero and yields;

$$\dot{V}_1(t) \leq \sum_{l=1}^{3}\sum_{k=1}^{2}\left(\mu_1^l(x_1(t))m_1^k(t)\left(x_1^T(t)\left({Y_1^{lk}}^T P_1 + P_1 Y_1^{lk} + \tau_1{P_1}^2 + P_1\right)x_1(t) + x_2^T(t)C_{12}^T P_1 x_1(t) + x_1^T(t)P_1C_{12}x_2(t)\right)\right) \quad (17)$$

Relation number#(18) is at the last page, this relation describe $m_i^k(t)$

Let $\begin{cases} {Y_1^{lk}}^T P_1 + P_1 Y_1^{lk} + \tau_1{P_1}^2 + P_1 = -Q_1^{lk} \\ x_2^T(t)C_{12}^T P_1 x_1(t) + x_1^T(t)P_1C_{12}x_2(t) = F_{12}(t) \end{cases}$

$$\dot{V}_1(t) \leq \left(m_1^1(t)\left(\sum_{k=1}^{2}\mu_1^l(x_1(t))(-x_1^T(t)Q_1^{l1}x_1(t) + F_{12}(t))\right) + m_1^2(t)\left(\sum_{k=1}^{2}\mu_1^l(x_1(t))(-x_1^T(t)Q_1^{l2}x_1(t) + F_{12}(t))\right)\right) \quad (19)$$

$$\Rightarrow \dot{V}_1(t) \leq -m_1^1(t)\left(\sum_{k=1}^{2}\mu_1^l(x_1(t))(x_1^T(t)Q_1^{l1}x_1(t) - F_{12}(t))\right) - \frac{(\sum_{l=1}^{3}\mu_1^l(x_1(t)^T Q_1^{l2}x_1(t) - F_{12}(t)))^2}{\sum_{h=1}^{2}\sum_{l=1}^{3}\left|\mu_1^l(x_1(t)^T Q_1^{l2}x_1(t) - F_{12}(t))\right|} \leq -m_1^1(t)\left(\sum_{k=1}^{2}\mu_1^l(x_1(t))(x_1^T(t)Q_1^{l1}x_1(t) - F_{12}(t))\right) \quad (20)$$

And similar to above procedure for $\dot{V}_2(t)$, following inequality is obtained

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \leq -m_1^1(t)\left(\sum_{k=1}^{2}\mu_1^l(x_1(t))(x_1^T(t)Q_1^{l1}x_1(t) - F_{12}(t))\right) - m_2^1(t)\left(\sum_{k=1}^{2}\mu_2^l(x_2(t))(x_2^T(t)Q_2^{l1}x_2(t) - F_{21}(t))\right) \quad (21)$$

$for\ \ each\ subsystem\ (S_i\ , i = 1{,}2)\ ,if$

$$-Q_i^{11} \leq -\eta_i^1$$
$$-Q_i^{21} \leq -\eta_i^2$$
$$-Q_i^{31} \leq -\eta_i^3 \quad (22)$$

$\left(\mu_i^l = \mu_i^l(x_i(t))\ , for\ abbreviation\right)$, then

$$-\mu_i^1 x_i(t)Q_i^{11}{x_i}^T(t) \leq -\mu_i^1\eta_i\|x_i\|^2$$

$$-\mu_i^2 x_i(t)Q_i^{21}{x_i}^T(t) \leq -\mu_i^2\eta_i\|x_i\|^2$$

$$-\mu_i^3 x_i(t) Q_i^{31} x_i{}^T(t) \le -\mu_i^3 \eta_i \|x_i\|^2 \tag{23}$$

$$\eta_i^l > 0, \eta_i = \min_l \eta_i^l, (i = 1,2 \;\; l = 1,2,3)$$

And Since

$$\sum_{l=1}^{3} \mu_i^l = 1$$

Then

$$-\sum_{l=1}^{3} \mu_i^l \left(x_i(t)^T Q_i^{l1} x_i(t)\right) \le -\eta_i \|x_i\|^2 \sum_{\substack{l=1 \\ i\neq j}}^{3} \mu_i^l = -\eta_i \|x_i\|^2 \tag{24}$$

Also,

$$\mu_i^l F_{ij}(t) = \mu_i^l \left(x_j^T(t) C_{ij}{}^T P_i x_i(t) + x_i(t)^T P_i C_{ij} x_j(t)\right) \le 2\|x_i\| \, \|x_j\| \|P_i\|_2 \|C_{ij}\| \tag{25}$$

Based on ***Lemma1***, (24) and(25), it is yielded

$$\dot V(t) \le \sum_{i=1}^{2}\left(-m_i^1(t)\left(\eta_i \|x_i\|^2 + \sum_{\substack{j=1 \\ i\neq j}}^{2} 2\|x_i\| \, \|x_j\| \|P_i\|_2 \|C_{ij}\|\right)\right)$$

$$= -[\|x_1\| \quad \|x_2\|] \times \begin{bmatrix} \eta_1 m_1^1(t) & -2m_1^1(t)\|P_1\|_2\|C_{12}\| \\ -2m_2^1(t)\|P_2\|_2\|C_{21}\| & \eta_1 m_1^1(t) \end{bmatrix} \times \begin{bmatrix} \|x_1\| \\ \|x_2\| \end{bmatrix} \tag{26}$$

From [13], $\|P_i\|_2 = \lambda_{max}(P_i) = {}^1/_{\lambda_{min}(q_i)}$,Thus

if

$$M = \begin{bmatrix} \eta_1 m_1^1(t) & -2m_1^1(t)\, {}^1/_{\lambda_{min}(q_1)} \|C_{12}\| \\ -2m_2^1(t)\, {}^1/_{\lambda_{min}(q_2)} \|C_{21}\| & \eta_1 m_1^1(t) \end{bmatrix} > 0 \tag{27.a}$$

and

$$\begin{bmatrix} -q_i & \left(\alpha_1^l - B_i^l \gamma_i\right) \\ \left(\alpha_i^l - B_i^l \gamma_1\right)^T & \tau_i {\beta_i}^2 {D_i^l}^T D_i^l \end{bmatrix} < 0 \quad (i = 1,2 \;\; \& \;\; l = 1,2,3) \tag{27.b}$$

Then $\dot V(t) < 0$ and then, the Fuzzy Inverted Double Pendulum is stable.

Remark1: For satisfying $M > 0$, J. J. Sylvester criterion [14] for positive definite is used then, this condition becomes independent of $m_i^1(t)$ $(i = 1,2)$ and conditions of η_i $(i = 1,2)$ are obtained easily as:

$$m_1^1(t)\eta_1 > 0 \quad \Rightarrow \quad \eta_1 > 0 \tag{28}$$

$$and$$

$$\begin{vmatrix} \eta_1 m_1^1(t) & -2m_1^1(t)\, {}^1/_{\lambda_{min}(q_1)} \|C_{12}\| \\ -2m_2^1(t)\, {}^1/_{\lambda_{min}(q_2)} \|C_{21}\| & \eta_1 m_1^1(t) \end{vmatrix} > 0 \tag{29}$$

$$\Rightarrow m_1^1(t)\, m_2^1(t) \begin{vmatrix} \eta_1 & -{}^2/_{\lambda_{min}(q_1)} \|C_{12}\| \\ -{}^2/_{\lambda_{min}(q_2)} \|C_{21}\| & \eta_1 \end{vmatrix} > 0$$

$$\Rightarrow \eta_1 \eta_2 > {}^2/_{\lambda_{min}(q_1)} \|C_{12}\| \; {}^2/_{\lambda_{min}(q_2)} \|C_{21}\| \tag{30}$$

IV. Results

Using LMI TOOLBOX OF MATLAB for solving $(27.a), (27.b)$ following results are obtained (Where

$$v_1(t) = sin(t), v_2(t) = cos\,(t))$$

Thus

$$K_1^1 = (1.0e + 004\,)[-0.0704 \quad 6.2805]$$

$$K_2^1 = (1.0e + 004\,)[0.0784 \quad 0.6293]$$

$$\gamma_1 = -12.3708$$

$$\gamma_2 = -12.3708$$

$$\tau_1 = 1.8400$$

$$\tau_2 = 2.5060$$

$$q_1 = \begin{bmatrix} 20.750 & 0.856 \\ 0.856 & 10019.17 \end{bmatrix}$$

$$q_2 = \begin{bmatrix} 70.01 & -2.500 \\ -2.500 & 100000 \end{bmatrix}$$

And $K_1^2$and K_2^2 are optional as:

$$K_1^2 = (1.0e + 004\,)[-1.5895 \quad -10]$$

$$K_2^2 = (1.0e + 004\,)[-1.5895 \quad -10]$$

Fig. 4, Fig. 5, Fig. 6 and Fig. 7 show responses of systems when $x_1(0) = [50, -35]^T$ and $x_2(0) = [65, -75]^T$

Remark2: Consider that horizontal axes in the figures are not "time" but they are "time-step" then, it is reasonable if they are large numbers (time-steps are steps in solving equations with numeric methods).

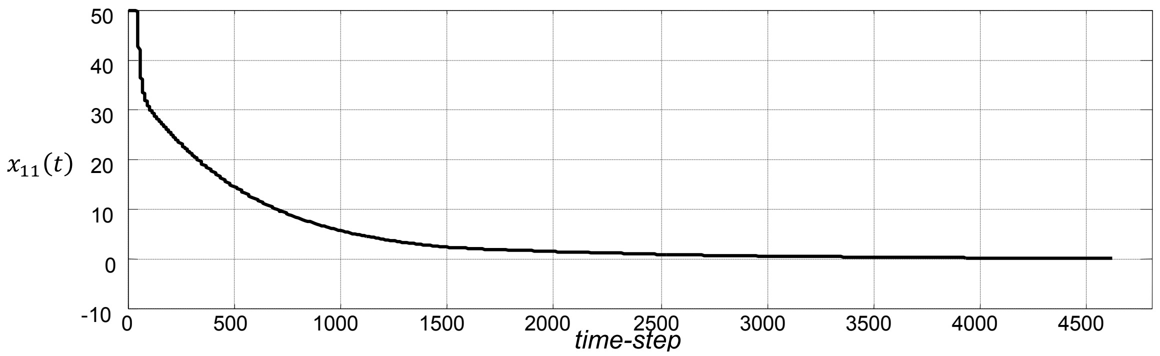

Fig. 4 **Response of $\mathbf{x_{11}(t)}$ when $\mathbf{x_{11}(0) = 50}$**

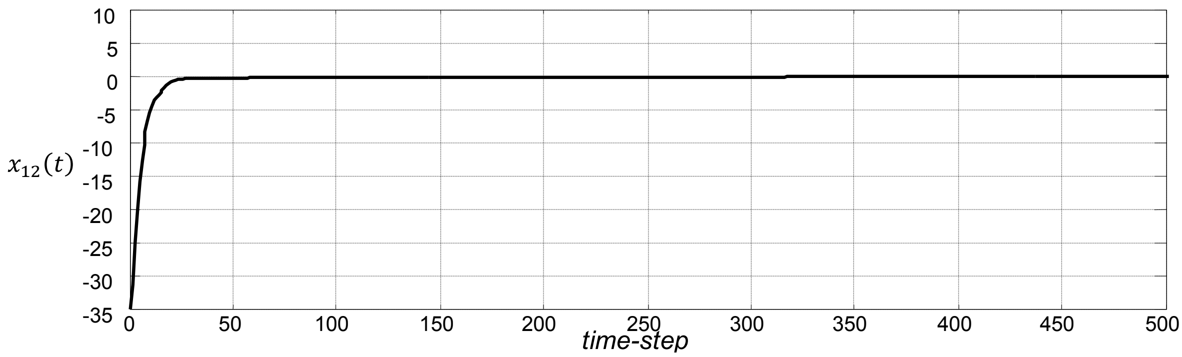

Fig. 5 **Response of $\mathbf{x_{12}(t)}$ when $\mathbf{x_{12}(0) = -35}$**

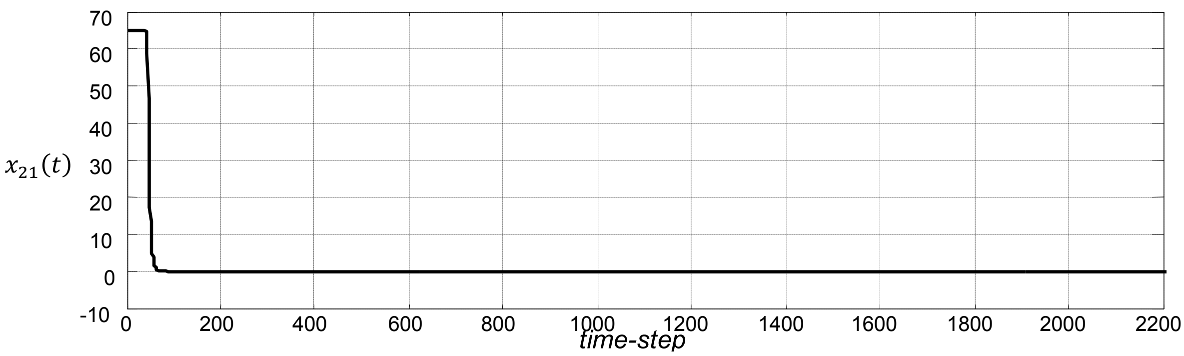

Fig. 6 **Response of $\mathbf{x_{21}(t)}$ when $\mathbf{x_{21}(0) = 65}$**

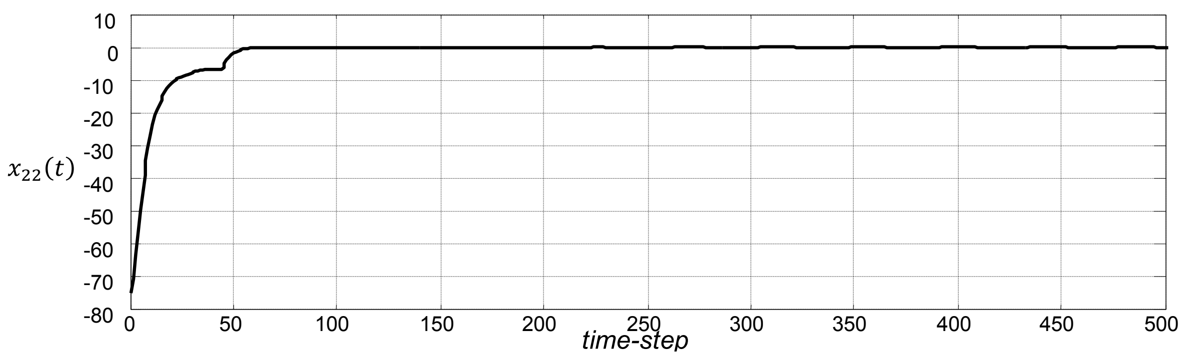

Fig. 7 **Response of $\mathbf{x_{22}(t)}$ when $\mathbf{x_{22}(0) = -75}$**

Definitions of $m_1^k(t)$

$$m_1^1(t) = \begin{cases} 1 - \dfrac{\sum_{l=1}^{3} \mu_1^l\left(x_1(t)^T Q_1^{l2} x_1(t) - F_{12}(t)\right)}{\sum_{h=1}^{2}\sum_{l=1}^{3}\left|\mu_1^l\left(x_1(t)^T Q_1^{lh} x_1(t) - F_{12}(t)\right)\right|} & , if \sum_{h=1}^{2}\sum_{l=1}^{3}\left|\mu_1^l\left(x_1(t)^T Q_1^{lh} x_1(t) - F_{12}(t)\right)\right| \neq 0 \\ \dfrac{1}{2} & if \quad \sum_{h=1}^{2}\sum_{l=1}^{3}\left|\mu_1^l\left(x_1(t)^T Q_1^{lh} x_1(t) - F_{12}(t)\right)\right| = 0 \end{cases}$$

$$(18.a)$$

$$m_1^2(t) = \begin{cases} \dfrac{\sum_{l=1}^{3} \mu_1^l\left(x_1(t)^T Q_1^{l2} x_1(t) - F_{12}(t)\right)}{\sum_{h=1}^{2}\sum_{l=1}^{3}\left|\mu_1^l\left(x_1(t)^T Q_1^{lh} x_1(t) - F_{12}(t)\right)\right|} & , if \sum_{h=1}^{2}\sum_{l=1}^{3}\left|\mu_1^l\left(x_1(t)^T Q_1^{lh} x_1(t) - F_{12}(t)\right)\right| \neq 0 \\ \dfrac{1}{2} & if \quad \sum_{h=1}^{2}\sum_{l=1}^{3}\left|\mu_1^l\left(x_1(t)^T Q_1^{lh} x_1(t) - F_{12}^l(t)\right)\right| = 0 \end{cases}$$

$$(18.b)$$

V. CONCLUSION

This paper introduced Fuzzy Double Inverted Pendulums, then presented a nonlinear controller for this system and analyzed stability and showed results. In other hand, this system is a fuzzy large-scale that includes deterministic offset terms and disturbances that satisfy a matching condition. This controller is simple in computation and guarantees stability.

However, it is mentioned that industrial systems are large-scale and stochastic and fuzzy model of Double Inverted Pendulums can be proper primary models for analyzing of industrial systems and military. Also, it is mentioned that we can use above idea for more complicated fuzzy large-scale system $(namely\ i > 2)$ easily, where J is the number of subsystems thus, significance of this method is obvious perfectly.

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