New delay-dependent stability conditions for neutral systems with nonlinear perturbations

Lianglin Xiong, Xiuyong Ding, and Shouming Zhong

Abstract—In this paper, the problem of asymptotical stability of neutral systems with nonlinear perturbations is investigated. Based on a class of novel augment Lyapunov functionals which contain free-weighting matrices, some new delay-dependent asymptotical stability criteria are formulated in terms of linear matrix inequalities (LMIs) by using new inequality analysis technique. Numerical examples are given to demonstrate the derived condition are much less conservative than those given in the literature.

Keywords—Asymptotical stability, neutral system, nonlinear perturbation, Delay-dependent, linear matrix inequality(LMI)

I. Introduction

It is well known that neutral systems are frequently encountered in various engineering systems, including population ecology, distributed networks containing lossless transmission lines, heat exchangers, and repetitive control [13], [9], [18]. There are many reports about stability conditions for neutral systems in the literature, such as [16], [7], [10], [11], [12], [17], [8], and the references therein. Currently efforts on the problem for stability of neutral systems can be divided into two categories, namely delay-dependent stability criteria and delay-independent stability criteria. Generally speaking, the delay-dependent stability conditions are less conservative than the delay-independent stability conditions for the neutral systems with small time delay.

In recent decades, the problem of robust stability of time-delay systems with nonlinear perturbations has also received considerable attention. To deal with the stability of systems with time varying delays and nonlinear perturbations, Cao and Lam proposed a model transformation technique [1]. By using a descriptor transformation method combined with a matrix decomposition approach, [4] presented the stability conditions for uncertain systems including time-varying delays, and both nonlinear perturbations and norm-bounded uncertainties are considered. The results in [4] were less conservative than those of [1],[4]. In order to reduce the conservatism, some free-weighting matrices (slack matrices) were introduced together with a descriptor transformation method [23]. Using the Lyapunov functional technique combined with matrix inequality technique, [14] presented a novel asymptotic stability criterion for neutral systems with nonlinear perturbations. [6] also studied the neutral systems with nonlinear parameter perturbations with a model transformation technique, by constructing Lyapunov-functionals. To reduce the conservatism, a new integral inequality which is particularly suitable for the analysis of the stability of the neutral systems was introduced in [21]. However, both the results of time-delay bounds in [6] and [21] are so small that can be improved with another novel approach, and some novel integrate inequalities which were introduced in [22] might also be considered into the stability of neutral systems with nonlinear perturbation, all of which motivates this paper.

In this paper, the delay-dependent asymptotic stability for uncertain neutral systems with nonlinear perturbations is studied. Owing to a class of novel augmented Lyapunov-Krasovskii functionals, improved delay-dependent asymptotical stability criteria for the neutral systems are derived by using the inequalities analysis technique and introducing some free weighting matrices. Note that these advantages are not obtained at the cost of high computational complexity. Finally, numerical examples are given to illustrate the superiority of present result to those in the literature.

II. Problem statement

Nomenclature

R^n n-dimensional real space

$R^{n\times n}$ set of all real n by n matrices

x^T or A^T transpose of vector x (or matrix A)

$P>0$ (respectively, $P<0$) matrix P is symmetric positive (respectively, negative) definite

$P\geq 0$ (respectively, $P\leq 0$) matrix P is symmetric positive (respectively, negative) semi-definite

$*$ the elements below the main diagonal of a symmetric block matrix.

Consider the following uncertain nonlinear with mixed time-varying delay system:

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-\tau_2) = Ax(t) + Bx(t-\tau_1(t)) \\ \qquad +f_1(t, x(t)) \\ \qquad +f_2(t, x(t-\tau_1(t))) \\ \qquad +f_3(t, \dot{x}(t-\tau_2)) \\ x(t_0+\theta) = \varphi(\theta), \forall\theta\in[-\rho, 0] \end{cases} \tag{1}$$

where $x(t)\in R^n$ is the state vector , the time-varying delays $h(t)$ and $\tau(t)$ satisfy

$$0\leq\tau_1(t)\leq\tau_1<\infty, \dot{\tau}_1(t)\leq\tau_{1d}, \rho=\max\{\tau_1,\tau_2\}.$$

$\varphi(\theta)$ is the initial condition function, $A\in R^{n\times n}$, $B\in R^{n\times n}, C\in R^{n\times n}$ are uncertain matrices, and the function

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$f_1(t,x(t)), f_2(t,x(t-\tau_1(t)))$ and $f_3(t,\dot{x}(t-\tau_2))$ represent the nonlinear time-varying perturbations. It is assumed that $f_i(t,0)=0(i=1,2,3)$,and

$$\|f_1(t,x(t))\| \le \beta_1 \|x(t)\| \quad (2a)$$
$$\|f_2(t,x(t-\tau_1(t)))\| \le \beta_2 \|x(t-\tau_1(t))\| \quad (2b)$$
$$\|f_3(t,\dot{x}(t-\tau_2))\| \le \beta_3 \|\dot{x}(t-\tau_2)\| \quad (2c)$$

where $\beta_1 \ge 0$, $\beta_2 \ge 0$ and $\beta_3 \ge 0$ are given constants.

Constraint (2) can be rewritten as following:

$$f_1^T(t,x(t)) f_1(t,x(t)) \le \beta_1^2 x^T(t) x(t) \quad (3a)$$
$$f_2^T(t,x(t-\tau_1(t))) f_2(t,x(t-\tau_1(t))) \le \beta_2^2 x^T(t-\tau_1(t)) x(t-\tau_1(t)) \quad (3b)$$
$$f_3^T(t,\dot{x}(t-\tau_2)) f_3(t,\dot{x}(t-\tau_2)) \le \beta_3^2 \dot{x}^T(t-\tau_2) \dot{x}(t-\tau_2) \quad (3c)$$

for the sake of simplicity, let $f_1 := f_1(t,x(t)), f_2 := f_2(t,x(t-\tau_1(t))), f_3 := f_3(t,\dot{x}(t-\tau_2))$.

Lemma 1: [22] For any constant symmetric matrix $Q \in R^{n\times n}$, $Q = Q^T > 0$, and any appropriate dimensional matrices, $M_1 \in R^n$, $M_2 \in R^n$, $Z = \begin{pmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{pmatrix} \in R^{2n\times 2n}$, $Y = \begin{bmatrix} M_1 & M_2 \end{bmatrix} \in R^{n\times 2n}$, if $\begin{pmatrix} Q & Y \\ * & Z \end{pmatrix} > 0$, $0 \le \tau(t) \le \tau < \infty$, such that the integrations in the following are well defined, then

$$-\int_{t-\tau}^{t} \dot{x}^T(s) Q \dot{x}(s) ds \le \xi(t)^T \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ * & \mathcal{M}_{22} \end{pmatrix} \xi(t)$$

where,

$$\mathcal{M}_{11} = M_1 + M_1^T + \tau Z_{11},$$
$$\mathcal{M}_{12} = -M_1^T + M_2 + \tau Z_{12},$$
$$\mathcal{M}_{22} = -M_2 - M_2^T + \tau Z_{22},$$
$$\xi(t) = col\begin{pmatrix} x(t) & x(t-\tau(t)) \end{pmatrix}.$$

Lemma 2: [3] For any constant symmetric matrix $M \in R^{n\times n}$, $M = M^T > 0$, scalar $r > 0$, vector function $g:[0,r] \to R^n$, such that the integrations in the following are well defined, then

$$r\int_0^r g^T(s) M g(s) ds \ge \left[\int_0^r g(s) ds\right]^T M \left[\int_0^r g(s) ds\right]$$

III. Main results

In general, the following assumption is satisfied as considering the stability of neutral systems.

A1. All the eigenvalues of matrix C are inside the unit circle.

For the asymptotically stability of systems described by (1), we have the following result.

Theorem 1: Under **A1**, the systems (1) is asymptotically stability, if there exist matrices $L = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{12}^T & L_{22} & L_{23} \\ L_{13}^T & L_{23}^T & L_{33} \end{pmatrix} \ge 0$ with $L_{11} > 0$, $R = \begin{pmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{pmatrix} \ge 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $N_{99} > 0$ and any appropriate dimensional matrices $N_{ij}(i,j=1,\cdots,9)$, $M_1 \in R^n$, $M_2 \in R^n$, $Y = \begin{bmatrix} M_1 & M_2 \end{bmatrix} \in R^{n\times 2n}$, $Z = \begin{pmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{pmatrix}$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, such that the following LMIs holds:

$$\begin{pmatrix} Q_3 & Y \\ * & Z \end{pmatrix} > 0 \quad (4)$$

$$N = \begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & \cdots & N_{19} \\ * & N_{22} & N_{23} & N_{24} & \cdots & N_{29} \\ * & * & N_{33} & N_{34} & \cdots & N_{39} \\ * & * & * & N_{44} & \cdots & N_{49} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & N_{99} \end{pmatrix} > 0 \quad (5)$$

$$\phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \cdots & \phi_{19} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \cdots & \phi_{29} \\ * & * & \phi_{33} & \phi_{34} & \cdots & \phi_{39} \\ * & * & * & \phi_{44} & \cdots & \phi_{49} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & \phi_{99} \end{pmatrix} > 0 \quad (6)$$

where

$$\phi_{11} = (L_{11}+R_{12})A + A^T(L_{11}+R_{12})^T + L_{13} + L_{13}^T + N_{19} + N_{19}^T + R_{11} + Q_1 + \tau_2 Q_2 + M_1 + M_1^T + \tau_1 Z_{11} + \varepsilon_1\beta_1^2 I + \tau_1 N_{11},$$
$$\phi_{12} = A^T L_{12} + N_{29}^T - L_{13} + L_{23}^T + \tau_1 N_{12},$$
$$\phi_{13} = (L_{11}+R_{12})C + L_{12} + N_{39}^T + \tau_1 N_{13},$$
$$\phi_{14} = (L_{11}+R_{12})B + N_{49}^T - M_1^T + M_2 + \tau_1 Z_{12} + \tau_1 N_{14},$$
$$\phi_{15} = L_{11} + R_{12} + N_{59}^T + \tau_1 N_{15},$$
$$\phi_{16} = L_{11} + R_{12} + N_{69}^T + \tau_1 N_{16},$$
$$\phi_{17} = L_{11} + R_{12} + N_{79}^T + \tau_1 N_{17},$$
$$\phi_{18} = \tau_2 A^T L_{13} + \tau_2 L_{33}^T + N_{89}^T + \tau_1 N_{18},$$
$$\phi_{19} = A^T(\tau_1 N_{99} + R_{22} + \tau_1 Q_3),$$
$$\phi_{22} = -L_{23} - L_{23}^T - R_{11} + \tau_1 N_{22},$$
$$\phi_{23} = L_{22} + L_{12}^T C - R_{12} + \tau_1 N_{23},$$
$$\phi_{24} = L_{12}^T B - N_{29}^T + \tau_1 N_{24},$$
$$\phi_{25} = L_{12}^T + \tau_1 N_{25},$$
$$\phi_{26} = L_{12}^T + \tau_1 N_{26},$$
$$\phi_{27} = L_{12}^T + \tau_1 N_{27},$$
$$\phi_{28} = -\tau_2 L_{33}^T + \tau_1 N_{28}, \quad \phi_{29} = 0$$
$$\phi_{33} = -R_{22} + \varepsilon_3\beta_3^2 I + \tau_1 N_{33},$$
$$\phi_{34} = -N_{39} + \tau_1 N_{34},$$
$$\phi_{3j} = \tau_1 N_{3j}\,(j=5,6,7),$$
$$\phi_{38} = \tau_2 C^T L_{13} + \tau_2 L_{23} + \tau_1 N_{38},$$
$$\phi_{39} = C^T(\tau_1 N_{99} + R_{22} + \tau_1 Q_3),$$
$$\phi_{44} = -(1-\tau_{1d})Q_1 - N_{49} - N_{49}^T - M_2 - M_2^T + \tau_1 Z_{22} + \varepsilon_2\beta_2^2 I + \tau_1 N_{44},$$

$$\begin{aligned}
\phi_{45} =& -N_{59}^T + \tau_1 N_{45},\\
\phi_{46} =& -N_{69}^T + \tau_1 N_{46},\\
\phi_{47} =& -N_{79}^T + \tau_1 N_{47},\\
\phi_{48} =& -N_{89}^T + \tau_2 B^T L_{13} + \tau_1 N_{48},\\
\phi_{49} =& B^T \left(\tau_1 N_{99} + R_{22} + \tau_1 Q_3\right),\\
\phi_{55} =& -\varepsilon_1 I + \tau_1 N_{55},\\
\phi_{56} =& \tau_1 N_{56}, \quad \phi_{57} = \tau_1 N_{57},\\
\phi_{58} =& \tau_2 L_{13} + \tau_1 N_{58},\\
\phi_{59} =& \tau_1 N_{99} + R_{22} + \tau_1 Q_3,\\
\phi_{66} =& -\varepsilon_2 I + \tau_1 N_{66},\\
\phi_{67} =& \tau_1 N_{67}, \quad \phi_{68} = \tau_2 L_{13} + \tau_1 N_{68},\\
\phi_{69} =& \tau_1 N_{99} + R_{22} + \tau_1 Q_3,\\
\phi_{77} =& -\varepsilon_3 I + \tau_1 N_{77},\\
\phi_{78} =& \tau_2 L_{13} + \tau_1 N_{78},\\
\phi_{79} =& \tau_1 N_{99} + R_{22} + \tau_1 Q_3,\\
\phi_{88} =& -\tau_2 Q_2 + \tau_1 N_{88}, \quad \phi_{99} = -\tau_1 N_{99} - R_{22} - \tau_1 Q_3.
\end{aligned}$$

Proof. Firstly, from (3), we obtain for any scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$.

$$\varepsilon_1 \left[\beta_1^2 x^T(t)\, x(t) - f_1^T(t, x(t))\, f_1(t, x(t))\right] \geq 0 \tag{7a}$$

$$\varepsilon_2 \left[\beta_2^2 x^T(t-\tau_1(t))\, x(t-\tau_1(t)) - f_2^T f_2\right] \geq 0 \tag{7b}$$

$$\varepsilon_3 \left[\beta_3^2 \dot{x}^T(t-\tau_2)\, \dot{x}(t-\tau_2) - f_3^T f_3\right] \geq 0 \tag{7c}$$

Choose a class of augmented Lyapunov-Krasovskii functional candidate for systems (1) as following:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t),$$

where,

$$V_1(t) = \begin{pmatrix} x(t) \\ x(t-\tau_2) \\ \int_{t-\tau_2}^t x(s)\, ds \end{pmatrix}^T L \begin{pmatrix} x(t) \\ x(t-\tau_2) \\ \int_{t-\tau_2}^t x(s)\, ds \end{pmatrix},$$

$$V_2(t) = \int_{t-\tau_2}^t \begin{pmatrix} x(s) \\ \dot{x}(s) \end{pmatrix}^T R \begin{pmatrix} x(s) \\ \dot{x}(s) \end{pmatrix} ds,$$

$$V_3(t) = \int_{t-\tau_1(t)}^t x^T(s) Q_1 x(s)\, ds,$$

$$V_4(t) = \int_{t-\tau_2}^t (\theta - t + \tau_2) x^T(\theta)\, Q_2 x(\theta)\, d\theta,$$

$$V_5(t) = \int_{t-\tau_1}^t (\theta - t + \tau_1) \dot{x}^T(\theta)\, Q_3 \dot{x}(\theta)\, d\theta,$$

$$V_6(t) = \int_{t-\tau_1}^t (\theta - t + \tau_1) \dot{x}^T(\theta)\, N_{99} \dot{x}(\theta)\, d\theta,$$

and L, Q_1, Q_2, Q_3, R and N_{99} are defined in theorem1.

The time derivative of $V(t)$ along the trajectory of system (1) is given by:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t) + \dot{V}_6(t),$$

where

$$\begin{aligned}
\dot{V}_1(t) &= 2 \begin{pmatrix} x(t) \\ x(t-\tau_2) \\ \int_{t-\tau_2}^t x(s)\, ds \end{pmatrix}^T L \begin{pmatrix} \dot{x}(t) \\ \dot{x}(t-\tau_2) \\ x(t) - x(t-\tau_2) \end{pmatrix}\\
&= 2 \begin{pmatrix} x(t) \\ x(t-\tau_2) \\ \int_{t-\tau_2}^t x(s)\, ds \end{pmatrix}^T L \begin{pmatrix} Ax(t) + Bx(t-\tau_1(t)) \\ +C\dot{x}(t-\tau_2) + f_1 \\ +f_2 + f_3 \\ \dot{x}(t-\tau_2) \\ x(t) - x(t-\tau_2) \end{pmatrix},
\end{aligned} \tag{8}$$

$$\begin{aligned}
\dot{V}_2(t) =& \begin{pmatrix} x(t) \\ \dot{x}(t) \end{pmatrix}^T \begin{pmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{pmatrix} \begin{pmatrix} x(t) \\ \dot{x}(t) \end{pmatrix}\\
& - \begin{pmatrix} x(t-\tau_2) \\ \dot{x}(t-\tau_2) \end{pmatrix}^T \begin{pmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{pmatrix}\\
& \times \begin{pmatrix} x(t-\tau_2) \\ \dot{x}(t-\tau_2) \end{pmatrix}\\
=& x^T(t) R_{11} x(t) + \dot{x}^T(t) R_{22} \dot{x}(t)\\
& - \begin{pmatrix} x(t-\tau_2) \\ \dot{x}(t-\tau_2) \end{pmatrix}^T \begin{pmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{pmatrix}\\
& \times \begin{pmatrix} x(t-\tau_2) \\ \dot{x}(t-\tau_2) \end{pmatrix}\\
& + 2x^T(t) R_{12}[Ax(t) + Bx(t-\tau_1(t))\\
& + C\dot{x}(t-\tau_2) + f_1 + f_2 + f_3],
\end{aligned} \tag{9}$$

$$\begin{aligned}
\dot{V}_3(t) =& x^T(t) Q_1 x(t) - (1 - \dot{\tau}_1(t))\, x^T(t-\tau_1(t))\\
& \times Q_1 x(t-\tau_1(t))\\
\leq & x^T(t) Q_1 x(t) - (1 - \tau_{1d})\, x^T(t-\tau_1(t))\\
& \times Q_1 x(t-\tau_1(t)),
\end{aligned} \tag{10}$$

It's from the Lemma1 and Lemma2 that we have

$$\begin{aligned}
\dot{V}_4(t) &= x^T(t) \tau_2 Q_2 x(t) - \int_{t-\tau_2}^t x^T(s) Q_2 x(s)\, ds\\
&\leq x^T(t) \tau_2 Q_2 x(t) - \left(\frac{1}{\tau_2}\int_{t-\tau_2}^t x(s) ds\right)^T\\
&\quad \times \tau_2 Q_2 \left(\frac{1}{\tau_2}\int_{t-\tau_2}^t x(s) ds\right),
\end{aligned} \tag{11}$$

$$\begin{aligned}
\dot{V}_5(t) &= \dot{x}^T(t) \tau_1 Q_3 \dot{x}(t) - \int_{t-\tau_1}^t \dot{x}^T(s) Q_3 \dot{x}(s)\, ds\\
&\leq \dot{x}^T(t) \tau_1 Q_3 \dot{x}(t)\\
&\quad + \xi(t)^T \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ * & \mathcal{M}_{22} \end{pmatrix} \xi(t),
\end{aligned} \tag{12}$$

with

$$\begin{aligned}
&\mathcal{M}_{11} = M_1 + M_1^T + \tau Z_{11},\\
&\mathcal{M}_{12} = -M_1^T + M_2 + \tau Z_{12},\\
&\mathcal{M}_{22} = -M_2 - M_2^T + \tau Z_{22},\\
&\xi(t) = col\begin{pmatrix} x(t) & x(t-\tau(t)) \end{pmatrix}.
\end{aligned}$$

$$\dot{V}_6(t) = \dot{x}^T(t) \tau_1 N_{99} \dot{x}(t) - \int_{t-\tau_1}^t \dot{x}^T(s) N_{99} \dot{x}(s)\, ds$$

$$\leq \dot{x}^T(t) \tau_1 N_{99} \dot{x}(t) - \int_{t-\tau_1(t)}^t \dot{x}^T(s) N_{99} \dot{x}(s)\, ds. \tag{13}$$

From the Leibniz-Newton formula, the following equation is true for any appropriate dimensional matrices $N_{i9}\,(i=1,\cdots,8)$

$$\begin{aligned}&2\left\{x^T(t)N_{19}+x^T(t-\tau_2)N_{29}+\dot{x}^T(t-\tau_2)N_{39}\right.\\&+x^T(t-\tau_1(t))N_{49}+f_1^TN_{59}\\&\left.+f_2^TN_{69}+f_3^TN_{79}+\left(\frac{1}{\tau_2}\int_{t-\tau_2}^{t}x(s)ds\right)^TN_{89}\right\}\\&\times\left\{x(t)-x(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t}\dot{x}(s)ds\right\}=0\end{aligned} \quad (14)$$

And consider the fact that, for any $m>0$ and any function $f(t)$,

$$mf(t)-\int_{t-m}^{t}f(t)ds=0,$$

the following inequality is also true for any appropriate dimensional matrices $N_{ij}\;(i,j=1,\cdots,8)$

$$\begin{aligned}&\tau_1\xi^T(t)\begin{pmatrix}N_{11}&N_{12}&N_{13}&\cdots&N_{18}\\ *&N_{22}&N_{23}&\cdots&N_{28}\\ *&*&N_{33}&\cdots&N_{38}\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ *&*&*&*&N_{88}\end{pmatrix}\xi(t)\\&-\int_{t-\tau_1(t)}^{t}\xi^T(t)\begin{pmatrix}N_{11}&N_{12}&N_{13}&\cdots&N_{18}\\ *&N_{22}&N_{23}&\cdots&N_{28}\\ *&*&N_{33}&\cdots&N_{38}\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ *&*&*&*&N_{88}\end{pmatrix}\\&\times\xi(t)\,ds\geq 0\end{aligned} \quad (15)$$

where

$$\begin{aligned}\xi^T(t)=\big[&\;x^T(t)\quad x^T(t-\tau_2)\quad \dot{x}^T(t-\tau_2)\\&x^T(t-\tau_1(t))\quad f_1^T\quad f_2^T\quad f_3^T\quad \left(\frac{1}{\tau_2}\int_{t-\tau_2}^{t}x(s)ds\right)^T\;\big].\end{aligned}$$

Choosing $M=\tau_1N_{99}+R_{22}+\tau_1Q_3$, use systems (1) to obtain

$$\begin{aligned}&\dot{x}^T(t)(\tau_1N_{99}+R_{22}+\tau_1Q_3)\dot{x}(t)=\\&[Ax(t)+Bx(t-\tau_1(t))+C\dot{x}(t-\tau_2)+f_1+f_2+f_3]^T\\&\times M[Ax(t)+Bx(t-\tau_1(t))+C\dot{x}(t-\tau_2)+f_1+f_2+f_3]\end{aligned} \quad (16)$$

Then, we add the terms on the left sides of (14) and (15) to $\dot{V}(t)$, and use the Schur's complement in [15] on the term of (16), we obtain

$$\dot{V}(t)\leq\xi^T(t)\varphi\xi(t)-\int_{t-\tau_1(t)}^{t}\zeta^T(t,s)N\zeta(t,s)ds,$$

where

$$\begin{aligned}\zeta^T(t,s)=\big[&\;x^T(t)\quad x^T(t-\tau_2)\quad \dot{x}^T(t-\tau_2)\\&x^T(t-\tau_1(t))\quad f_1^T\quad f_2^T\quad f_3^T\\&\left(\frac{1}{\tau_2}\int_{t-\tau_2}^{t}x(s)ds\right)^T\quad \dot{x}^T(s)\;\big],\end{aligned}$$

and most elements of φ are the same as the elements of ϕ, except that the following:

$$\begin{aligned}\varphi_{11}=&(L_{11}+R_{12})A+A^T(L_{11}+R_{12})^T+L_{13}+L_{13}^T\\&+N_{19}+N_{19}^T+R_{11}+Q_1+\tau_2Q_2+M_1+M_1^T\\&+\tau_1Z_{11}+\tau_1N_{11},\\ \varphi_{33}=&-R_{22}+\tau_1N_{33},\\ \varphi_{44}=&-(1-\tau_{1d})Q_1-M_2-M_2^T+\tau_1Z_{22}\\&-N_{49}-N_{49}^T+\tau_1N_{44},\\ \varphi_{55}=&\tau_1N_{55},\qquad \varphi_{66}=\tau_1N_{66},\qquad \varphi_{77}=\tau_1N_{77}\end{aligned}$$

By the theorem 9.8.1 in [13], the system (1) with **A1** is asymptotically stable if there exist $L>0$, $R\geq 0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $N_{99}>0$ and $N>0$ which were defined in Theorem1 such that:

$$\dot{V}(t)\leq\xi^T(t)\varphi\xi(t)-\int_{t-\tau_1(t)}^{t}\zeta^T(t,s)N\zeta(t,s)ds<0 \quad (17)$$

for all $\xi(t)\neq 0,\zeta(t,s)\neq 0$ satisfying (3). Using the S-procedure [19], we see that this condition is implied by (6) such that:

$$\begin{aligned}&\xi^T(t)\varphi\xi(t)-\int_{t-\tau_1(t)}^{t}\zeta^T(t,s)N\zeta(t,s)ds\\&+\varepsilon_1\left[\beta_1^2x^T(t)x(t)-f_1^T(t,x(t))f_1(t,x(t))\right]\\&+\varepsilon_2\left[\beta_2^2x^T(t-\tau_1(t))x(t-\tau_1(t))-f_2^Tf_2\right]\\&+\varepsilon_3\left[\beta_3^2\dot{x}^T(t-\tau_2)\dot{x}(t-\tau_2)-f_3^Tf_3\right]\\&<0\end{aligned}$$

for all $\xi(t)\neq 0,\zeta(t,s)\neq 0$. Therefore, there exist $L>0$, $R\geq 0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $N_{99}>0$ and $N>0$ which were defined in Theorem1, and scalars $\varepsilon_1>0$, $\varepsilon_2>0$, $\varepsilon_3>0$, such that the LMIs (4), (5) and (6) are satisfied, then systems (1), with uncertainty (2), is asymptotically stability. This completes the proof.

Remark 1: Many existing delay-derivative-dependent stability criteria of system with severely time-varying delay generally require a constraint $\tau_{1d}<1$. In this paper, we omit this assumption and obtained a less conservative stability condition. As a matter of fact, the chosen Lyapunov-Krasovskii functional in this theorem is the same as our latest article [20], however, in the process of the derivative of the functional, the lemma 1 is very important to our less conservative results, which will be shown subsequently in the examples.

If we set $\beta_3=0$, similar to the proof of Theorem 1, we can obtain the following Corollary.

Corollary 1: Under **A1**, the systems (1) is asymptotically stability, if there exist matrices $L=\begin{pmatrix}L_{11}&L_{12}&L_{13}\\L_{12}^T&L_{22}&L_{23}\\L_{13}^T&L_{23}^T&L_{33}\end{pmatrix}\geq 0$ with $L_{11}>0$, $R=\begin{pmatrix}R_{11}&R_{12}\\R_{12}^T&R_{22}\end{pmatrix}\geq 0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $N_{88}>0$ and any appropriate dimensional matrices $N_{ij}\,(i,j=1,\cdots,8)$, $M_1\in R^n$, $M_2\in R^n$, $Y=\begin{bmatrix}M_1&M_2\end{bmatrix}\in R^{n\times 2n}$, $Z=\begin{pmatrix}Z_{11}&Z_{12}\\ *&Z_{22}\end{pmatrix}$ and scalars $\varepsilon_1>0$, $\varepsilon_2>0$, $\varepsilon_3>0$, such that the following LMIs holds:

$$\begin{pmatrix}Q_3&Y\\ *&Z\end{pmatrix}>0 \quad (18)$$

$$N=\begin{pmatrix} N_{11} & N_{12} & N_{13} & \cdots & N_{18} \\ * & N_{22} & N_{23} & \cdots & N_{28} \\ * & * & N_{33} & \cdots & N_{38} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & N_{88} \end{pmatrix} > 0 \qquad (19)$$

$$\phi=\begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \cdots & \phi_{18} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \cdots & \phi_{28} \\ * & * & \phi_{33} & \phi_{34} & \cdots & \phi_{38} \\ * & * & * & \phi_{44} & \cdots & \phi_{48} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & \phi_{88} \end{pmatrix} > 0 \quad (20)$$

where

$$\begin{aligned}
\phi_{11} =& \left(L_{11}+R_{12}\right)A + A^T\left(L_{11}+R_{12}\right)^T + L_{13} + L_{13}^T \\
& + N_{18} + N_{18}^T + R_{11} + Q_1 + \tau_2 Q_2 + M_1 + M_1^T \\
& + \tau_1 Z_{11} + \varepsilon_1 \beta_1^2 I + \tau_1 N_{11}, \\
\phi_{12} =& A^T L_{12} + N_{28}^T - L_{13} + L_{23}^T + \tau_1 N_{12}, \\
\phi_{13} =& \left(L_{11}+R_{12}\right)C + L_{12} + N_{38}^T + \tau_1 N_{13}, \\
\phi_{14} =& \left(L_{11}+R_{12}\right)B + N_{48}^T + \tau_1 N_{14} - M_1^T + M_2 + \tau_1 Z_{12}, \\
\phi_{15} =& L_{11} + R_{12} + N_{58}^T + \tau_1 N_{15}, \\
\phi_{16} =& L_{11} + R_{12} + N_{68}^T + \tau_1 N_{16}, \\
\phi_{17} =& \tau_2 A^T L_{13} + \tau_2 L_{33}^T + N_{78}^T + \tau_1 N_{18}, \\
\phi_{18} =& A^T\left(\tau_1 N_{88} + R_{22} + \tau_1 Q_3\right), \\
\phi_{22} =& -L_{23} - L_{23}^T - R_{11} + \tau_1 N_{22}, \\
\phi_{23} =& L_{22} + L_{12}^T C - R_{12} + \tau_1 N_{23}, \\
\phi_{24} =& L_{12}^T B - N_{28}^T + \tau_1 N_{24}, \\
\phi_{25} =& L_{12}^T + \tau_1 N_{25}, \\
\phi_{26} =& L_{12}^T + \tau_1 N_{26}, \\
\phi_{27} =& -\tau_2 L_{33}^T + \tau_1 N_{27}, \quad \phi_{28} = 0 \\
\phi_{33} =& -R_{22} + \varepsilon_3 \beta_3^2 I + \tau_1 N_{33}, \\
\phi_{34} =& -N_{38} + \tau_1 N_{34}, \quad \phi_{3j} = \phi_{3j} = \tau_1 N_{3j}\left(j=5,6\right), \\
\phi_{37} =& \tau_2 C^T L_{13} + \tau_2 L_{23} + \tau_1 N_{37}, \\
\phi_{38} =& C^T\left(\tau_1 N_{88} + R_{22} + \tau_1 Q_3\right), \\
\phi_{44} =& -\left(1-\tau_{1d}\right)Q_1 - N_{48} - N_{48}^T - M_2 - M_2^T \\
& + \tau_1 Z_{22} + \varepsilon_2 \beta_2^2 I + \tau_1 N_{44}, \\
\phi_{45} =& -N_{58}^T + \tau_1 N_{45}, \\
\phi_{46} =& -N_{68}^T + \tau_1 N_{46}, \\
\phi_{47} =& -N_{78}^T + \tau_2 B^T L_{13} + \tau_1 N_{47}, \\
\phi_{48} =& B^T\left(\tau_1 N_{88} + R_{22} + \tau_1 Q_3\right), \\
\phi_{55} =& -\varepsilon_1 I + \tau_1 N_{55}, \qquad \phi_{56} = \tau_1 N_{56}, \\
\phi_{57} =& \tau_2 L_{13} + \tau_1 N_{57}, \\
\phi_{58} =& \tau_1 N_{88} + R_{22} + \tau_1 Q_3, \\
\phi_{66} =& -\varepsilon_2 I + \tau_1 N_{66}, \\
\phi_{67} =& \tau_2 L_{13} + \tau_1 N_{67}, \\
\phi_{68} =& \tau_1 N_{88} + R_{22} + \tau_1 Q_3, \\
\phi_{77} =& -\tau_2 Q_2 + \tau_1 N_{77}, \\
\phi_{88} =& -\tau_1 N_{88} - R_{22} - \tau_1 Q_3.
\end{aligned}$$

If $C \equiv 0$ and$f_3\left(t, \dot{x}\left(t-\tau_2\right)\right) \equiv 0$, then system (1) reduces to the following system:

$$\begin{cases} \dot{x}\left(t\right) = Ax\left(t\right) + Bx\left(t-\tau_1\left(t\right)\right) \\ \qquad + f_1\left(t, x\left(t\right)\right) + f_2\left(t, x\left(t-\tau_1\left(t\right)\right)\right) \\ x\left(t_0+\theta\right) = \varphi\left(\theta\right), \forall\theta \in \left[-\tau_{1m}, 0\right] \end{cases} \qquad (21)$$

According to Theorem1, we have the following corollary for the delay-dependent stability of system (22).

Corollary 2: Under **A1**, the systems (1) is asymptotically stability, if there exist matrices $L > 0$, $Q_1 > 0$, $Q_2 > 0$, $N_{55} > 0$ and any appropriate dimensional matrices $N_{ij}\left(i,j=1,\cdots,5\right)$, $M_1 \in R^n$, $M_2 \in R^n$, $Y = \left[\begin{array}{cc} M_1 & M_2 \end{array}\right] \in R^{n\times 2n}$, $Z = \begin{pmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{pmatrix}$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that the following LMIs holds:

$$\begin{pmatrix} Q_2 & Y \\ * & Z \end{pmatrix} > 0 \qquad (22)$$

$$N=\begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} \\ * & N_{22} & N_{23} & N_{24} & N_{25} \\ * & * & N_{33} & N_{34} & N_{35} \\ * & * & * & N_{44} & N_{45} \\ * & * & * & * & N_{55} \end{pmatrix} > 0 \qquad (23)$$

$$\phi=\begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & A^T S \\ * & \phi_{22} & \phi_{23} & \phi_{24} & B^T S \\ * & * & \phi_{33} & \phi_{34} & S \\ * & * & * & \phi_{44} & S \\ * & * & * & * & -S \end{pmatrix} > 0, \qquad (24)$$

where,

$$\begin{aligned}
\phi_{11} =& LA + A^T L^T + N_{15} + N_{15}^T + Q_1 + M_1 + M_1^T \\
& + \tau_1 Z_{11} + \varepsilon_1 \beta_1^2 I + \tau_1 N_{11}, \\
\phi_{12} =& LB + N_{25}^T - N_{15} + \tau_1 N_{12} - M_1^T + M_2 + \tau_1 Z_{12}, \\
\phi_{13} =& L + N_{35}^T + \tau_1 N_{13}, \\
\phi_{14} =& L + N_{45}^T + \tau_1 N_{14}, \\
\phi_{22} =& -\left(1-\tau_{1d}\right)Q_1 - N_{25} - N_{25}^T - M_2 - M_2^T \\
& + \tau_1 Z_{22} + \varepsilon_2 \beta_2^2 I + \tau_1 N_{22}. \\
\phi_{23} =& -N_{35}^T + \tau_1 N_{23}, \\
\phi_{24} =& -N_{45}^T + \tau_1 N_{24}, \\
\phi_{33} =& -\varepsilon_1 I + \tau_1 N_{33}, \\
\phi_{34} =& \tau_1 N_{34}, \phi_{44} = -\varepsilon_2 I + \tau_1 N_{44}, \\
S =& \tau_1 N_{55} + \tau_1 Q_2.
\end{aligned}$$

Remark 2: Theorem1, Collary1 and Collary2 are novel delay-dependent asymptotically stability conditions for nonlinear systems (1) with different cases. And the results are both delay-dependent and delay-derivative-dependent. They are expected to be less conservative than some results in the literature, because we make good use of the integrate inequalities technique and free-weighting matrix which can be selected by solving the LMIs in Theorem 1, Corollary 1 and Corollary 2. In contrast, previous methods employed fixed weighting matrices, which are not usually the optimal ones and may bring some conservatism. The comparisons of their conservatism with some existing methods will be presented in Section 4.

IV. Numerical examples

In order to show the effectiveness of the approaches presented in Section 3, in this section, two numerical examples are provided.

Example1. Consider the neutral systems (1) which was considered in [21] with

$$A = \begin{pmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{pmatrix},$$

$$C = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix},$$

$$\|f_1(t, x(t))\| \le \alpha_1 \|x(t)\|,$$

$$\|f_2(t, x(t - \tau_1(t)))\| \le \alpha_2 \|x(t - \tau_1(t))\|,$$

$$\|f_3(t, \dot{x}(t - \tau_2))\| \le \alpha_3 \|\dot{x}(t - \tau_2)\|,$$

where $\alpha_1 \ge 0$,$\alpha_2 \ge 0$,$\alpha_3 \ge 0$and $0 \le c < 1$.

We now also consider the effect of the bound α_3 on the maximal allowable value τ_{1m}.For $c = 0.1$, $\tau_2 = 1$, $\tau_{1d} = 0.5$, $\alpha_2 = 0.1$, and different values of α_3, we apply theorem1 and Corollary to calculate the maximal allowable value τ_{1m} that guarantees the asymptotical stability of the system.

Table I gives the comparison of our results with those in [6] and [21]. We can see from Table 1 that the upper bound of α_3 has a remarkable effect on τ_{1m}, τ_{1m} decreases as α_3 increases. In conclusion, the results obtained in this paper are less conservative than that presented in [6] and [21].

For $c = 0$ and $f_3(t, \dot{x}(t - \tau_2)) \equiv 0$, the system under consideration reduces to the system studied in [1].Applying criteria in [1], [4], [6] and in this work, the maximum value of τ_{1m} for the stability of the system is listed in Table II. It is easy to see that our proposed stability criterion gives a much less conservative result than one in [1], [4] and [6].

One should be noted that, on the one hand, from the comparison in table I, our results is inferior to our latest results in [20], however, it is also less conservative than those conditions in [6] and [21]. On the other hand, the less conservativeness is also shown in the table II.

TABLE I
THE MAXIMAL ALLOWABLE DELAYS τ_{1m} OF EXAMPLE1 FOR DIFFERENT VALUES OF α_3.

α_3	0	0.1	0.2	0.3
[6](α_1=0)	0.9328	0.7402	0.5637	0.4042
[21](α_1=0)	0.9488	0.7695	0.6087	0.4667
This paper(α_1=0)	1.5603	1.3460	0.9686	0.7326
[6](α_1=0.1)	0.8418	0.6439	0.4864	0.3433
[21](α_1=0.1)	0.8408	0.6841	0.5420	0.4144
This paper(α_1=0.1)	1.4531	1.2805	0.9466	0.7303

Example2. Consider the neutral system

$$\frac{d}{dt}[x(t) - Cx(t - \tau)] = Ax(t) + Bx(t - \tau) + f_1(t, x(t)) + f_2(t, x(t - \tau)) \quad (25)$$

$$A = \begin{pmatrix} -2 & 0.5 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0.4 \\ 0.4 & -1 \end{pmatrix},$$

$$C = \begin{pmatrix} 0.2 & 1 \\ 0 & 0.2 \end{pmatrix},$$

TABLE II
THE MAXIMAL ALLOWABLE DELAYS τ_{1m} OF EXAMPLE1 FOR DIFFERENT VALUES OF α_3.

	$\alpha_1 = 0, \alpha_2 = 0.1$		$\alpha_1 = 0.1, \alpha_2 = 0.1$	
	τ_{1d}=0	τ_{1d}=0.5	τ_{1d}=0	τ_{1d}=0.5
[1]	0.6811	0.5467	0.6129	0.4950
[4]	1.3279	0.6743	1.2503	0.5716
[6]	2.7424	1.1365	1.8753	0.9953
This work	1.7565×10^5	7.3488	1.2404×10^5	5.8188

with $\|f_1(t, x(t))\| \le \alpha_1 \|x(t)\|$, $\|f_2(t, x(t - \tau))\| \le \alpha_2 \|x(t - \tau)\|$ where $\alpha_1 = 0.2$, $\alpha_2 = 0.1$.

This system was studied in [14], where it is found that the admissible bound of the time delay τ for the stability of systems (25) is 0.583. Applying the criteria in this paper, the upper bound of the delay τ has been obtained as 1.9391. This also shows that the criterion given in this paper is much less conservative than that in [14].

V. Conclusion

The asymptotical stability for uncertain neutral systems with nonlinear perturbations has been investigated. Based on a new class of Lyapunov-Krasovskii functionals, and combined with the use of novel integrate inequalities and the Leibniz-Newton formula, some novel stability criteria have been obtained. Numerical examples have shown significant improvements over some existing results.

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