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Ultra-Precise Hybrid Lens Distortion Correction

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***Abstract*—**A new hybrid method to realise high-precision distortion determination for optical ultra-precision 3D measurement systems based on stereo cameras using active light projection is introduced. It consists of two phases: the basic distortion determination and the refinement. The refinement phase of the procedure uses a plane surface and projected fringe patterns as calibration tools to determine simultaneously the distortion of both cameras within an iterative procedure. The new technique may be performed in the state of the device "ready for measurement" which avoids errors by a later adjustment. A considerable reduction of distortion errors is achieved and leads to considerable improvements of the accuracy of 3D measurements, especially in the precise measurement of smooth surfaces.

***Keywords*—**3D Surface Measurement, Fringe Projection, Lens Distortion, Stereo.

I. Introduction

OPTICAL high precision measurement systems based on active light projection and image observation using stereo camera pairs require high precision optical components including lenses showing a minimum of distortion effects. However, there are no really distortion free cameras. Especially compact systems which realize short distances between optical system and measurement object and relative large measurement volumes are reliant on the use of wide angle lenses usually suffering considerably on lens distortion. Additionally, these systems also show a distance dependent variation of the distortion.

This fact has already been investigated by Magill [1] more than 50 years ago. He developed a formula for the description of the distance dependence of distortion which was refined by Brown [2], Fryer and Brown [3], Fraser and Shortis [4], and Shortis et al. [5]. Dold [6] introduces an own model for distance dependent distortion variation and Luhmann [7] gives an extensive survey over distance depending distortion variation.

In order to realize high precision measurements distortion variation must not be neglected. However, the accuracy of the distortion determination for constant measurement distance has to be as precise as the consideration of distance dependence becomes worth anyway.

Here, recent works dealing with lens distortion determination and correction usually do not consider this fact, and, are appropriate not as precise as necessary for high precision measurement applications. Recently an enormous number of works treating the problem of lens distortion and correction has been published. Accordingly it is not possible to cite all relevant papers, but, nevertheless, some contributing and original papers should be cited here [8 to 13]. A step towards increasing measurement accuracy is gone by the work of Grompone et al. [14]. However, the achieved accuracy documented in [14] seems to be not sufficient for high-precision 3D measurement tasks as the comparison with our experimental results will show.

In this work a brief overview over existing distortion models together with an evaluating discussion is given. The importance of accuracy estimation and measurement error determination is outlined. A new hybrid methodology for ultra-precise distortion determination is introduced which is especially designed for optical 3D measurement systems based on active light production technique.

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II. State of the Art

As already mentioned lens distortion correction plays an important role in the field of photogrammetry. For the description of distortion several models are used. First let C be a camera system following the pinhole camera model [7] described by the intrinsic camera parameters principal distance c, and principal point $p = (x_0, y_0)$. The position and orientation of camera C in a 3D world co-ordinate system is described by the three extrinsic camera parameters X, Y, Z (position of the projection center), and ω, φ, and κ (orientation angles). Using this camera model a 3D scene can be mapped onto an image in the image plane of our camera C by central projection of the 3D points through the projection center (X, Y, Z). Distortion should be defined as the deviation of the actual mapping from the ideal central projection defined by the set of intrinsic and extrinsic camera parameters. The source of the distortion is the fact that lens systems usually do not exactly realize an ideal central projection mapping. The most typical distortion effect is radial symmetric lens distortion caused by the shape of the lenses. Additionally, deviations of the camera chip from the quadratic pixel size, pixel distance, or flatness are also sources of distortion effects.

Hence distortion is a function of a 2D point in the camera image plane characterized by the picture co-ordinates (pixels) of the camera and can be described by a 2D vector $\Delta(x,y) = (\Delta x, \Delta y)$. This function is interesting for all image points (pixels) of the visible part of the image plane (the camera chip).

A. The Functional Representation

Using the functional representation, or, as denoted by Grompone et al. [14] the parametric model, the distortion function will be approximated by a sum function $F(x,y)$ including radial symmetric, decentering, affine, and other parts:

$$F(x,y) = F_1(x,y) + F_2(x,y) + \ldots + F_n(x,y). \tag{1}$$

Let us, for example consider radial symmetric, decentering, and affine distortion. Then we obtain:

$$\begin{aligned} \Delta x = \ & \Delta x_r + \Delta x_d + \Delta x_a = x'\left(a_2 r'^2 + a_4 r'^4 + \ldots\right) \\ & + b_1 \cdot \left(r'^2 + 2x'^2\right) + b_2 \cdot x' y' + c_1 x' + c_2 y' \\ \Delta y = \ & \Delta y_r + y_d = y'\left(a_2 r'^2 + a_4 r'^4\right) + b_2 \cdot \left(r'^2 + 2y'^2\right) + 2b_1 \cdot x' y' \end{aligned} \ . \tag{2}$$

The functional description of the describing distortion function may be classified by certain characteristics into radial, division, FOV, polynomial and rational models. An example for the division model which is the inverse of the radial one is either

$$r' = f(r) = \frac{r}{1 + a_0 + a_2 r^2 + a_4 r^4 + \ldots} \quad \text{or}$$
$$r = f(r') = \frac{r'}{1 + d_0 + d_2 r'^2 + d_4 r'^4 + \ldots} \ , \tag{3}$$

where r and r' are the undistorted and the distorted distance to the symmetry point $P=(x_0, y_0)$ of distortion and $a_0, a_1, \ldots, a_n$ or $d_0, d_1, \ldots, d_n$, respectively are the distortion coefficients. The FOV model (see Devernay & Faugeras [12]) describes the distortion usually by the first order parameter of the field of view. An example for the FOV model is

$$r' = f(r) = r\frac{\tan(r\omega)}{2r\tan(\omega/2)} \ . \tag{4}$$

The polynomial model is the description of the distortion as a polynomial in x and y (the undistorted image coordinates) and the rational model the extension of the polynomial model to a quotient of two polynomials (see Claus & Fitzgibbon [13]). An example for the rational model of third order is

$$\begin{aligned} x' &= \frac{a_1 x^3 + a_2 x^2 y + a_3 x y^2 + a_4 y^3 + a_5 x^2 + a_6 xy + a_7 y^2 + a_8 x + a_9 y + a_{10}}{c_1 x^3 + c_2 x^2 y + c_3 x y^2 + c_4 y^3 + c_5 x^2 + c_6 xy + c_7 y^2 + c_8 x + c_9 y + c_{10}} \\ y' &= \frac{b_1 x^3 + b_2 x^2 y + b_3 x y^2 + b_4 y^3 + b_5 x^2 + b_6 xy + b_7 y^2 + b_8 x + b_9 y + b_{10}}{c_1 x^3 + c_2 x^2 y + c_3 x y^2 + c_4 y^3 + c_5 x^2 + c_6 xy + c_7 y^2 + c_8 x + c_9 y + c_{10}} \end{aligned} \ . \tag{5}$$

Kruck [15] uses a set of up to 30 parameters for the description of the distortion. However, the more parameters are used the more correlation between these parameters occurs. Hence, numeric determination may be sensitive against errors.

B. The Distortion Matrix Representation

Another description of the distortion is a sampling point based on an explicit 2D distortion vector representation. Typically, this vector should be given for every pixel. For sub-pixel exact processing, an interpolation rule must be defined. This may be e.g. a bilinear or spline interpolation. This representation leads to a distortion matrix D (in the dimensions of the image size). Distortion correction may be easily achieved by application of the distortion matrix D as a cumulative (or subtraction) operator. This representation is used e.g. by Grompone et al. [14], Bräuer-Burchardt et al. [16], and Hanning [17].

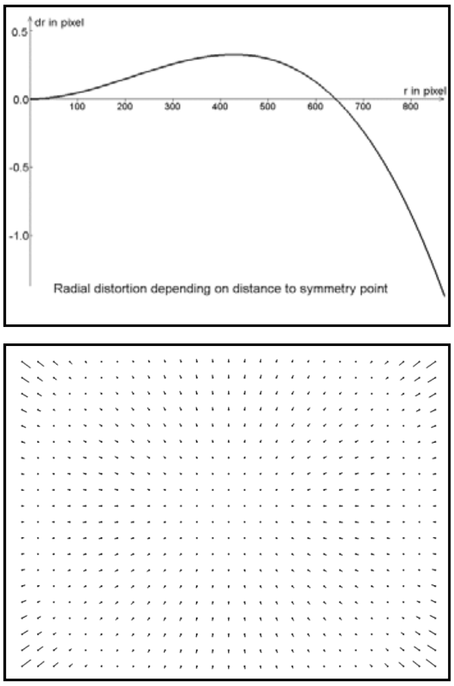

Fig. 1 Example of a radial symmetric distortion function according to radial distance to the symmetry point of distortion (above) and the same distortion function in matrix representation showing selected sampling points and distortion vector scaling (below)

III. Accuracy Model

The camera distortion is usually determined within the context of camera calibration. Let us consider the typical case of using the pinhole camera model and an additional distortion model which can be either a functional model or a matrix representation. Depending on the type of the current distortion model, a correlation between the distortion and one or more of the camera parameters may be present.

For all functional models it must be noticed that the model fits the actual distortion only until a certain accuracy threshold. The higher the number of distortion function parameters the closer the distortion description may be to the actual distortion. Additionally, the correlation between the parameters is also higher.

Comparing the functional model with the matrix representation of the distortion two main aspects determine the advantages or disadvantages of the model choice. The main advantage of the functional model *F* is the representation of the distortion at every point of the image plane by *F*. If the actual distortion is represented sufficiently by *F* the points for determination of *F* have not be necessarily distributed consistently over the image area. The main disadvantage is the fact that the model usually does not fit exactly the actual distortion. Conversely, the main advantage of the matrix model is the exact description of the distortion at every point of the image plane. The main disadvantage is, respectively, that the determination must be realized for all regions of the image plane by a consistent distribution of the calibration points. Outliers should be prevented because their influence is much bigger according to their local effect on the distortion.

When distortion is analyzed and removed it is of essential interest how accurate the current correction compensates the actual distortion effects and how large is the remaining error due to distortion. Here, unfortunately, most work in the literature content a presentation of a remaining *rms* error of the corrected 2D image coordinates concerning straight lines or reference points. However, according to our experimental results, this is not sufficient for high precision 3D measurements. In order to perform a meaningful error analysis and description, there are given some error definitions in the following section.

A. Distortion Error Description

Let $p=(x,y)_i$ be the ideal coordinates of a set of reference points in the image with known coordinates or known linearity, respectively. Let $p'=(x',y')_i$ be the distorted or corrected measured coordinates of the reference points. Then we define the 2D *rms point error E2D* as follows:

$$E2D = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - x_i^{'}\right)^2 + \left(y_i - y_i^{'}\right)^2}\,. \tag{6}$$

As the deviation from a straight line is often the only measure for the quality of distortion correction, we define the 2D *rms line error LE2D* as

$$LE2D = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i^{L} - x_i^{'}\right)^2 + \left(y_i^{L} - y_i^{'}\right)^2}\,, \tag{7}$$

where $p_i^{\,L}=(x_i^{\,L}, y_i^{\,L})$ are those points on a fitted straight line with the shortest distance to the $p_i'=(x_i',y_i')$. Note that this error may be small although the distortion correction may be still erroneous. See fig. 2 for illustration of *E2D* and *LE2D*. In order to evaluate the quality in 3D measurements we define

$$E3D = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - x_i^{'}\right)^2 + \left(y_i - y_i^{'}\right)^2 + \left(z_i - z_i^{'}\right)^2} \tag{8}$$

as the root of squared 3D distances between measured points (x',y',z') and their ideal 3D coordinates (x,y,z) and, analogously to the 2D case, the 3D *rms plane error PE3D* as

$$PE3D = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - x_i^{E}\right)^2 + \left(y_i - y_i^{E}\right)^2 + \left(z_i - z_i^{E}\right)^2}\,. \tag{9}$$

These errors (8) and (9) include all parts of the remaining error as noise and reference point uncertainty. Consequently, in order to describe the deviation from flatness, absolute deflection error DE_{abs} is defined as the noise reduced height difference between the highest $Z_{\max}^{av}$ and the lowest $Z_{\min}^{av}$ measured 3D point concerning a fitted plane. The normalized deflection error DE_n is defined describing the maximal deflection of a plane (with removed noise) normalized to the maximum measured distance *diam* on the plane.

$$DE_{abs} = Z_{\max}^{av} - Z_{\min}^{av} \quad \text{and} \quad DE_n = DE_{abs}/diam\,. \tag{10}$$

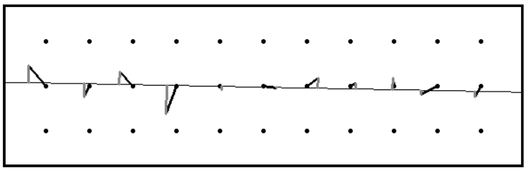

Fig. 2 Illustration of *E2D* and *LE2D*: *rms* of the length of black line segments (distortion vectors - strongly scaled) lead to *E2D* whereas *rms* of grey line segments length lead to *LE2D*

B. Error Analysis

Let us, first, consider the 2D case considering the image coordinates. In the ideal case the correction of a distorted image leads to error free image coordinates. If the correct coordinates of a set of certain reference points are known, the remaining error of a corrected point can be determined. However, usually neither the reference points are known exactly, nor the determination of the (corrected) points in the image can be obtained error free. Nevertheless, the measure of the deviation between the measured points and the reference points should be the *E2D* error as defined by (6).

However, this error includes the remaining error E_{dc} of the current distortion correction, the 2D reference point location error E_{ref}, and the image coordinate determination error E_{dc}, too. Assume a normal distribution of these three error amounts. Then the variances of the errors sum up. Hence we get

$$E2D_{rms}^{\;\;2} = E_{dc}^{\;\;2} + E_{ref}^{\;\;2} + E_{cd}^{\;\;2} \tag{11}$$

and thus for the distortion correction error

$$E_{dc} = \sqrt{E2D_{rms}^{\;\;2} - E_{ref}^{\;\;2} - E_{cd}^{\;\;2}}\,. \tag{12}$$

Usually, both the 2D reference point location error E_{ref} and the image coordinate determination error E_{cd} can only be estimated by experimental analysis.

In the documentation of our experiments we will provide the *E2D* error, the *LE2D* error, the *PE3D* error, and the deflection error *DE*.

IV. The New Methodology

A. Situation

The newly developed methodology is designed for application in stereo systems for 3D reconstruction based on active pattern projection. However, as we will see later it can be generalized for any camera system. Nevertheless, we assume a sensor head consisting of a pair of stereo cameras and a projection unit in a fix arrangement.

Our algorithm is separated into two parts. The first part contains the whole procedure of camera calibration including distortion determination using any arbitrary method. The second and novel part realizes a refinement of the distortion determination of the two cameras leading to an ultra-precise distortion description.

We use the matrix representation of the distortion. Let D_1 and D_2 the distortion matrices determined so far. Let all intrinsic and extrinsic camera parameters for both cameras be known. Notice, that only the relative orientation between the cameras is of interest.

B. The Approach

For the initial distortion determination we used the following new method which is similar to the techniques as proposed by Grompone et al. [14] and Bräuer-Burchardt [16]. We used a plane grid pattern on an actual plane ceramic tile of the type as shown by fig. 3 (left). This pattern which is absolutely flat (bending is below 1 µm in the field of view) was recorded by each of the two cameras at least ten times with some rotation around the optical axis. The normal angle and the optical axis angle differ at most 15°.

In the recorded images all detectable cross points of the grid were localized with sub-pixel accuracy. To these points a projective transform T from the set of original point co-ordinates was estimated and applied such that the squared differences $dv_i = (dx, dy)_i$ between the transformed original and the measured points are minimal. The differences dv_i are defined as the distortion vectors which can be used as sampling points to produce a distortion matrix D. The matrix D is obtained by smoothing all produced sampling points by a suitable low pass filter operator.

The refinement procedure used two criteria towards an improvement of the existing distortion representation: first, the 2D residuals should be minimized and, second, the deflection error *DE* as defined previously must be minimized. If a refinement of the distortion description is achieved by the proposed method, the calibration has to be renewed, too, and the process should be performed iteratively.

The input data of the algorithm are:

- Complete calibration data including D_1 and D_2
- A set of point correspondences $PC = \{(x_1, y_1, x_2, y_2)_i\}$ between the two camera images describing points of a really flat scene (we used a plane surface without texture, points are obtained by phase determination of a projected sinusoidal fringe pattern, see [18])

The corresponding points define together with the calibration data a set of reconstructed 3D points $P = \{p_i = (x_i, y_i, z_i\}$ obtained by triangulation (see [7]). In order to reduce errors due to noise the phase values are suitably filtered by a mean or Gaussian operator. A plane E is fitted to P in the 3D space. The projection of the points p_i to E in plane normal direction lead to corrected points $p_i' = (x_i', y_i', z_i')$ which are assumed to be "correct". Back-propagation from the corrected points p_i' lead to 2D vectors $dv_j = (dx_j, dy_j)$ corresponding to the image co-ordinates of the set PC. The vectors dv_j are sampling points of the distortion correction matrices A_1 and A_2 for the two cameras. An average or Gaussian operator should be applied to the vectors dv_j in order to reduce noise. The complete matrices A_1 and A_2 are obtained by inter- and extrapolation. Finally, D_1 and D_2 are corrected by $D_1 := D_1 + A_1$, $D_2 := D_2 + A_2$.

If the amount of the necessary correction (A_i) is considerable, calibration of the intrinsic camera parameters and the relative orientation between the cameras must be updated. The whole procedure will be repeated iteratively until $A_1 \sim A_2 \sim \mathbf{0}$ (zero matrix).

C. The Algorithm

The algorithm can be briefly summarized as follows:

0. Realize image acquisition of a flat and not bended surface by projection of a sequence of structured light images, producing point correspondences between the images of the two cameras
1. Perform 3D reconstruction using the corresponding points of the surface, fit a plane to the reconstructed surface, correct the measured 3D points into the fitted plane
2. Determine the distortion correction matrices by corrected 3D point back-propagation
3. Check finishing criterion, go to 4 or 5
4. Update calibration parameters using the corrected distortion matrices, go to 1
5. End of algorithm

D. Generalization of the Algorithm

If only one camera should be considered and no active light projection unit is available, the method may be generalized as follows: instead of the surface without texture an arbitrary plane pattern with well-known exact position, data of the identified points may be used. This may be for example a grid pattern, a chessboard pattern or a dot pattern as shown by Fig.3. However, the accuracy of the point localization in the camera image is crucial for the accuracy of the distortion determination. Additionally, the number n of localized points should be as high as possible because the random error of the determination is proportional to $\sqrt{n}$.

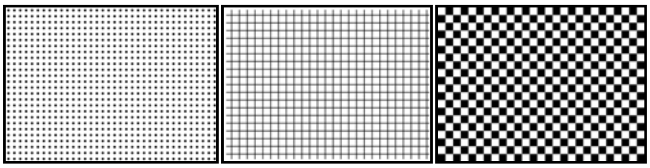

Fig. 3 Several plane patterns suitable for the described methodology: grid pattern (left), dot pattern (middle), and chess board (right)

A higher number of points can be achieved e.g. by repetition of the procedure including a shift of the recorded grid pattern.

In order to obtain a 3D measurement the camera should be calibrated (intrinsic parameters) in a preprocessing step and brought into two different positions observing the plane grid pattern with a meaningful triangulation angle. Here, the relative orientation between the two positions must be determined by a suitable calibration procedure (see e.g. [7]). After calibration the 3D reconstruction can be performed and the algorithm can be applied analogously.

V. Experiments and Results

In order to evaluate the presented methodology the following experiments were performed. The whole method was applied according to the description in the previous section. Our system is a sensor head consisting of two cameras (AVT Guppy, 1392 x 1040 pixels, pixel size 4.65 μm) with 17mm lenses (self-designed and self-produced) and a projection unit in a fix arrangement. The calibration of the system including initial distortion determination was a-priori performed using the BINGO-Software [15]. Alternatively, the distortion was also determined according to the suggested method as described in the previous section.

Distortion was determined using a plane grid pattern according our new methodology and for comparison by applying the BINGO-Software.

For evaluation of the distortion correction first the several distortion matrices were applied to images of a grid pattern (similar as shown by fig. 3). The *E2D* error to the estimated "correct" points was determined as well as the *LE2D* error to the fitted straight lines. The dimension of the errors is given in pixels. Notice, that these errors contain not only the remaining distortion error but also the random error of pixel localization and the systematic error caused by deviations of the ideal 3D point co-ordinates.

An absolute flat plane without bending (below 1μm) was the next measurement object. Active light projection was used in order to produce phase values leading to corresponding points arbitrarily close and well distributed over the whole images.

Using the calibration data 3D reconstruction of the corresponding points was performed. A plane was fitted to the resulting points and the *PE3D* and the deflection error DE_{abs} and DE_n as defined in section 2 were determined. The results of the measurement are summarized in table 1. For comparison the errors (Reference[Gr]) documented by Grompone et al. [14] are listed, too.

We guess that the amounts of the *E2D* and the *LE2D* errors are mainly influenced by the image coordinate determination error E_{cd}. The experimental estimation of E_{cd} yielded an amount of about E_{cd} ~ 0.04 pixel. Reference point error E_{ref} was estimated by E_{ref} ~ 0.02 pixel. Hence the estimated amount of our distortion correction error according to (12) is assumed to be not larger than 0.05 pixel.

Although the 2D errors are similar for all methods, the 3D deflection error is considerably reduced by our new refinement method according to both BINGO correction and our initial correction method, too. That means that for high precision 3D measurements the 2D criterions of the remaining error are too weak. Comparing BINGO with our initial correction, our method shows a lower deflection but larger noise. Note, that in the uncorrected case the deflection error is considerable, although the absolute distortion is weak (mean distortion <0.4 pixel, maximal distortion <1.5 pixel). If high precision measurements are performed, we strongly recommend the application of our new methodology.

Figures 4 and 5 show the determined distortion matrices for our two cameras compared to those obtained by the BINGO software, and fig. 6 illustrates the shape of the reconstructed surface.

TABLE I
Remaining Errors After Distortion Correction According to Formulas (6) to (9)

Method \ Error	*E2D* [pixel]	*LE2D* [pixel]	*PE3D* [μm]	DE_{abs} [μm]	DE_n[‰]
Uncorrected	0.37	0.20	23.7	96	1.60
BINGO	0.16	0.08	3.3	12	0.20
Initial	0.14	0.07	5,0	10	0.16
Refined	0.15	0.07	1.9	< 1	< 0.02
Reference [Gr]	-	0.08	-	-	

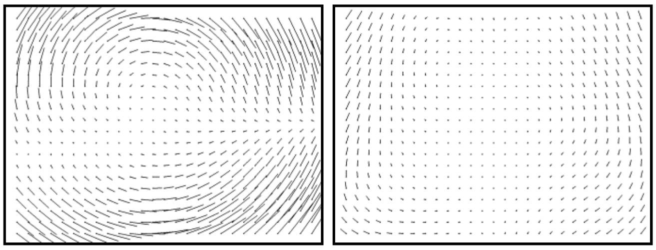

Fig. 4 Distortion matrices by BINGO for C_1 (left) and C_2 (right)

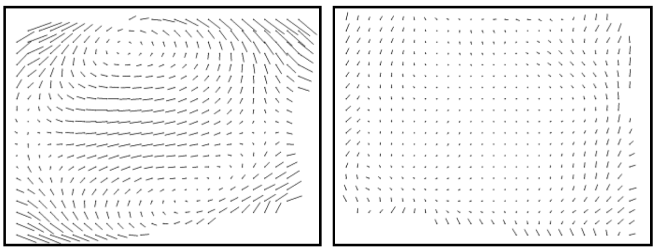

Fig. 5 Distortion matrices after refinement for C_1 (left) and C_2 (right)

As described by several authors [1 to 7] a distortion variation occurs especially in the case of short measurement distances. Because our magnification *m* was about *m* = 1:13, the distortion significantly changes depending on the distance of the object points. This effect could be proved clearly by our new method. We produced the distortion matrices for cameras C_1 and C_2 at three different distances between the plane and the cameras of 160 mm to 180 mm.

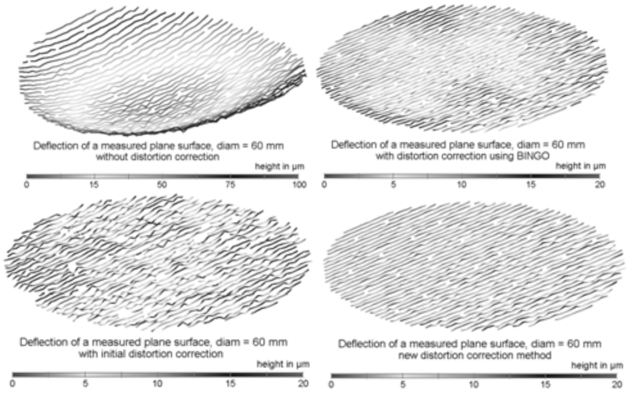

Fig. 6 3D measurement of a plane without correction (left above), after distortion correction by BINGO (right above), using our initial distortion correction (left below), and after refinement (right below). Deflection error is about 100 µm (without), 15 µm (BINGO correction), 12 µm (initial correction), and < 1 µm (new method). Note different scaling of the graph left above

VI. Summary, Discussion, and Outlook

We presented a new hybrid method to realise high-precision distortion determination for optical ultra-precision 3D measurement systems based on stereo cameras using active light projection.

As the results show the new methodology may provide a considerable improvement of the distortion correction of 3D measurement systems using active light projection or photogrammetric techniques. It could be shown that pure functional approach to distortion description may be insufficient for tasks of high precision measurement techniques. This is important if the flatness of the objects is the critical measurement quantity or if the shape of smooth objects (for example lenses) should be determined with high accuracy.

The procedure is quite complex because of the possible necessity of iterations. However, this effort is invested profitably. If the proposed procedure is applied an extensive error analysis is necessary.

With the achieved resulting accuracy the variation of the distortion concerning the distance to the measurement object may be analyzed adequately and applied subsequently in order to achieve a further improvement of 3D measurements.

Compared to the results documented by the literature it is difficult to evaluate the method because most of the authors do not give such detailed results. Taking e.g. the results of Grompone et al. [14] or our previous results [16] it seems that taking into account only the deviation from a straight line in the 2D image (error $LE2D$) distortion correction cannot be evaluated adequately because error $LE2D$ may be dominated by reference point location error and image co-ordinate determination error, respectively.

Future work will be pointed towards a separated determination of the distortion refinement matrices A_1 and A_2. More experiments concerning distance depending variation of the distortion should be performed.

VII. Conclusion

If high-precision measurements have to be performed the flatness error should be considered in order to evaluate both the measurement quality and the excellence of the calibration including lens distortion correction. The proposed methodology may be a step forward to the direction of achieving high-precision measurement results.

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