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Nonfactorizable Contributions to Weak $D \to \pi\pi$ Decay Modes

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Abstract—We investigate nonfactorizable contributions to $D \to \pi\pi$ decay modes. We perform isospin analysis of the nonfactorizable contributions to these decays. Obtaining the factorizable contributions from spectator-quark diagrams using $N_C = 3$, we determine nonfactorizable amplitudes for these decays and predict their branching ratios.

***Keywords*—**Mesons, Branching Ratios, Decay Amplitudes, Heavy Flavor Mesons, Nonfactorizable Contributions, Weak Decays.

I. Introduction

NOW extensive data [1] is available on two-body weak decays of heavy flavor mesons and it has become possible to explore the factorization model in more detail. In factorization hypothesis, the two undetermined coefficients are assigned to the effective charged current, a_1, and effective neutral current, a_2, parts of the weak Hamiltonian and are related to QCD Wilson coefficients through $a_{1,2} = c_{1,2} + \xi c_{2,1}$, where $\xi = 1/N_C$, N_C being the number of colors. In this approach, nonfactorizable part of the weak Hamiltonian is usually ignored, so ξ is treated as a parameter fixed from experiment. Data on $D \to \pi\overline{K}$ seems to favor $\xi \to 0$ limit [2-4], thereby fixing $a_1 = 1.26$ and $a_2 = -0.51$. Employing isospin formalism, in a strong - interaction phase independent manner, Kamal and Pham [5,6] has shown that the naïve factorization fails to account for isospin amplitudes for $D \to \pi\pi$ and $D \to K\overline{K}$ modes. One of the way to remove the discrepancy could be to use inelastic final-state interactions [5,6]. Moreover, the semi-phenomenological analysis of two-body decays of heavy flavor mesons indicates the presence of large non-factorizable contributions [2-6]. Two scales identifying these decays are short distance and long distance scales. The short distance effects are calculated using perturbative QCD [2-6] but long distance effects which involve nonfactorizable contributions are being, non-perturbative and cannot be calculated from first principles. An alternative to bridge the gap between theory and experiment, may be to include the non-factorizable contributions [7] arising due to soft gluon exchange, which are generally ignored in the factorization model. Therefore a reinvestigation of charm meson decays is called for, by considering the significant non-factorizable terms. Many attempts have been made to explore such contributions in hadronic decays of charm and bottom mesons [5-12].

We in this report investigate the non-factorizable contributions to $D \to \pi\pi$ decays. Employing the isospin formalism in a phase independent manner, we determine these contributions in the respective isospin amplitudes 0 and 2 for $D \to \pi\pi$ decay modes by taking $N_C = 3$ to evaluate their factorizable terms and predict the branching ratios of these modes.

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II. Formalism

To the lowest order in weak interaction, the nonleptonic Hamiltonian has the usual $current \otimes current$ form:

$$H_W = \frac{G_F}{\sqrt{2}} J_\mu^+ J^\mu + H.C., \tag{1}$$

where G_F is the Fermi coupling constant and the weak current J_μ is given by:

$$J_\mu = (\; \bar{u} \;\; \bar{c} \;\; \bar{t}\;)\gamma_\mu(1-\gamma_5)\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}, \tag{2}$$

where the eigenstates d', s', and b' are the Cabibbo-Kobayashi-Maskawa (CKM) mixture of the mass eigenstates d, s, and b which are related through CKM-matrix V as:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{3}$$

where

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \tag{4}$$

The effective weak Hamiltonian generating the Cabibbo-angle-singly suppressed ($\Delta C = -1, \Delta S = 0$) charm changing decays is given by [6]

$$H_W = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \left[c_1 \left\{ (\bar{u}d)(\bar{d}c) - (\bar{u}s)(\bar{s}c) \right\} + c_2 \left\{ (\bar{d}d)(\bar{u}c) - (\bar{s}s)(\bar{u}c) \right\} \right] \quad (5)$$

However, the effective weak Hamiltonian generating $D \to \pi\pi$ decays is given by

$$H_W^{\Delta C=-1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \left[c_1 (\bar{u}d)(\bar{d}c) + c_2 (\bar{d}d)(\bar{u}c) \right] \quad (6)$$

where $\bar{q}_1 q_2 \equiv \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$ represents the color singlet $V - A$ current, and the QCD coefficients at the charm mass scale [1,6] are

$$c_1 = 1.26 \pm 0.04, \qquad c_2 = -0.51 \pm 0.05, \quad (7)$$

In the standard factorization scheme, hadronic matrix elements of an operator O receives contributions from the operator itself and from Fierz transformation of O. Separating the factorizable and nonfactorizable parts and using the Fierz identity

$$(\bar{u}d)(\bar{d}c) = \frac{1}{N_C}(\bar{d}d)(\bar{u}c) + \frac{1}{2}\sum_{a=1}^{8}(\bar{d}\lambda^a d)(\bar{u}\lambda^a c), \quad (8)$$

where $\bar{q}_1 \lambda^a q_2 \equiv \bar{q}_1 \gamma_\mu (1-\gamma_5) \lambda^a q_2$ represent color octet current, matrix element of the operator $(\bar{u}d)(\bar{d}c)$ in (5) between initial and final states can be written as:

$$\begin{aligned} \langle P_1 P_2 | (\bar{u}d)(\bar{d}c) | D \rangle &= \langle P_1 | (\bar{u}d) | 0 \rangle \langle P_2 | (\bar{d}c) | D \rangle \\ &+ \langle P_1 P_2 | (\bar{u}d)(\bar{d}c) | D \rangle_{nonfac} \\ &+ \frac{1}{N_C}\langle P_2 | (\bar{d}d) | 0 \rangle \langle P_1 | (\bar{u}c) | D \rangle + \frac{1}{N_C}\langle P_1 P_2 | (\bar{d}d)(\bar{u}c) | D \rangle_{nonfac} \\ &+ \frac{1}{2}\langle P_1 P_2 | \sum_{a=1}^{8} (\bar{d}\lambda^a d)(\bar{u}\lambda^a c) | D \rangle_{nonfac} \end{aligned} \quad (9)$$

Performing a similar treatment to the other operator $(\bar{u}s)(\bar{s}c)$ in (5), the decay amplitude becomes

$$\begin{aligned} \langle P_1 P_2 | H_W | D \rangle &= \tilde{G}_F \left[a_1 \left\{ \langle P_1 | (\bar{u}d) | 0 \rangle \langle P_2 | (\bar{d}c) | D \rangle - \langle P_1 | (\bar{u}s) | 0 \rangle \langle P_2 | (\bar{s}c) | D \rangle \right\} \right. \\ &+ a_2 \left\{ \langle P_2 | (\bar{d}d) | 0 \rangle \langle P_1 | (\bar{u}c) | D \rangle - \langle P_2 | (\bar{s}s) | 0 \rangle \langle P_1 | (\bar{u}c) | D \rangle \right\} \\ &+ \left\{ c_1 \langle P_1 P_2 | H_W^8 | D \rangle + c_2 \langle P_1 P_2 | \tilde{H}_W^8 | D \rangle \right\}_{nonfac} \\ &+ \left\{ a_1 \langle P_1 P_2 | (\bar{u}d)(\bar{d}c) - (\bar{u}s)(\bar{s}c) | D \rangle \right. \\ &\left. + a_1 \langle P_1 P_2 | (\bar{d}d)(\bar{u}c) - (\bar{s}s)(\bar{u}c) | D \rangle \right\}_{nonfac} \end{aligned} \quad (10)$$

where

$$a_{1,2} = c_{1,2} + \frac{c_{2,1}}{N_C}, \quad (11)$$

$$H_W^8 = \frac{1}{2}\sum_{a=1}^{8} \left\{ (\bar{u}\lambda^a d)(\bar{d}\lambda^a c) - (\bar{u}\lambda^a s)(\bar{s}\lambda^a c) \right\}, \quad (12)$$

$$\tilde{H}_W^8 = \frac{1}{2}\sum_{a=1}^{8} \left\{ (\bar{d}\lambda^a d)(\bar{u}\lambda^a c) - (\bar{s}\lambda^a s)(\bar{u}\lambda^a c) \right\}, \quad (13)$$

and other quantities have the usual meaning. In addition to the effects considered so far there may be factorizable effects from W-exchange or W-annihilation diagrams, but such contributions are suppressed due to the helicity arguments [3]. At $N_C = 3$, the relation (11) allow us to calculate the values of QCD Wilson coefficients as

$$a_1 = 1.09, \qquad a_2 = -0.09. \quad (14)$$

To illustrate our procedure we now discuss $D \to \pi\pi$ decays.

III. Isospin Analysis of Nonfactorizable Contribution To $D \to \pi\pi$ Decays

As these decays involves elastic final state interactions (FSI) due to their isospin amplitudes 0 and 2 develop different phases:

$$\begin{aligned} A(D^0 \to \pi^+\pi^-) &= \frac{1}{\sqrt{12}}\left[\sqrt{2} A_0^{\pi\pi} e^{i\delta_0} + A_2^{\pi\pi} e^{i\delta_2} \right], \\ A(D^0 \to \pi^0\pi^0) &= \frac{1}{\sqrt{6}}\left[A_0^{\pi\pi} e^{i\delta_0} - \sqrt{2}\, A_2^{\pi\pi} e^{i\delta_2} \right], \\ A(D^+ \to \pi^0\pi^+) &= -\frac{\sqrt{3}}{2\sqrt{2}} A_2^{\pi\pi} e^{i\delta_2} \end{aligned} \quad (15)$$

These leads to the following relations:

$$4\left| A(D^0 \to \pi^+\pi^-) \right|^2 + 2\left| A(D^0 \to \pi^0\pi^0) \right|^2 = \left| A_0^{\pi\pi} \right|^2 + \left| A_2^{\pi\pi} \right|^2, \quad (16)$$

$$\left|A(D^0 \to \pi^0\pi^+)\right|^2 \quad = \quad \frac{3}{8}\left|A_2^{\pi\pi}\right|^2, \qquad (17)$$

which allow us to work without the phases. Writing the total decay amplitude as sum of the factorizable and nonfactorizable parts

$$A(D \to \pi\pi) = A^f(D \to \pi\pi) + A^{nf}(D \to \pi\pi). \qquad (18)$$

Using factorization scheme, the factorizable part of the decay amplitude can be written as

$$A^f\ (D^0 \to \pi^+\pi^-) \quad = \quad a_1 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2),$$

$$A^f\ (D^0 \to \pi^0\pi^0) \quad = -\ a_2 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2),$$

$$A^f\ (D^+ \to \pi^+\pi^0)$$
$$= -\frac{1}{\sqrt{2}}\{a_1 + a_2\} f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2). \qquad (19)$$

Using the numerical inputs $f_\pi = 0.132$, $F_0^{D\pi}(0) = 0.83$ and masses of mesons from [1,2,11], we calculate the factorizable part of $D \to \pi\pi$ decay modes. We write nonfactorizable part of the decay amplitude in terms of isospin C. G. coefficients as scattering amplitudes for spurion + $D \to \pi + \pi$ process:

$$A^{nf}\ (D^0 \to \pi^+\pi^-) =$$
$$\frac{1}{6}c_2\{(\frac{1}{2} < \pi\pi\|H_W^8\ \|D>_2) + (< \pi\pi\|H_W^8\ \|D>_0)\},$$

$$A^{nf}\ (D^0 \to \pi^0\pi^0) =$$
$$\frac{1}{6}c_1\{(< \pi\pi\|\tilde{H}_W^8\ \|D>_0) + (< \pi\pi\|\tilde{H}_W^8\ \|D>_2)\},$$

$$A^{nf}\ (D \to \pi^0\pi^-) =$$
$$c_2 < \pi\pi\|H_W^8\ \|D>_2 + c_1 < \pi\pi\|H_W^8\ \|D>_2. \qquad (20)$$

In order to reduce the number of unknown reduced amplitudes further in (11), we assume the following constraints:

$$< \pi\pi\|\tilde{H}_W^8\ \|D>_0 \quad = \quad < \pi\pi\|H_W^8\ \|D>_0, \qquad (21)$$

$$< \pi\pi\|\tilde{H}_W^8\ \|D>_2 \quad = \quad < \pi\pi\|H_W^8\ \|D>_2, \qquad (22)$$

as both H_W^8 and $\tilde{H}_W^8$ behave like components of an isovector spurions. Hence (15) can be written as:

$$A_0^{nf}(D \to \pi\ \pi) = \frac{\sqrt{2}}{\sqrt{3}}[\ 2A^{nf}\ (D^0 \to \pi^+\pi^-) +$$
$$A^{nf}\ (D^0 \to \pi^0\pi^0)\ \ ], \qquad (23)$$

$$A_2^{nf}(D \to \pi\ \pi) = \frac{2}{\sqrt{3}}[\ A^{nf}\ (D^0 \to \pi^+\pi^-)\ -$$
$$A^{nf}\ (D^0 \to \pi^0\pi^0)\ \ ], \qquad (24)$$
$$= -\frac{2\sqrt{2}}{\sqrt{3}} A^{nf}\ (D^+ \to \pi^0\pi^+). \qquad (25)$$

These relations then lead to following predictions:

$$\frac{A_0^{nf}(D \to \pi\pi)}{A_2^{nf}(D \to \pi\pi)} = \frac{c_1^2 + 2c_2^2}{\sqrt{2(c_2^2 - c_1^2)}}$$
$$= -1.123 \pm 0.158, \qquad (26)$$

bearing same universal ratio as obtained for $D \to \pi\bar{K}$ modes [11].

IV. Numerical Results and Discussion

Using the experimental values [1] for branching:

$$Br\ (D^+ \to \pi^0\pi^+) = (0.128 \pm\ 0.009)\%,$$

$$Br\ (D^0 \to \pi^-\pi^+) = (0.1364 \pm\ 0.0032)\%,$$

$$Br\ (D^0 \to \pi^0\pi^0) = (0.079 \pm\ 0.008)\ \%, \qquad (27)$$

and D-meson lifetimes

$$\tau_{D^0} = 0.4101 ps,$$

$$\tau_{D^+} = 1.040 ps, \qquad (28)$$

the decay rate formula

$$\Gamma(D \to \pi\pi) = \left|\frac{G_F^2}{\sqrt{2}} V_{ud} V_{cd}^*\right|^2 \frac{p}{8\pi m_D^2}\left|A(D \to \pi\pi)\right|^2, \qquad (29)$$

(where p is the three momentum of the final state particles in the rest frame of D-meson and m_D the mass of parent D-meson) and (16,17) yields

$$\left|A_0^{\pi\pi}\right|_{\exp} = \pm 0.4823 \text{ GeV}^3 ,$$

$$\left|A_2^{\pi\pi}\right|_{\exp} = \pm 0.2531 \text{GeV}^3. \tag{30}$$

By taking positive signs for $\left|A_0^{\pi\pi}\right|_{\exp}$ and $\left|A_2^{\pi\pi}\right|_{\exp}$ in (30), the nonfactorizable isospin amplitudes are then determined as

$$A_0^{nf}\ (D \to \pi\pi) = -0.2166 \text{GeV}^3 ,$$

$$A_2^{nf}\ (D \to \pi\pi) = -0.1822 \text{GeV}^3 . \tag{31}$$

This then yields

$$\frac{A_0^{nf}(D \to \pi\pi)}{A_2^{nf}(D \to \pi\pi)} = 1.1886 . \tag{32}$$

which shows that the ratio of nonfactorizable isospin amplitudes in different isospin channels for Cabibbo-suppressed $D \to \pi\pi$ modes is comparable in magnitude as predicted in equation (26). Using isospin relations we derive the formula for the sum of branching ratios of $D \to \pi\pi$ decays as

$$2Br_{-+} + Br_{00} = \frac{4\tau_{D^0}}{3\tau_{D^+}} D_{0+}\left[1+\left\{\alpha+\frac{(2-\sqrt{2}\alpha)A_{-+}^{fac}+(1+\sqrt{2}\alpha)A_{00}^{fac}}{2A_{0+}}\right\}^2\right] , \tag{33}$$

with $\alpha \equiv A_0^{nf} / A_2^{nf}$, where subscripts -+, 00, 0- denote the charge states of the non charmed mesons emitted in each case. A_{-+}^{fac} and A_{00}^{fac} denote the factorized amplitudes of D^0 decays, A_{0-} is obtained from the D^+-decay branching ratio D_{0+} as

$$A_{0+} = \sqrt{\frac{Br_{0+}}{\tau_{D^+} \times (\text{Kinematic factors})}} . \tag{34}$$

The relation (33) and the branching ratio of $D^+ \to \pi^+\pi^0$ allow us to predict the sum of branching ratios of $D \to \pi\pi$ decays as:

$$2Br_{-+} + Br_{00} = 0.312\% , \tag{35}$$

which is in close agreement with the experimental value 0.352%[1].

V. Conclusion

We study the nonfactorizable contributions to $D \to \pi\pi$ decay modes. We perform isospin analysis for the nonfactorizable contributions to these decays. Employing the spectator quark model we evaluate the factorizable parts of the decay amplitude to these decays using $N_C = 3$. Then we determine the useful nonfactorizable part of the decay amplitudes for these decays. We find that the isospin amplitudes 0 and 2 for these decays $A_0^{nf}(D \to \pi\pi)$, $A_2^{nf}(D \to \pi\pi)$ bear the same universal ratio A_0^{nf} / A_2^{nf} = 1.1886. Further, we predict the sum of branching ratios of $D \to \pi\pi$ (00 and -+ charged states) modes to be 0.312% and this predicted value is in the right direction and in close agreement with the experimental value 0.352%, thereby justifying the inclusion of nonfactorizable contributions to these decays. Therefore the evaluation of nonfactorizable contributions to the decay amplitudes of charm mesons can play a significant role to bridge the gap between theory and experiment.

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