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The Leaves of a Tree

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***Abstract*—**In this article, models based on quantitative analysis, physical geometry and regression analysis are established, by using analytic hierarchy process analysis, fuzzy cluster analysis, fuzzy photographic and data fitting. The reasons of various leaf shapes among different species and the differences between the leaf shapes on same tree have been solved by using software, such as Eviews, VB and Matlab. We also successfully estimate the leaf mass of a tree and the correlation with the tree profile.

***Keywords*—**Leaf shape; Mass; Fuzzy cluster; Regression analysis; Eviews; Matlab

I. Introduction

TREE is the name of all the woody plants which can be further divided into arbor, shrub and liana. Trees are mainly consists of seed plants and only the tree fern among all the fern are trees. There are approximately 35,841 species of trees in the world. Trees are normally consist of roots, branches and leaves[1].

Leaves are the most important organ of the tree which can synthesize sugar or other substances by means of photosynthesis and provide energy for the tree. Then 'How much is the leaf mass of a tree?' The different kinds of the trees have obvious different leaf mass. Given the size of the tree, even one species can has various statistics of leaf mass. Therefore, qualitative analysis and quantitative analysis are significant and meaningful. The collections of various leaves of some tree species and the data of normal trees are necessary.

Considering the operability of the data, we decided to focus on the environment elements and ignore the contribution of other factors, which are difficult to model accurately and have minimal effect on the leaf shape. There are approximately 35,841 types of trees. Based on the comparison of theses trees, we divided the 35,841 species into12 types of leaf shape and study on the presentations of the 12 types[2]. As the growth of the tree is mainly determined by temperature, water and sunlight, we can analyze the latitude, elevation, sunlight, temperature, moisture and other factors which are easy observed to determine the leaf shape.

Preliminary, to simplify and standardize the model, we assume that the same kind of trees shares the common leaf shape. Then, analyze the factors which have significant effects on the leaf shape by principal component analysis. Use the cluster analysis to design the quantitative analysis model. Through the analysis of thousands of trees, we can establish the mostly representative trees to prove the process.

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Our findings indicate that the leaf shape matches the tree itself. Finally, we can get the error caused by the ignored factors and the influences on the final results.

II. Model of Different Trees

A. Assumptions

In order to simplify the complex real situations, we have made several key assumptions to exclude the uncertainty.

1. The leaf shape of one species is certain and the leaves of a tree have the same shape.
2. Assume that the longitude cannot influence the basic leaf shape.
3. The other factors which are difficult to estimate are included in the factors we observed.
4. Ignore the interference of human.
5. The source of data is reliable and realistic.

B. Factor Analysis and Data Collection

Under the premise of rational assumptions, we conclude that the temperature, moisture leaf length and leaf width determine the environment of trees which directly influence the growth of trees.

Explain in details: The high temperature and large density of moisture contributes to increasing the absorption of sunlight and carbon dioxide. They can also increase the frequency of exchange with the water in the air. Trees in the tropical area generally have large leaves which grow quickly. Oppositely, trees in the cold area generally have small and hard leaves, such as the needle-shaped. Furthermore, the latitude and elevation have strong influence on the temperature and the moisture[3].

Based on the above analysis, we should comprehensively consider the four factors.

After filtering thousands of tree species, we select some representative species and then make the Table I.

TABLE I
Factors' Data of Different Leaf Shapes [4][5]

Tree Name (leaf Shape)	Latitude α_1	Elevation α_2	Temperature α_3	Moisture α_4	Length β_1 cm	Width β_2 cm	Real leaf models
Cotinus coggygria (Round)	28.1	300	18	0.5	2-5	2.5-6	
Cinnamomum camphora (L.) Presl(Ellipse)	32	550	26	0.7	5-10	3.5-5.5	
Magnolia (Obovate)	30.85	100	20	0.8	10-18	6-12	
Ginkgo (Fan-shaped)	30	1500	15	0.5	5-8	2.5-3.5	
Chinese tallow (Diamond)	28	900	25	0.7	3-8	3-9	

China Redbud (Heart-shaped)	20	350	30	0.9	6-14	5-14	
Phoenix Tree (Palmate)	30	2300	25	0.7	9-15	9-17	
Willow (Lanceolate)	45	650	30	0.9	6-14	5-11	
Morus alba (Oval)	34	600	21	0.7	6-15	4-12	
Oriental Arborvitae (Squamous)	34	650	6	0.1	1-2	0.2-0.5	
Pinus tabulaeformis Carr.(Needle)	45	1050	1	0.1	2-4	0.1-0.2	
Betula platyphylla Suk(Triangle)	47.55	2000	10	0.5	3-7	2.5-5.5	

C. Establishment and Solving the Model

We established the model by fuzzy cluster analysis.

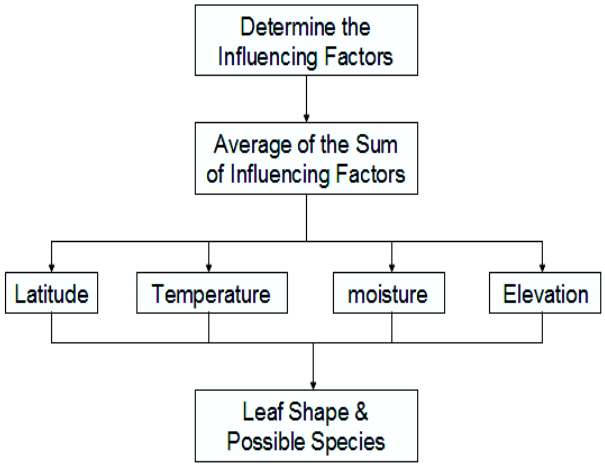

Fig. 1 the stratified analysis of the element which affects the leaf shape

Following, we conduct the analysis of the effects of the factors and get the weights of classification index.

TABLE II
THE WEIGHTS OF THE FACTORS' CLASSIFICATION INDEX

Factor	Latitude	Temperature	Moisture	Elevation
Latitude	1	2	1/3	2
Temperature	1/2	1	2/3	3
Moisture	3	3/2	1	2
Elevation	1/2	1/3	1/2	1

Based on the dealing of weights of classification index, we can calculate the quantitative value r_i of the similarity between the classified objects. We can take correlation coefficient method, cosine method, Euclidean distance method and so on to calibration the leaf shape samples. Through comparison of these methods, we select the cosine method[6].

$$r_i = \frac{\sum_{i,j=1}^{4} \left| x_{ik} - x_{jk} \right|}{\sqrt{\sum_{i=1}^{4} x_{ik}^2} \sqrt{\sum_{j=1}^{4} x_{jk}^2}}$$

We can obtain weighted similarity matrix by the method, and then we can get different species based on the matrix and the data.

The set of classified objects: $X = \{x_1, x_2, \cdots, x_n\}$

The matrix of characteristic

$$R = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{41} & x_{42} & \cdots & x_{4n} \end{pmatrix}$$

To divide thousands of trees into 12 types, we need to set up 12 cluster center vector:

$$V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}$$

The principle of cluster: Obtain suitable fuzzy classification matrix R and cluster center vector R to minimize the square of the sum of the distances from objective function and cluster centers, which can be established as:

$$J(R, V) = \min[\sum_{i=1}^{4} \sum_{k=1}^{4} (r_{jk})^{ik} \left\| x_k - V_i \right\|^2]$$

In the above formula, V_i represents the number i cluster center and $\left\| x_k - V_i \right\|^2$ represents the square of distance between x_k and V_i.

D. Test of the Model

Use VB to examine the leaf shapes which are known.

Already know the latitude, elevation, temperature and humidity, enter the figures in the designed program page.

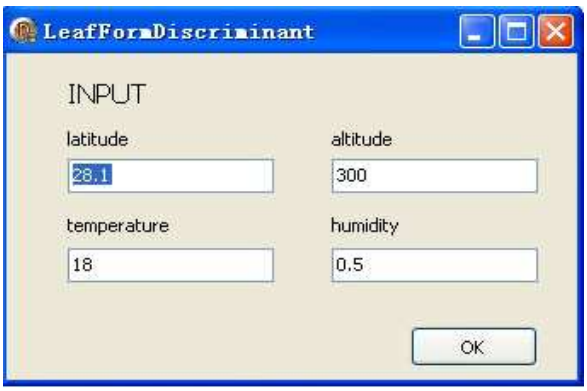

Fig. 2 the program page

We take fitting cluster filter method, use the latitude as an example and the code is the following:

```
clc,clear
x=1;
while x~=0
   x=input('\n please input the value :');
   if x>=20&&x<=28
      fprintf('\n round\n');
   else if x>28&&x<30
         fprintf('\n round or oval\n');
      else if x>=30&&x<=35
         fprintf('\n oval\n');
         else
            fprintf('\nillgel!\n');
         end
      end
   end
end
```

Similarly, we can do the same operation of other factors. Finally, we get the following page:

Fig. 3 final page

III. Conclusion

We can also operate the program to get leaf shape according to other factors which have been known. By statistical analysis and the information we got, we can preliminarily determine the leaf shape and the possible species.

Acknowledgment

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