Estimating the Runoff using the Simple Tank Model and Comparing it with the SCS-CN Model- A case Study of the Dez River Basin

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Abstract**—**Run-offs are considered as important hydrological factors in feasibility studies of river engineering and irrigation-related projects under arid and semi-arid condition. Flood control is one of the crucial factor, the management of which while mitigates its destructive consequences, abstracts considerable volume of renewable water resources. The methodology applied here was based on Mizumura, which applied a mathematical model for simple tank to simulate the rainfall-run-off process in a particular water basin using the data from the observational hydrograph. The model was applied in the Dez River water basin adjacent to Greater Dezful region, Iran in order to simulate and estimate the floods. Results indicated that the calculated hydrographs using the simple tank method, SCS-CN model and the observation hydrographs had a close proximity. It was also found that on average the flood time and discharge peaks in the simple tank were closer to the observational data than the CN method. On the other hand, the calculated flood volume in the CN model was significantly closer to the observational data than the simple tank model.

Keywords**—**Simple Tank, Dez River, run-off, lag time, excess rainfall.

I. INTRODUCTION

SUSTAINABLE water resources use and management particularly those in the storage reservoirs in arid and semi-arid regions like Dezful, Iran. For this reason run-off, if managed appropriately, can play a crucial role as a potential source of water to meet the ever increasing demand made by the competing sectors [5]. Run-off and floods emerge under circumstance where the rainfall intensity is greater than the soil capacity to absorb the rainwater. Estimation and forecast of surface run-off s are important aspect of decision concerning the dimensions of the river engineering -related projects. There are various mathematical and experimental methods for estimating the flood. The present paper investigates and compares run-off with the simple tank and SCS-CN models in the Dez water basin.

There can be valuable hydrographic data which can used to develop a sound mathematical model for a catchment. Linsley et al. [1958] and Bras [1990] specified that the flood curve can be expressed by an equation as follows:

$$q_t = q_o K_r^t \qquad (1)$$

Where,

q_o is the flow at each period,

q_t the flow in time t,

And k_r is a parameter which is derived from the curve.

Bames [1940] suggestion was that the descending curve can be approximated by three straight lines over a semi-logarithmic curve. Sugawara [10] provided a simulation of water basin by a conceptual water tank. The aim was to simulate the rainfall-runoff process using a combination of several tanks. The model incorporated two orifices on the tank wall to simulate the outflow of water [Fig 1]. Although there was some success in estimating the run-off from the rainfall, it was nonetheless hard to explain the physical phenomenon in the tank and identifying the parameters in the tank model [10]. Mizumura and Chiu [8] developed their model based on the run-offs resulting from the rain and melting snow by using a Kalman filter [6]. Shidawara [1972] used an equation to approximate the descending curve as follows:

$$q_t = q_o (t + t_0)^{\alpha} \qquad (2)$$

Where;

F_o, the time,

q_0 and α are the parameters which are derived from the hydrographic observational data.

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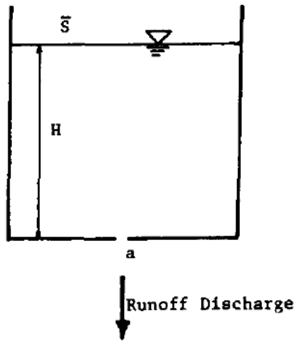

Fig.1. the schematic view of a simple tank

Hasebe and Hino [1984] divided the outflow run-off into overland, subsurface and ground water flows and estimated each parameter with relative accuracy [Fig.2]. They showed if the run-off outflow is divided into three components, a linear relation will exist between the rainfall and run-off. This study the water basin is assumed as a simple water tank, the cross-section of which is derived from the hydrographic data. It is a simple conceptual model, the parameters of which have physical base and it can easily be used to estimate the run-off flows.

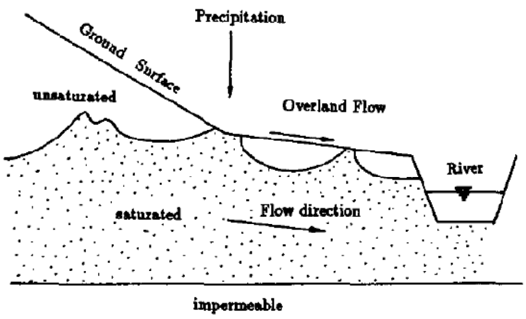

Fig. 2. schematic view of the rainfall-runoff phases

II. SIMPLE TANK MODEL

Considering the figure (1) we have:

$$u = \sqrt{2gH} \tag{3}$$

In which u= flow velocity, H= water depth in the tank, g= gravitational acceleration.

If the sectional area is a, discharge is given:

$$Q = au = a\sqrt{2gH} \tag{4}$$

So if considering C_d is discharge coefficient, modified discharge is given:

$$Q = C_d au = C_d a\sqrt{2gH} \tag{5}$$

Using the continuity equation between the water surface and the outlet of the water tank, we have:

$$C_d au = -\frac{d(\bar{S}H)}{dt} = -\bar{S}\frac{dH}{dt} \tag{6}$$

In which $\bar{S}$ = horizontal cross-sectional area of the water tank; C_d = discharge coefficient of the outlet; a= cross-sectional area of the outlet; and t= time. Defining

$h = C_d{}^2 a^2 H$

$S = \bar{S} / C_d^2 a^2$

So

$$-S\left(\frac{dh}{dt}\right) = \sqrt{2gh} \tag{7}$$

Recession curve equation is:

$$Q = C\exp\{-kt\} \tag{8}$$

In which c = discharge at t = 0; and k = recession coefficient.

Solving equations for water depth h gives

$$h = C_1 \cdot \exp\{-\sqrt{2gt} / A\} \tag{9}$$

In which C_1= an integration constant.

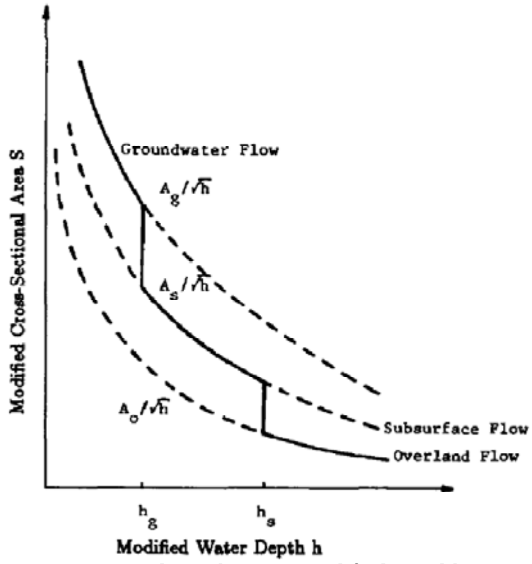

Fig. 3. Relation between modified S and *h*

A comparison of (7) and (9) gives[Fig.3]

$$2K = \frac{\sqrt{2g}}{A} \tag{10}$$

Model construction and computational method

A time lag (t_L) is defined as the travel time of water from the top of the tank to present water surface level in the tank.

$$t_L = (h_{\max} - h) / C_i \tag{11}$$

In which $h_{\max}$= total depth of the water tank, *h*= present water depth and C_i= infiltration velocity of water in the water tank over the water surface.

The continuity equation for the catchment is written as:

$$R(t - t_L) - Q(t) = \frac{dv}{dt} \tag{12}$$

In which $R(t - t_L)$ = total rainfall excess in this catchment at time $t - t_L$.

The water storage *V* in the catchment is also defined by

$$V = \int_0^h S dh = \int_0^h f(h) dh = 2A\sqrt{h} \tag{13}$$

Integration the continuity equation, and considering $dv = sdh$ may be rewritten as

$$\int_t^{t+\Delta t} \{R(t - t_L) - Q(t)\} dt = \int_h^{h'} S dh \tag{14}$$

In which h, h' water depth in the tank at times $t, t+\Delta t$, respectively.

Then, the water depth h' at time $t+\Delta t$ is given by

$$h' = \left[\frac{\bar{R}\Delta t + 2A\sqrt{h} - \Delta t\sqrt{gh/2}}{2A + \Delta t\sqrt{g/2}} \right]^2 \tag{15}$$

and the runoff discharge is

$$Q' = \sqrt{2gh'} = \frac{2K\bar{R}\Delta t + Q(2 - k\Delta t)}{2 + k\Delta t} \tag{16}$$

The parameters k_o, k_s, k_g, Q_s and Q_g are illustrated in Fig.4. Units in Fig.4 dependent upon the units of the observation data

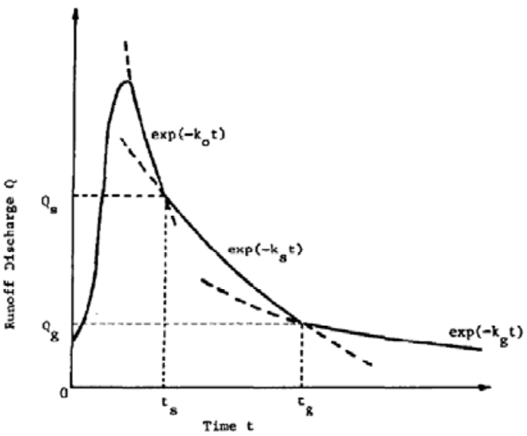

Fig.4. Definition of all parameters Q_g, Q_S, K_g, K_S, K_O

The substitution of the catchment water storage, into the continuity equation yields the following equation:

$$R(t - t_l) - Q(t) = (1/k)(dQ/dt) \tag{17}$$

This equation shows that the solution Q(t) satisfies the linear differential equation if k and t_l are constant. The recession coefficient k is determined from each recession curve of the runoff flow component.

If the time lag t_l is assumed to be constant for different water depth the solution of (17) is derived as [7][Fig.5]

$$Q(t) = Q\Big|_{t=0} \exp(-kt); \quad 0 \le t \le t_l \tag{18}$$

$$Q(t) = \exp(-kt)\left[\int_{t_l}^{t} kR(t - t_l)\exp(k\tau)\, d\tau + Q\Big|_{t=0}\right] \tag{19}$$
$$t_l \le t \le t_l + t_r$$

$$Q(t) = Q\Big|_{t = t_l + t_r} \exp(-k(t - t_l - t_r)); \quad t \ge t_l + t_r \tag{20}$$

in which $K = \frac{\sqrt{2g}}{2A}$ and t_r= rainfall duration. Considering the intensity of rainfall excess R to be constant, Eq. (19) is written as:

$$Q(t) = [1 - exp\{-k(t - t_l)\}]R + Q\Big|_{t=0} \exp(-kt) \tag{21}$$

The first and second terms in (21) represent the effect of the present and the previous rainfall, respectively. Eq. (17) is considered to be linear, but it contains four parameters k_o, k_s, k_g and t_l. These four parameters result in a "deformed hydrograph".

If compute the runoff discharge when the time lag is zero, we have:

$$Q_o(t) = [1 - \exp(-kt)]R + \exp(-kt)\, Q\Big|_{t=0} \tag{22}$$

This equation is zeroth-order solution and if time lag is not constant, the first solution is given:

$$Q(t) = Q\Big|_{t=0} \exp(-kt); \quad t \le t_o^* \tag{23}$$

$$Q_l(t) = R[1 - exp\{-k(t - t_o^*)\}] + Q\Big|_{t = t_o^*} \exp\{-k(t - t_o^*)\}$$
$$t_o^* \le t \le t_r^* \tag{24}$$

$$Q_1 = Q\Big|_{t - t_r^*} \exp(-k(t - t_r^*)); \quad t \ge t_r^* \tag{25}$$

in which $t_o^* = \left[h_{max} - \left\{Q\Big|_{t=0}\right\}^2 / 2g\right] / c_i$ and

$t_r^* = \left[h_{max} - \left\{Q\Big|_{t = t_r}\right\}^2 / 2g\right] / c_i$.

The zeroth- and first-order solutions are schematically represented in Fig.6.

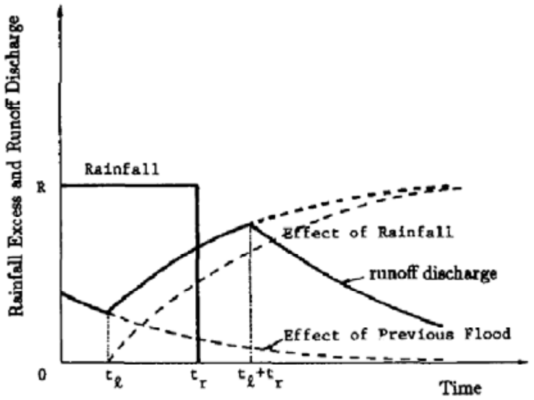

Fig.5. Relation between rainfall and runoff

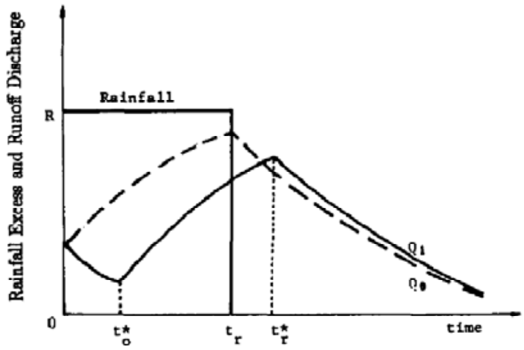

Fig. 6. Zeroth- and first order solution

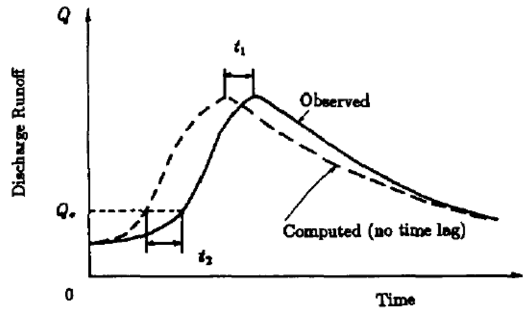

Fig. 8. Determination of time lag of hydrograph

III. MATERIALS AND METHOD

The study was carried out in the Dez water basin which expand in an area of about 21,720 km^2 having a circumference of about 900 km, the Dez water basin is located in an area of the Zagros range with an average altitude of 1676m and the average gradient of 12.1% [1,5].

The method involved computation of the parameters which were derived from the observational recession curves hydrographs in the Dez basin. Recession curve approximately divided into three straight lines with different slopes, K_o, K_g and K_s [Fig.7].

The runoff discharges in locations where the slopes suddenly change are shown by Q_s and Q_g. The time lag was computed by Eq.11. An example of the computed hydrographs excluding the time lag and observation hydrographs is shown in fig. 8. The hydrograph time lag in peak discharge and a given discharge Q_* are t_1 and t_2, respectively. If h, h_p are assumed as the water depth in peak discharge and given discharge Q_*, respectively, assuming C_i is constant, the equation is expressed:

$$t_1 = \frac{h_{\max} - h_p}{C_i} \tag{26}$$

$$t_2 = \frac{h_{\max} - h_*}{C_i} \tag{27}$$

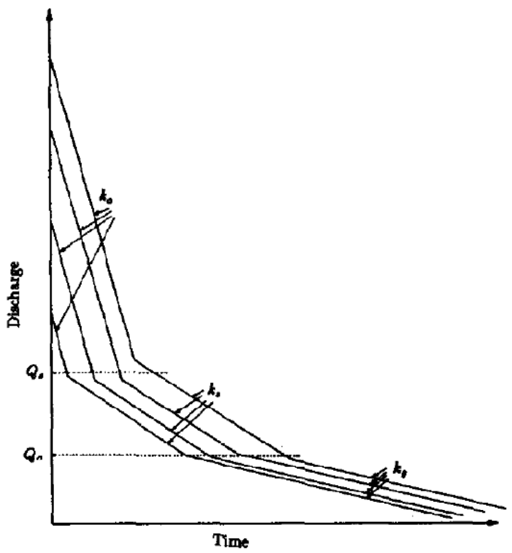

Fig . 7 Schematic plot of recession curve on semi logarithmic graph

The rainfall excess on each time is given by

$$R(t) = R_{obs} - R_{LOSS} \tag{28}$$

For the purpose of this study the results of two computational models of simple tank and SCS-CN models were compared with the observation hydrograph in the Dez water basin in Iran. In order to carry out the investigation, three flooding events were in three different periods in 1979.14.2, 1986.9.10 and 2001.18.2 were considered.

The Q_g, Q_s, K_g, K_S, K_O values for the 1979 flooding periods were arrived at 0.1571, 0.1378, 0.0516, 210 and 90 and the computed CN value was derived at 79 [1][Fig.9]. The Q_g, Q_s, K_g, K_S, K_O values were arrived at 0.1238, 0.1118, 0.089, 40 and 80 respectively and the CN values of 83 were considered for 1986 [1][Fig.10].

The Q_g, Q_s, K_g, K_S, K_O values for the year 2001 were 0.1826, 0.1267, 0.0257, 130 and 41 respectively while the CN values for this purpose was considered at 67 [1][Fig.11].

IV. RESULTS AND DISCUSSION

Results [fig.9, 10, 11] showed that the estimated discharge flow has a close proximity with the hydrographic observational data except for the cases of relatively high discharge flow.

Results also showed a linear relation between the rainfall and run-off. However, those which showed to have a non-linear relation were mainly due to the three discharge-flow components and the lag time between the rainfall and the run-off [1,7].

It was found that the recession curve slope is constant for the subsurface and underground flows under different conditions, quite similar to the results arrive at by [1,7]. There was however a considerable variation in the surface run-off from one flooding to the other.

It can be deducted from the findings [figs. 9, 10, and 11] that there was no significant statistical difference between the hydrographic observations and the simulation data and thus a close proximity exists between the two sets of data. The results also showed that on average the peak time and peak discharge for the tank model [figs. 9, 10 and 11] relative to CN method were closer to the peak hour in the observational data. Although these were substantiated by [1], others [i.e., 7] did not reach such conclusion.

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Results [figs.9, 10 and 11] showed a close proximity between the estimated flood volume in the CN method and the data obtained from the hydrographic observations than those in the simple tank model. Although this finding has been substantiated by research elsewhere [1] no major research nonetheless, has yet substantiated it.

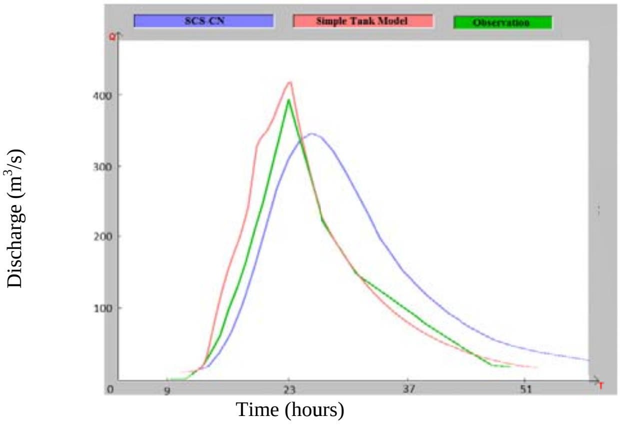

Fig.9. Flood in 1976

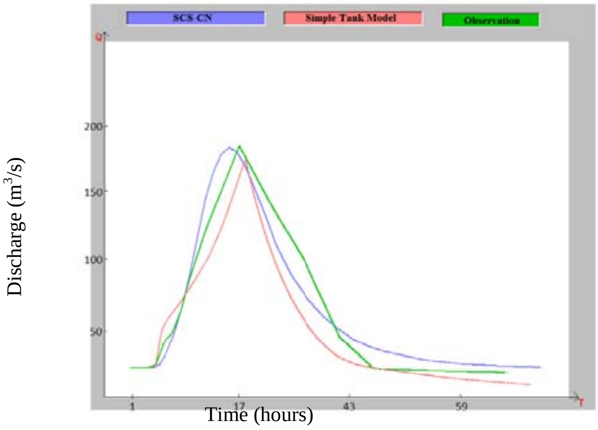

Fig.10. Flood in 1986

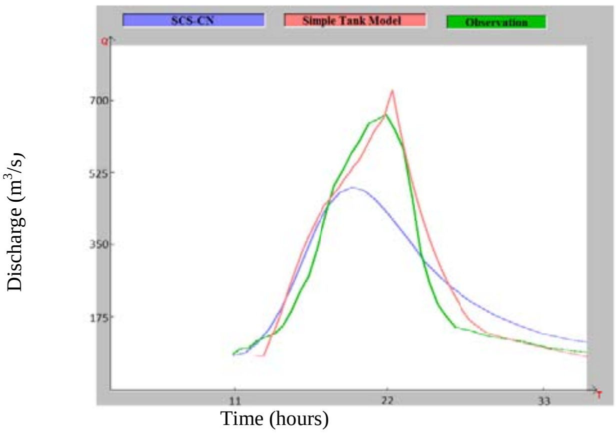

Fig.11.Flood in 2001

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