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Phase Control Array Synthesis Using Constrained Accelerated Particle Swarm Optimization

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***Abstract*—**In this paper, the phase control antenna array synthesis is presented. The problem is formulated as a constrained optimization problem that imposes nulls with prescribed level while maintaining the sidelobe at a prescribed level. For efficient use of the algorithm memory, compared to the well known Particle Swarm Optimization (PSO), the Accelerated Particle Swarm Optimization (APSO) is used to estimate the phase parameters of the synthesized array. The objective function is formed using a main objective and set of constraints with penalty factors that measure the violation of each feasible solution in the search space to each constraint. In this case the obtained feasible solution is guaranteed to satisfy all the constraints. Simulation results have shown significant performance increases and a decreased randomness in the parameter search space compared to a single objective conventional particle swarm optimization.

***Keywords*—**Array synthesis, Sidelobe level control, Constrained optimization, Accelerated Particle Swarm Optimization.

I. Introduction

ADAPTIVE antenna arrays are considered the key to increase the capacity and efficiency of the wireless communication systems. Each element in the antenna array contributes to the total array output power to create a certain radiation pattern. The performance of the adaptive antenna arrays can be altered using digital beamforming techniques [1].

Beamforming or array synthesis is a process of directing the antenna array radiated power in specified sectors called the main beam sectors while attenuating the radiated power in other directions called sidelobes. The interference that may interfere with transmission and reception can occupy narrow or wide beam sectors, in both cases, a null or successive beam nulls have to be introduced in order to minimize the sidelobe levels in that sector relative to the main beam level [2].

Recently with the fast growth of computing devices and digital signal processors, beamforming technique has enjoyed great interest to enhance the performance of wireless systems. The design of the antenna array tends to find the optimal array parameters to achieve the desired radiation pattern.

The antenna array parameters that can be optimized for a given performance are the excitation current magnitude [3], phase [4] and both [5] and the inter-element spacing [12]. The optimization process for a specified performance involves dealing with a min-max convex problem that is usually difficult to solve analytically [7]. Therefore, several numerical techniques such as Gradient based, Nelder-Mead and Newton-like methods have been proposed to deal with array synthesis. The performance of these techniques has been presented in [8]. However, these techniques generally need initial solution that must be close to the optimal parameters, otherwise, it is not generally easy to find these optimal parameters, especially if the function to be optimized is a multimodal and contains many local minima and local maxima.

In order to obtain good estimate of the array parameters, Metaheuristic methods such as Genetic algorithm and particle swarm optimization have been proposed. Particle Swarm Optimization (PSO) [9] and [10], simulates natural behaviors of bees (particles) living in a swarm and searching for flowers. Each particle in the swarm updates its position over time according to the fitness of its previous position, current position and the position of the particle with best fitness in the swarm.

PSO has been applied to array synthesis. In [11], the array phases were optimized to place nulls in specified interfering directions. However, the null depth and the sidelobe level have not been controlled. In [12], the position of the antenna elements is estimated to place nulls in a specified direction while minimizing the sidelobe level. It has also been applied to different array geometry such as in [13] where the relative distance of circular antenna array elements are optimized for a given first null beamwidth and sidelobe level.

Generally, placing a null in a certain direction will increase the sidelobe levels relative to the main beam. Therefore, when a null needs to be placed in a certain direction, the sidelobe level have to be optimized.

To control both nulls and sidelobe level, there will be more than one function to optimize (minimized or maximized), these functions are usually combined into a single fitness function with appropriate scaling factors determined by trial and error [14]. In this case, the obtained solution is not necessary a global minima or maxima that optimize the fitness function and all the constraints, especially in the case of conflicting constrained optimization.

In this paper, the array synthesis problem will be formulated as a constrained optimization problem. The constraints will be added to the problem sequentially and each constraint will be assigned a penalty factor that measures its violation level to a prescribed specification. The phase of the excitation current

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will be considered to control the nulls and their levels, in addition to sidelobe level for a specified array pattern.

Although the PSO has been exploited in array pattern synthesis, to our knowledge, the accelerated particle swarm optimization (APSO) has not been used in array synthesis problem. APSO has been presented in [12] for business optimization and applications and it has shown to have several promising features including the speed of the algorithm, reduced memory requirements in addition to have an increasing deterministic behavior over time which increases the convergence rate compared to the conventional PSO. Therefore, the APSO algorithm will be used in the optimization of the array design presented in this paper.

The rest of the paper is organized as follows, in section II, the array factor of a uniform linear array is presented, then the problem formulation as a constrained optimization is given in section III. In section IV, the accelerated particle swarm technique is explained. Problem is mapped into APSO algorithm via appropriate fitness in section V followed by the by the simulation results. Finally, conclusion is drawn in section VII.

II. Uniform Linear Array

Consider a uniform linear array consisting of N isotropic radiator with fixed inter-element spacing d as shown in Fig. 1.

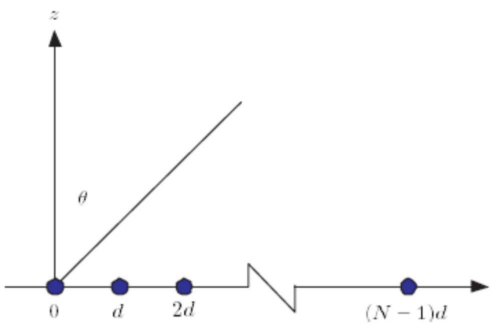

Fig. 1 N- element Linear Array

The array factor is the contribution of all antenna elements at the output of the array [15] and is given by

$$AF(\theta) = \sum_{n=1}^{N} \alpha_n \, e^{j[(n-1)kd \sin \theta + \beta_n]} \qquad (1)$$

where

$\theta =$ *is the angle measured from the z − axis*

$\alpha_n =$ *is the excitation amplitude*

$\beta_n =$ *is the excitation phase*

$k = \frac{2\pi}{\lambda}$ $[rad/m]$

$\lambda =$ *is the wavelength*

$d = \frac{\lambda}{2}$ *is the fixed inter − element spacing*

If the array is symmetric with even number of elements, N as shown in Fig. 2, then the array factor can be simplified as

$$AF(\theta) = 2\sum_{n=1}^{\frac{N}{2}} \alpha_n \cos\left[\left(\frac{2n-1}{2}\right) kd \sin \theta + \beta_n\right] \qquad (2)$$

In Eq. (2) the number of unknown excitation parameters is reduced to instead of , so the problem dimensionality is. Setting and , Eq. (2) can be written as

$$AF(\theta) = 2\sum_{n=1}^{\frac{N}{2}} \alpha_n \cos\left[\pi\left(n-\frac{1}{2}\right) \sin \theta + \beta_n\right] \qquad (3)$$

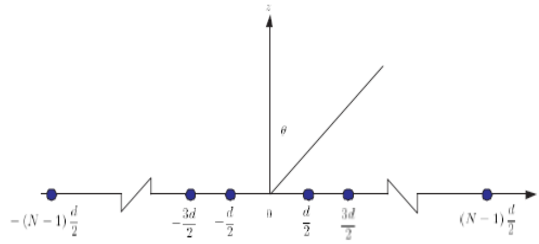

Fig. 2 Symmetric Linear Array

In this paper, the excitation phase β_n, n∈(1,N/2) will be optimized. Therefore, it is assumed that the excitation amplitude $\alpha_n=1, \forall n \in (1, N/2)$. The array factor at any angle θ can be calculated relative to the main beam in decibels, dB as

$$[AF(\theta)]_{dB} = 20\log_{10}\left(\frac{AF(\theta)}{AF(\theta_0)}\right) \qquad (4)$$

where θ_0 is the main lobe direction at which the array factor is maximum.

III. Problem Formulation

Given the array factor in Eq. (3), we need to determine the phase excitation vector, $\beta_n=\{\beta_1, \beta_2, . . . , \beta_{N/2}\}$ that minimizes Eq. (3) at the null directions and the sidelobe levels relative to the main beam.

Assume k nulls located at the directions, $\Theta_n = \{\theta_{n1}, \theta_{n2}, \dots, \theta_{nk}\}$, then the fitness function F to minimize is,

$$F(\boldsymbol{\theta}) = \sum_{i=1}^{k} |AF(\theta_{ni})|^2 - |AF(\theta_0)|^2 \qquad (5)$$

where θ_0 is the direction of the main beam.

To specify a null depth of $NULL_{0k}$ at the k^{th} null direction relative to the main beam, we use the following constraint

$$[AF(\theta_{nk})]_{dB} = 20\log_{10}\left(\frac{|AF(\theta_{nk})|}{|AF(\theta_0)|}\right) \leq NULL_{0k} \qquad (7)$$

where θ_s belongs to the angular sector of the sidelobe regions.

IV. Accelerated Particle Swarm Optimization

Accelerated Particle Swarm Optimization (APSO), is a variation of the conventional Particle Swarm Optimization (PSO) proposed by Kennedy and Eberhart [9]. In the PSO technique a swarm of a certain size, typically (20-30), particle is created. Each particle consists of number of components equal to the problem dimension. The position and velocity of the i^{th} particle with k components are initialized randomly and updated with time. Each particle updates its position and velocity based on the fitness of the individual position, p_{ik} and the group global best position, p_{gk} as follows

$$v_{ik}(n+1) = wv_{ik}(n) + C_1 r_1(n)\big(p_{ik}(n) - x_{ik}(n)\big) + C_2 r_2(n) \quad (8)$$

$$x_{ik}(n+1) = x_{ik}(n) + v_{ik}(n+1) \quad (9)$$

where w is the inertial weight used to stabalize the particle movements. C_1 and C_2 are the social parameters, usually set as $C_1 + C_2 \leq 4$ [6]. r_1 and r_2 are random constants selected from a uniform distribution, $r_1, r_2 \sim U[0,1]$. n is the discrete time for the update. The purpose of using the second term in Eq. (8) which contains the previous personnel best position, p_{ik}, is to provide the ability to pull the particle back to its previous position which increases diversity in the solution. Therefore, it can be replaced by introducing some randomness in the update relations [14]. In this case, there is no need for algorithm memory to keep the previously explored individual best position.

APSO algorithm looks similar to that of the PSO except in the previous best position term and the position update. The position of the i^{th} particle at time $n+1$ is updated as,

$$x_{ik}(n+1) = (1-\gamma)x_{ik}(n) + \gamma p_{gk} + \eta r \quad (10)$$

the parameter γ is selected in the range $\gamma \in (0.1\ ,\ 0.7)$, as γ increases particle is encouraged to move toward the position of the best particle in the swarm, while as γ decreases the particle tends to remain in its position. The last term in Eq. (10) is to give some randomness especially in the first few iterations to increase the diversity of the solution. Therefore, to increase the convergence, η is selected as exponentially decreasing function of the discrete time n, $\eta = \eta_0^n$, $\eta_0 < 1$ and $r \sim U[0,1]$. It should be noted from Eq. (10) that there is no velocity term in the equation which simplifies the update process. The pseudo-code of the APSO is shown in algorithm 1.

Algorithm 1: Accelerated Particle Swarm Optimization

```
repeat
    for i = 1 to PopSize do
        compute fitness(x_i)
    end
    compute global best: P_gk = arg min_{x_ik}(fitness);
    for i = 1 to PopSize do
        PositionUpdate:
        x_ik(n+1) = (1 - γ) x_ik(n) + γ p_gk + η r
    end
until Maximum iteration or termination condition is met ;
```

V. Fitness Function

The fitness (cost) function to be optimized using the APSO algorithm to estimate the phase excitation vector, $\boldsymbol{\beta}_n$, $n \in (1, N/2)$ is the combined function in Eq. (5) and the constraints in Eq. (6) and Eq. (7). The problem can be stated as

$$\begin{aligned} &Minimize: \quad F(\beta_n) \\ &\quad Subject\ to: \ [AF(\theta_{nk})]_{dB} \leq NULL_{0k} \\ &\qquad\qquad [\max\{SLL(\theta_s)\}]_{dB} \leq SLL_0 \end{aligned} \quad (11)$$

Eq. (11) is a constrained optimization problem that can be solved using either the solution feasiblity preservation technique in which only the feasible solution that satisfies all the contrtaints is preserved during iterations [16] and [17], or using the penalty functions technique where each constraint is weighted by a penalty factor to emphasize its violation to a prescribed value, in this case, the problem can be transformed into unconstrained optimization problem [16] and [18]. It has been shown in [19], that the constraints handling using penalty functions acheives faster convergence rate.

In [19], the penality functions are classified into two catogories, the first is the static penality method which assigns a fixed penality to the violated constraint regardless of the violation value. A more effective technique, is to penalize the constrains according to its violation value through scaling its value by a fixed penaliy factor. The later technique will be used in this paper.

To convert Eq. (11) into unconstrained optimization problem, the quadratic augmented fitness function $\Phi(\beta_n)$ is introduced as

$$\Phi(\beta_n) = F(\beta_n) + cP(\beta_n) \quad (12)$$

where $P(\beta_n)$ is the penalty function given by

$$P(\beta_n) = \left(\max\{0, [\max\{SLL(\theta_s)\}]_{dB} - SLL_0\}\right)^2 + \sum_{i=1}^{k}\left(\max\{0, [AF(\theta_{ni})]_{dB} - NULL_{0k}\}\right)^2$$

c is the penalty parameter selected to be a very large number to avoid that $F(\beta_n)$ from dominating the fitness function in Eq. (12), otherwise minimizing the fitness will not be the optimal solution, β_n, that satisfies all the constraints. The $\max\{.\}$ operator is used such that the satisfied constraint is excluded from the overall fitness function.

VI. Simulation Results

A. The first design scenario is to synthesize the array with 20 elements. The main beam is at $\theta_0 = 0^o$, null is placed at an angle $\theta_{n1} = -14.3^o$ which corresponds to the second peak of the uniform weighting - zero phase array pattern, ($\alpha_n = 1$, $\beta_n = 0$). The required null depth is $NULL_{01} = -50\ \ dB$ and without sidelobe level control. The APSO is invoked with $\gamma = 0.3$, $\eta_0 = 0.8$, $Swarm\ size\ of\ 20$ and 150 iterations. The excitation phase vector to be estimated is $\boldsymbol{\beta}_n = \{\beta_1, \dots, \beta_{10}\}$. The generated array pattern is shown in Fig. 3.

It can be seen from Fig. 3, that the null depth is $-50.2\ \ dB$ which is nearly the same as specified. The maximum sidelobe level is $-13.3\ \ dB$ which is approximately similar to that of the uniform weighting array with radiation pattern plotted on the same graph.

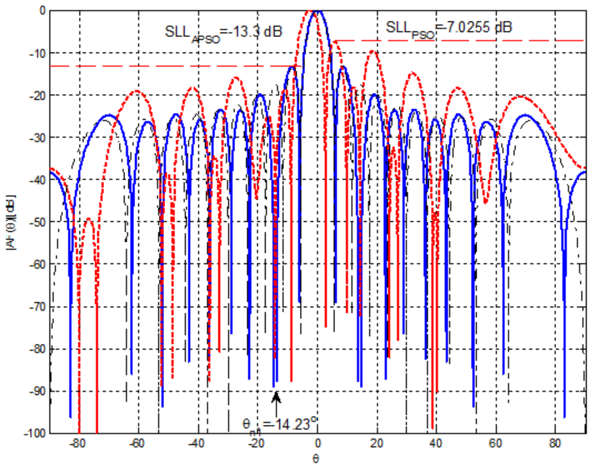

Fig. 3 20 $elements\ array$: radiation pattern using APSO (solid) vs. PSO (dashed) and uniform (dashed-dotted)

To compare the performance of the APSO to that of the PSO. The PSO is invoked with fitness proposed in [20] for 150 iterations,

$$Fitness_{PSO} = \sum_{i=1}^{k} |AF(\theta_{ni})|^2 + \sum_{\theta_s} \frac{1}{\Delta\theta_s} \int_{\theta_{\ell s}}^{\theta_{us}} |AF(\theta_s)|^2 \, d\theta_s \quad (14)$$

$\Delta\theta_s$ is the angular sector where the sidelobe is to be suppressed, $\Delta\theta_s = \theta_{us} - \theta_{\ell s}$ and θ_{ni} is the i^{th} null direction.

The obtained result have shown significant variations in the estimated parameters and the optimal fitness values. Therefore, the estimated parameters are averaged over 500 runs and the result is plotted in Fig 3. As it can be seen from the radiation pattern plot, the null is placed at the correct location but the main beam is shifted by 3^o to the left, in addition to a relatively large sidelobe level at $-7\ \ dB$.

B. The mean Fitness function is computed for both techniques with iterations. As it can be seen from Fig. 4 that the APSO technique (solid) converges faster with less oscillations than that of the PSO technique (dashed) as a result of the particles being affected by the best particle in the swarm regardless of their personel best postion.

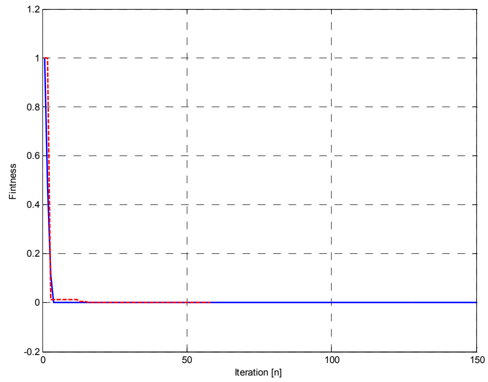

Fig. 4 20 $elements\ array$: APSO (solid) and PSO (dashed) convergence rate

C. The second design scenario is to control the null and the sidelobe level. The sidelobe level is set $SLL_0 = -20\ \ dB$, null with same specifications as in the first scenario. Fig. 5, shows the obtained radiation pattern with null depth $-50\ \ dB$ and sidelobe level of $-20.5\ \ dB$, the first null beamwidth is at $\pm 5.96°$ with depth $-70\ \ dB$, close to uniform weighting first null of $\pm 5.7°$. To compare with PSO, the same specifications are used. The radiation pattern is shown in Fig. 6, the null depth is $-77\ \ dB$ and sidelobe level is optimized to $-17.3\ \ dB$ with first null beamwidth $\pm 6.8°$.

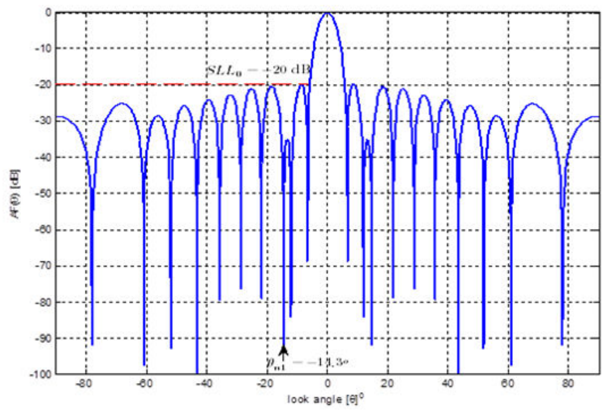

Fig. 5 APSO with optimized null and sidelobe level

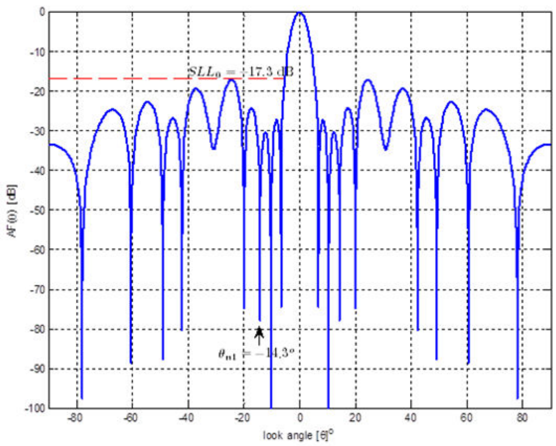

Fig. 6 PSO with optimized null and sidelobe level

D. Finally, the third scenario is to test the algorithm with more than one null. A null angles of $\theta_{n1} = -14.3^o$, $\theta_{n2} = 10^o$ with null depths $NULL_{01} = NULL_{02} = -70\ dB$ are specified. The sidelobe level is set to $SLL_0 = -20\ dB$. The radiation pattern is generated using the phases determined by the APSO with same parameters used in the above two scenarios. Fig. 7, shows the APSO pattern with null placed at the desired directions with attenuation of -80, $-78\ dB$, respectively, while the maximum sidelobe is maintained at $-20\ dB$. The obtained radiation pattern is compared with uniform weighting pattern using same number of antenna elements, i.e. 20, to show the superimposed nulls. The estimated phase shifts in degrees for the APSO in the three design scenarios are shown in Table I.

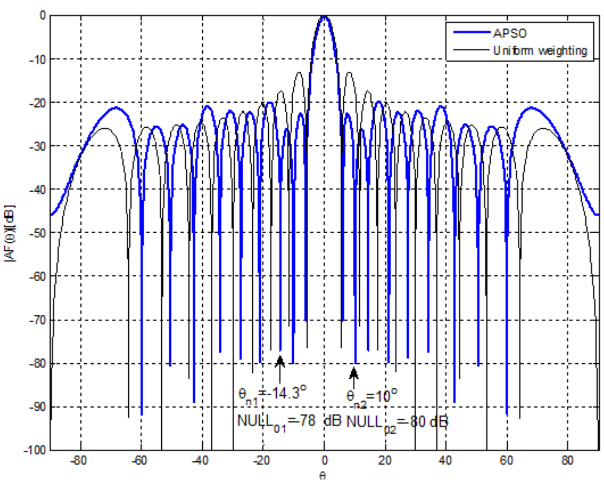

Fig. 7 APSO radiation pattern (solid) with $\theta_{n1} = -14.3^o$, $\theta_{n2} = 10^o$ and $SLL_{max} = -20\ dB$ and the uniform weighting (dashed)

TABLE I
APSO ARRAY ESTIMATED PHASES

Phases	Scenario 1	Scenario 2	Scenario 3
β1	-7.6168	-1.7647	-2.6986
β2	-13.7979	-0.4183	1.6730
β3	-11.8411	-0.0688	3.2830
β4	-5.0774	4.2743	6.3484
β5	3.4948	13.0348	11.2185
β6	11.2955	15.9511	16.2434
β7	14.8886	22.5402	8.3881
β8	9.0160	7.6203	1.8048
β9	-5.4139	-11.3388	-23.9382
β10	-13.8919	-27.1066	-34.6353

As it can be seen from the simulation results, the proposed fitness function with appropriate constraints gives an improved radiation pattern compared to that proposed in [9] for the PSO algorithm. Although the APSO used in the estimation process does not use the personal best position of the particles, so there is no need to keep them in memory, the results have shown a performance increase over that of the PSO and an improved deterministic behavior over time that gives almost the same estimated parameters over many repeated experiments. Therefore, the use of the APSO in the above setup is considered an advantage due to an efficient use of the algorithm memory.

VII. CONCLUSION

This paper presents the formulation of a constrained phase control array synthesis. Null directions with pre specified levels were imposed on the radiation pattern while maintaining sidelobe level at a desired value. The presented method uses the accelerated particle swarm optimization for efficient algorithm memory allocation. The method used the penalty functions constrained optimization. Simulation results have shown a performance increase over the single objective particle swarm optimization proposed in literature which uses the algorithm memory inefficiently.

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