A Intelligent Inference Model about Complex Systems’ Stability: Inspiration from Nature

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Abstract—A logic model for analyzing complex systems’ stability is very useful to many areas of sciences. In the real world, we are enlightened from some natural phenomena such as “biosphere”, “food chain”, “ecological balance” etc. By research and practice, and taking advantage of the orthogonality and symmetry defined by the theory of multilateral matrices, we put forward a logic model of stability of complex systems with three relations, and prove it by means of mathematics. This logic model is usually successful in analyzing stability of a complex system. The structure of the logic model is not only clear and simple, but also can be easily used to research and solve many stability problems of complex systems. As an application, some examples are given.

Keywords—Complex system, logic model, relation, stability.

I. INTRODUCTION

THE logic analysis of stability is a concerned problem of many knowledge branches. Why can an atom be a comparatively stable system relatively to the substances world [1]? Why can a cell be the basic unit that makes a living thing? Why can an aggregate model of biological gene form a stable structure? Why can a special structure of gene be the regular cause making a genetic disease? What reason can make a special working procedure of a factory into the stable working procedure [2,3]? Like this, and so on, the problems of intelligent reasoning of stability are usually encountered, but how to define the logic analysis structure of stability; the views of different scholars are different from each other. In the real world, we are enlightened from some concepts and phenomena such as “biosphere”, “food chain”, “ecological balance” etc. With research and practice, by using the theory of multilateral matrices [4] and analyzing the conditions of symmetry [5] and orthogonality [6-8] what a stable system must satisfy, in particular, with analyzing the basic conditions [9,10] what stable working procedure of good product quality must satisfy, we are inspired and find some rules and methods, then present the logic model for analyzing the stability of a complex system. This logic model is usually successful in analyzing stability of a complex system. The structure of the logic model is not only clear and simple, but also can be easily used to research and solve many stability problems of complex systems.

This paper is structured as follows. Section 2 defines the concept of logic analysis model with three relations (Definition 3), and proves three basic properties of it. Section 3 builds the logic analysis model of stability (Definition 4), and presents three theorems about the model with proving them. In section 4, five examples are given to illustrate their simple applications about the concept and the model presented above. Finally, section 5 summarizes the paper, and conclusions are given.

II. A LOGIC ANALYSIS MODEL WITH THREE RELATIONS

A. Definitions of Three Relations and Logic Analysis Model

Definition 1. Let set $A \neq \emptyset$, and ~ be a relation on $A$. Then ~ is called an equivalence relation on $A$, if and only if $\forall x, y, z \in A$, satisfy:

1. $x \sim x$;
2. If $x \sim y$, then $y \sim x$;
3. If $x \sim y, y \sim z$, then $x \sim z$.

That is, ~ is reflexive, symmetric and conveyable.

Definition 2. Let set $A \neq \emptyset$, and $\Rightarrow$ and $\Rightarrow$ are two different relations on $A$. Then $\Rightarrow$ and $\Rightarrow$ is called a neighboring relation and a alternate relation on $A$ respectively, if and only $\forall x, y, z \in A$, satisfy:

1. First triangle reasoning (transition reasoning)
   (1). If $x \Rightarrow y, y \Rightarrow z$, then $x \Rightarrow z$, i.e. $\Rightarrow$ meets with developing transition phenomenon;
   (2). If $x \Rightarrow y, x \Rightarrow z$, then $y \Rightarrow z$;
   (3). If $x \Rightarrow z, y \Rightarrow z$, then $x \Rightarrow y$.

2. Second triangle reasoning (atavism reasoning)
   (1). If $x \Rightarrow y, y \Rightarrow z$, then $z \Rightarrow x$, i.e. $\Rightarrow$ meets with developing atavism;
   (2). If $z \Rightarrow x, x \Rightarrow y$, then $y \Rightarrow z$;
   (3). If $y \Rightarrow z, z \Rightarrow x$, then $x \Rightarrow y$.

The First triangle reasoning (transition reasoning) and the Second triangle reasoning (atavism reasoning) can be represented by the following Fig. 1, where to every triangle, any two sides determine the third side.
Similarly, we can prove that there are not both $x \Rightarrow y$ and $y \Rightarrow x$, both $x \Rightarrow y$ and $y \Rightarrow x$ simultaneously. The proof is complete.

**Property 2.** $\forall x, y, z \in V$, if $x \Rightarrow y$, $x \Rightarrow z$; then $y \Rightarrow z$; similarly, if $x \Rightarrow y$ and $x \Rightarrow z$, then $y \Rightarrow z$.

**Proof.** We adopt disproved method. Taking advantage of conditions $x \Rightarrow y$ and $x \Rightarrow z$, concerning the relation between $y$ and $z$. If $y \Rightarrow z$, then we can obtain $x \Rightarrow z$ by transition reasoning, but $x \Rightarrow z$ contradicts $x \Rightarrow z$. If $y \Rightarrow z$, then $z \Rightarrow x$ by atavism reasoning, this contradicts $x \Rightarrow z$. Similarly, we can prove that there are not both $z \Rightarrow y$ and $z \Rightarrow y$, therefore, $y \Rightarrow z$.

It is the similar process to prove another half of property 2.

**Property 3.** $\forall x, y, z \in V$, if $x \Rightarrow z$, $y \Rightarrow z$, then $x \Rightarrow y$; similarly, if $x \Rightarrow z$ and $y \Rightarrow z$, then $x \Rightarrow y$.

**Proof.** It is similar to that proof like property 2.

### III. Logic Analysis Model of Stability

#### A. Steady Logic Analysis Model

**Definition 4.** A logic analysis model is said to be steady, if at least for one of $\Rightarrow$ and $\Leftrightarrow$, such as $\Rightarrow$, there is a cycle chain (or causal circle) like the following form:

$$x_1 \Rightarrow x_2 \Rightarrow x_3 \Rightarrow \cdots \Rightarrow x_n \Rightarrow x_1$$

The definition given above, for a relatively stable system, is most essential. If there is not the chain or circle, then there will be some elements without causes or some elements without results in a system. Thus, this system is to be in the state of finding its results or causes, i.e. this system will fall into an unstable state, and there is not any stability to say.

From the stable logic analysis model of complex systems, we can obtain several interesting consequences given below.

**B. Three Important Theorems**

**Theorem 1.** In a stable logic analysis model, there must be the cycle chain that its length is five, and there is not the cycle chain that its length is less than 5.

**Proof.** The only need is to prove the three cases given below:

1. There are not the cycle chains: their length is 1, 2, 3 or 4;
2. There is a cycle chain that length is five;
3. For a stable logic analysis model, there must be a cycle chain that length is five.

Three cases given above are proved as follows:

1. Obviously, $x_1 \Rightarrow x_2 \Rightarrow x_3 \Rightarrow x_4 \Rightarrow x_5 \Rightarrow x_1$ are all impossible. Assume that there is $x_1 \Rightarrow x_2 \Rightarrow x_3 \Rightarrow x_4 \Rightarrow x_5$, we can obtain $x_1 \Rightarrow x_2 \Rightarrow x_3$ by transition reasoning and $x_5 \Rightarrow x_1$ by atavism reasoning. This result contradicts Property 1.
2. For the cycle chain whose length is 5: $x_1 \Rightarrow x_2 \Rightarrow x_3 \Rightarrow x_4 \Rightarrow x_5 \Rightarrow x_1$, can infer that $x_1 \Rightarrow x_2 \Rightarrow x_3 \Rightarrow x_4 \Rightarrow x_1$, there is not any contradiction here.
3. For any one of stable logic analysis models, by Definition 4, there is a cycle chain like this: \( x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow \cdots \rightarrow x_n \rightarrow x_1 \).

By proving step 1, we know that \( n \geq 5 \).

If \( n = 5 \), then case 3 has been proved.

If \( n > 5 \), then \( x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow \cdots \), so we can obtain \( x_i \Rightarrow x_{i+1} \) by transition reasoning and obtain \( x_i \rightarrow x_{i+1} \) by atavism reasoning. Therefore, we have \( x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow x_1 \). The proof is complete.

From proving Theorem 1, we can know that there are two different cycle chains, whose length is five simultaneously. That is, cycle chains \( x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow x_1 \) and \( x_1 \Rightarrow x_2 \Rightarrow x_3 \Rightarrow x_4 \Rightarrow x_5 \Rightarrow x_1 \) appear together at the same time.

**Theorem 2.** To any one of stable logic analysis model \( V \), we can divide all elements of \( V \) into 5 categories: \( V_1, V_2, V_3, V_4, V_5 \), in which \( V_i \cap V_j = \emptyset \), and \( \bigcup V_i = V \), \( i = 1, 2, \ldots, 5 \). Elements in the same category are equivalence each other, and there is the relation \( \Rightarrow \) or \( \Rightarrow \) between this category \( V_i \) and that category \( V_j \).

**Proof.** To any \( V \), by Theorem 1, there is a cycle chain as follows:

\[ x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow x_1 \]

Let \( V_i = \{ x : x \sim x_i, x \in V \}, i = 1, 2, \ldots, 5 \). Firstly, we will prove \( V_i \cap V_j = \emptyset \) by using disease of disproving. If \( V_i \cap V_j \neq \emptyset \), then \( \exists x \in V_i \cap V_j \), make \( x \sim x_i, x \sim x_j \), therefore, \( x_i \sim x_j \), leading to contradiction.

Secondly, we will prove \( \bigcup_i V_i = V \), i.e. \( \forall x \in V \), \( \exists x_i \), make \( x \sim x_i \). We know that there must be one of 5 relations

\[ x \sim x_i, x \rightarrow x_i \sim x \Rightarrow x_i \Rightarrow x \] between \( x \) and \( x_i \). If \( x \sim x_i \), then the proof is complete; if \( x \rightarrow x_i \), in addition, \( x_i \rightarrow x \), then \( x \sim x_i \); if \( x \Rightarrow x_i \), in addition, \( x_i \Rightarrow x \), then \( x \sim x_i \). Similarly, other cases can be proved too.

Theorem 2 indicates that we can research stability of a complex system with 3 relations by researching its 5 equivalence categories.

**Theorem 3.** To any logic analysis system \( V \) with 3 relations \( \sim, \rightarrow \) and \( \Rightarrow \), dividing its elements into categories according to equivalence relations, uniquely stable architecture is shown as follows (Fig. 2):

![Fig. 2 Uniquely stable architecture](image-url)

These theorems have very important significance. Please look at several examples given below.

### IV. Examples

**A. Example 1**

**Example 1.** In an on-line control system of product quality, both different relations—working procedure and management are considered generally. For example, let \( \rightarrow \) be working procedure, \( \Rightarrow \) be managing procedure. The on-line control system given below can be adopted:

Assume that \( x_1, x_2, \ldots \), etc. are inspection points of working procedure or managing procedure, then \( x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow \cdots \). Where, \( x_i \rightarrow x_{i+1} \) is called flow section of adopting \( i \)th working procedure. Suppose that, according to design, substandard products rate of each section is \( q \). Assume that the inspector discovers that there may be problems at \( x_1 \rightarrow x_3 \), then manager wants to inspect that weather working procedure section \( x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow \cdots \rightarrow x_6 \) is to be in a stable producing state. Then he or she can take an inspection of substandard products rate at \( x_2 \), recording it as \( q_1 \), and take an inspection of substandard products rate at \( x_4 \), recording it as \( q_2 \), in addition, take another inspection of the rate at \( x_6 \), recording as \( q_3 \).

Note that:

\[ r_i = 1 - \sqrt{(1-q_i)/(1-q_{i-1})} \]

\[ r_i > q \] will shows that there may be problems at working procedure section \( x_1 \rightarrow x_2 \rightarrow x_3 \). But there may be errors in inspection, so we inspect continuously. If finding \( r_i > q \) in the next inspection, then it is reasonable to think that there are some problems in working procedure section \( x_1 \rightarrow x_2 \rightarrow \cdots \rightarrow x_6 \), and the quality problems may probably be located at section \( x_2 \rightarrow x_3 \rightarrow x_4 \). Thus, productions or half-productions need to return from \( x_6 \) to \( x_2 \) to reproduce.

From the above analysis, to the above quality inspection management, \( \rightarrow \) can be understood as an error of some working procedure inspections, \( \Rightarrow \) as an error that found by some above inspectors, thinking the reasoning below (Fig. 3) reasonable:

![Fig. 3 Reasoning of Example 1](image-url)

The above reasoning rules form the inspection to stability of producing. The rules may be expressed as: “The same mistake can be permitted one or two times but three or four times.”
Example 2. In system design of dependability, people consider a risk function $\lambda(k)$ and scalar variable $r$:

$$\lambda(k) = \frac{P_i}{\sum_{i=k}^{P_i} P_i} = \frac{P_i}{P_i - P_{i+1}}$$

where $P_i$, $P_2$, $\cdots$, $P_n$ are scattered probability-distribution-density, and $n$ is the product life-span. Analyzing the state of products, we can see that they are to be in flow as follows: producing stage: $A_i : \lambda(0) = 0$; early stage: $B_i : 0 < \lambda(k) < 1, r < 0$; accidental stage: $C_i : 0 < \lambda(k) < 1, r = 0$; breakage stage: $D_i : 0 < \lambda(k) < 1, r > 0$; life-span stage: $E_i : \lambda(k) = 1$; another producing stage: $F_i : \lambda(0) = 0$; another early stage: $G_i : 0 < \lambda(k) < 1, r < 0$; ...

Although for designers of a product and for inspectors of product dependability the producing stages is the stage what they pay careful attention to together, the designers may be more concerned with design of accidental stage and life-span stage, and the inspectors may be more concerned with testing to early stage and breakage stage. We can regard the relation between the same kind of stage, for example, between $A_i, A_i^{*\cdots}$, as $\sim$, and the relation between the two continuous stage, for example, between $A_i, B_i^{\cdots}$, as $\Rightarrow$, and with the relation between two alternate stages, such as $B_i, D_i^{\cdots}$ as $\Leftrightarrow$. Obviously, the above system forms a stable logic analysis system, and it satisfies Theorem 1 to 3.

Example 3. Assume that $F=\{X_1, X_2\}$:

$0 \leq x_1 \leq 1, 0 \leq x_2 \leq 2$ is a plane rectangle, translation is regarded as $\sim$. Define that $\rightarrow$ is to turn $F \pi/5$ ant clockwise, that is:

$$x \rightarrow y : y = \begin{pmatrix} \cos \frac{\pi}{5} & -\sin \frac{\pi}{5} \\ \sin \frac{\pi}{5} & \cos \frac{\pi}{5} \end{pmatrix} x, x = (x_1, x_2)$$

In addition, define that $\Rightarrow$ is to turn $F 4 \pi/5$ ant clockwise, i.e.

$$x \Rightarrow y : y = \begin{pmatrix} \cos \frac{3\pi}{5} & -\sin \frac{3\pi}{5} \\ \sin \frac{3\pi}{5} & \cos \frac{3\pi}{5} \end{pmatrix} x, x = (x_1, x_2)$$

Going through the functions of $\sim$, $\rightarrow$, $\Rightarrow$, original plane rectangle will become many plane rectangles in the plane. All of them form a symmetric plane graph. Let $A=0, B=2\pi/5, C=4\pi/5, D=8\pi/5$, then it is correct to reason as fellows (Fig. 4):

<table>
<thead>
<tr>
<th>A</th>
<th>A</th>
</tr>
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<tbody>
<tr>
<td>↓</td>
<td>↘</td>
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<tr>
<td>C← B</td>
<td>C→ D</td>
</tr>
</tbody>
</table>

Fig. 4 Reasoning of Example 3

Example 4. Ancient Chinese theory “Yin Yang Wu Xing” [11] has been surviving for several thousands of years without dying out, proving it reasonable to some extent. If we regard $\sim$ as the same category, neighboring relation $\rightarrow$ as consistency and alternate relation $\Rightarrow$ as conflict, then the above defined logic analysis model of stability is consistent with the logic architecture of reasoning of “Yin Yang Wu Xing”. Yin and Yang mean that there are two opposite relations in the world: consistency $\rightarrow$ and conflict $\Rightarrow$, as well as general equivalence category $\sim$. There is only one of three relations $\sim$, $\rightarrow$, $\Rightarrow$ between every two objects. Everything makes something, and is made by something; everything restrains something, and is restrained by something; i.e. one thing is overcome by another thing. The ever changing world, following the relations, must be divided into five categories by equivalence relation, being called “Wu Xing”: wood, fire, earth, gold, water. The “Wu Xing” is to be “neighbor is friend”, and “alternate is foe”. We can see, from this, the ancient Chinese theory “Yin Yang Wu Xing” is a reasonable logic analysis system to stability of complex systems.

Example 5 A object is launched, with its elevation $\alpha$ (degree), and its mass $m$(kg), and momentum $G$(kg·m/s), then distance that it can arrive there in level is:

$$y = \frac{1}{g} \frac{G}{m} \sin 2\alpha = f(G, m, \alpha)$$

where $m=1.0kg$, $g=9.8m/s^2$(acceleration of gravity). When launching, $m$, $G$ and $\alpha$ go up and down within ranges of $\Delta m=0.01m, \Delta G=0.02G, \Delta \alpha=0.05\alpha$. The value of $G$ and $\alpha$ are unknown. The question is: what are the value of $G$ and $\alpha$ that is $f$. In system design of dependability, people find the stable center. The example demonstrates that this system forms a stable logic analysis model too.

Example 6 The above problem is a usually model in control area of missiles, with having important worth in theory and practice. There are many similar problems in many domains such as economic management and prediction, products quality control, stability of working procedure, online automatic control, physics, chemistry and biology, etc. The kind of problems is called the problem of stable center of complex systems. Although there is a lot of that kind of problems, there are few accurate mathematical models to use, and few good methods to find the stable center.

Finding the stable center of a complex system is a important problem on data analysis. In general, people now select the
stable center of a complex system by firstly selecting some criteria for judging the stability, then finding the most optimal criteria of stability. However, different school of thought selects different stability criteria, so that different stability criteria bring about different stability center. For improving the above problem, we present the above logic model to help people analyzing stability of complex systems, and proving that its cycle chain length of the stable structure is five. From this, we obtain a novel method analyzing the stable center of a complex system. This method needs only five criteria, and if we can make them the most optimal, then we will find its stable center.

Suppose that the value y of a complex system can be expressed as:

\[ y = f(x_1, x_2, \ldots, x_m) + \varepsilon \]

where \( f(x_1, x_2, \ldots, x_m) \) is a polybasic function, \( x_j \in [a_j, b_j] \), \( j = 1, 2, \ldots, m \), \( \varepsilon \) is a random variable. Suppose that mb is the goal value.

\( \forall x^0 = (x_{1}^0, x_{2}^0, \ldots, x_{m}^0) \in [a_{i}, b_{i}] \), set a permitted error \( \Delta x_i = \Delta x_2, \ldots, \Delta x_m \). Suppose that \( x_j = (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m) \) are some points in \( [x_j^0 - \Delta x_j, x_j^0 + \Delta x_j] \), satisfying \( |x_{i+2}^j - x_{i+1}^j| = |x_{i+1}^j - x_{ij}^j| \). We call \( x^0 \) as the center point of experiments, and \( x_j \) as the sequence points of experiments. If \( f \) is known by us, then we can obtain \( y_i = f(x_1, x_2, \ldots, x_m) \), \( i = 1, 2, \ldots, n \), through putting \( x_j \) into formula and computing them. If we don’t know \( f \), then we can make some experiments at \( x_j \) and get the observation value \( y_i \) of \( y = f(x_1, x_2, \ldots, x_m) \).

Thus, we just obtain the experiment data \( y_1, y_2, \ldots, y_n \) nearby the center point \( x^0 \) of experiments.

We select five criteria below to carve and paint this system’s stability at \( x^0 \):

1. \( x^2 \)-identification criterion. These observation values \( y_1, y_2, \ldots, y_n \) from experiments must sufficiently identify or include those information at point \( x^0 \). We have known that if orthogonal design method is adopted, then \( x^2 \) just will reach the most optimal;

2. \( \mu = f(x^0) \) — position criterion;

3. \( \sigma^2(x^0) = \frac{1}{n-1} \sum_{i=1}^{n} \left[ f(x_1, x_2, \ldots, x_m) - \mu \right]^2 \) — fluctuation criterion, representing this system’s fluctuation at point \( x^0 \);

4. \( R^2(f) = \frac{1}{n} \sum_{i=1}^{n} \left[ f(x_1, x_2, \ldots, x_m) - mb \right] \) — risk criterion;

5. \( SN = \frac{(E y)^2}{\sigma^2} \) — signal noise ratio criterion.

The five criteria and relations between them form a stable logic analysis system (fig. 5). Its stability at point \( x^0 \) can be controlled by using the above 5 criteria.

In fig. 5, \( \leq \) can be understood as positive function, and \( \Rightarrow \) can be understood as negative function. For helping reader comprehension, a example in true world is given in fig. 6. It is too simple and easy to explain more. Where, M and W express a Man and a Woman, respectively, while \( \rightarrow \) and \( \Rightarrow \) express “love” and “kill”, respectively.

![Fig. 5 The five criteria and relations between them form a stable logic analysis system](image-url)

Our purpose is finding a test center point \( x^0 \) in ranges of \( \prod_{j=1}^{n} [a_j, b_j] \), at this point, with satisfying that:

\[ \mu = f(x^0) = mb, \quad \sigma^2(x^0) = \min_{x_1, x_2, \ldots, x_m} \sigma^2(x), \quad SN = \max_{x_1, x_2, \ldots, x_m} SN(x) \]

We call the point \( x^0 \) stable center or stable point of this complex data system about target design. Like this, we just give a statistical model of the stable center of a complex data system.

In stability experiments of launched objects and experiment designs of products, the new logic analysis model have been already successfully applied many times, with efficiently reducing testing times and bring us many benefits.

V. Conclusion

In this paper, with enlightening from nature, we present a new logic model of intelligent reasoning for analyzing stability of a complex system, and we prove it by means of the mathematics. In the meantime we illustrate the applications of the logic model by using five examples.

The logic model presented by us has been already applied in some areas. For example, in the experiment design and in the analysis of stability of a weapon factory’s products, we have used the logic model with reducing the test times, promoting the stability of products, and deriving many economical benefits. Its application practice shows that the logic model is very much effectual to analyzing stability of a complex system. The logic model has very wide uses. Consequently, we can believe that it would bring many benefits for us. Its application algorithm of the logic analysis model will be written in another paper by the authors after.
REFERENCES


