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Analysis of Wave Propagation in Two-dimensional Phononic Crystals with Hollow Cylinders

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***Abstract*—**Large full frequency band gaps of surface and bulk acoustic waves in two-dimensional phononic band structures with hollow cylinders are addressed in this paper. It is well-known that absolute frequency band gaps are difficultly obtained in a band structure consisted of low-acoustic-impedance cylinders in high-acoustic-impedance host materials such as PMMA/Ni band structures. Phononic band structures with hollow cylinders are analyzed and discussed to obtain large full frequency band gaps not only for bulk modes but also for surface modes. The tendency of absolute frequency band gaps of surface and bulk acoustic waves is also addressed by changing the inner radius of hollow cylinders in this paper. The technique and this kind of band structure are useful for tuning the frequency band gaps and the design of acoustic waveguides.

***Keywords*—**Phononic crystals, Band gap, SAW, BAW.

I. Introduction

THE studies of the photonic crystals[1,2] have led to a rapidly growing interest in the analogous acoustic effects in phononic crystals or periodic elastic structures. The plane-wave expansion (PWE), finite-difference time-domain, and multiple-scattering theory are the the well-known methods used to analyze the investigations on frequency band gaps of bulk acoustic waves (BAW) in composite materials or phononic band structures [3-5]. For the PWE method, the dispersion relations and the frequency band-gap feathers of the transverse and mixed polarization modes have been studied. Except for the PWE method, the layered multiple scattering theory was applied to study the frequency band gaps of bulk acoustic waves in three-dimensional periodic acoustic composites and the band structure of a phononic crystal consisting of complex and frequency-dependent Lame′ coefficients [6-8]. In addition, the finite-difference time-domain method was applied to predict exactly the transmission properties of slabs of phononic crystals and to interpret the experimental data of two-dimensional systems [9]. Recently, Sun and Wu[10] investigated and analyzed the mode coupling in joined parallel phononic crystal waveguides using the finite-difference time-domain method with periodic boundary condition.

On the other hand, the frequency band-gap features of surface modes are studied mainly using PWE method [11-13]. Tanaka *et al*.[11] proposed the theory of surface waves propagating in two-dimensional phononic crystals consisting of two cubic materials in square lattice, and also explained the stop band distribution of the surface, pseudosurface, and bulk waves using the PWE method. Recently, Wu *et al*.[12,13] extended the theory to describe the phononic crystals consisting of materials with general anisotropy, and Wu and Huang[14] discussed the level repulsion of bulk acoustic waves in periodic composite materials. For considering the piezoelectric materials, Wu *et al*.[15] and Laude *et al*.[16] conducted the band gaps and electromechanical coupling coefficient of a surface acoustic wave in two-dimensional phononic band structures.

As we know, the techniques for tuning frequency band gaps of elastic/acoustic waves in phononic crystals are very important and they are still the excited research topics in physics community. Filling fraction, rotation of noncircular rods, different cuts of anisotropic materials, and temperature effect were discussed clearly to obtain large frequency band gaps of BAW modes in periodic structures. Among the literature materials, it is well-known that absolute frequency band gaps are difficultly obtained in a band structure consisted of low-acoustic-impedance cylinders in high-acoustic-impedance host materials such as PMMA/Ni band structures. The purpose of this study is to obtain large full frequency band gaps not only for bulk modes but also for surface modes in a two-dimensional phononic crystal consisted of PMMA hollow cylinders in Ni host materials. On the other hand, very low frequency band gaps are also discussed in the vacuum/Si phononic band structures with hollow cylinders. The plane-wave expansion method is adopted in this paper. By changing the inner radius of hollow cylinders in the example, the frequency band gaps of SAW and BAW are tuned and are larger than the same band structure with perfect array of circular cylinders. The full and large frequency band gaps of SAW modes are the necessary condition for the design of acoustic channels. The technique is suitable for obtaining large frequency band gaps in the band structures consisted of low-acoustic-impedance cylinders in high-acoustic-impedance

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host materials.

II. Theory

In the following calculations, the formulation based on the plane-wave expansion method presented in [12] was adopted. A brief introduction of the theory is given in the following. In an inhomogeneous linear elastic medium with no body force, the equation of motion of the displacement vector $\mathbf{u}(\mathbf{r},t)$ can be written as

$$\rho(\mathbf{r})\ddot{u}_i(\mathbf{r},t) = \partial_j[C_{ijmn}(\mathbf{r})\partial_n u_m(\mathbf{r},t)], \quad (1)$$

where $\mathbf{r} = (\mathbf{x}, z) = (x, y, z)$ is the position vector, t is the time variable; $\rho(\mathbf{r})$, $C_{ijmn}(\mathbf{r})$ are the position-dependent mass density and elastic stiffness tensor, respectively. In the following, we consider a phononic crystal composed of a two-dimensional periodic array (x-y plane) of material A, embedded in a background material B. Due to the spatial periodicity, the material constants, $\rho(\mathbf{x})$ and $C_{ijmn}(\mathbf{x})$ can be expanded in Fourier series, with respect to the two-dimensional reciprocal lattice vectors (RLVs), $\mathbf{G} = (G_1, G_2)$, as

$$\rho(\mathbf{x}) = \sum_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} \rho_{\mathbf{G}}, \quad (2)$$

$$C_{ijmn}(\mathbf{x}) = \sum_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} C_{\mathbf{G}}^{ijmn}, \quad (3)$$

where $\rho_{\mathbf{G}}$ and $C_{\mathbf{G}}^{ijmn}$ are the corresponding Fourier coefficients.

To utilize the Bloch's theorem and to expand the displacement vector $\mathbf{u}(\mathbf{r},t)$ in the Fourier series for the analyses of the surface and bulk waves, we have

$$\mathbf{u}(\mathbf{r},t) = \sum_{\mathbf{G}} e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}(e^{i\mathbf{G}\cdot\mathbf{x}}\mathbf{A}_{\mathbf{G}}e^{ik_z z}), \quad (4)$$

where $\mathbf{k} = (k_1, k_2)$ is the Bloch wave vector, ω is the circular frequency, k_z is the wave vector along the z direction, and A_G is the amplitude of the displacement vector. We note that as the component of the wave vector k_z equals to zero, Eq. (4) degenerates into the displacement vector of a bulk acoustic wave. On substituting Eqs. (2), (3) and (4) into Eq. (1), and after collecting terms systematically, we obtain the generalized eigenvalue problem as

$$(\mathbf{A}k_z^2 + \mathbf{B}k_z + \mathbf{C})\mathbf{U} = 0, \quad (5)$$

where A, B, and C are $3n \times 3n$ matrices, and are functions of the Bloch wave vector k, components of the two-dimensional RLV, circular frequency ω, the Fourier coefficients of mass density $\rho_{\mathbf{G}}$ and components of elastic stiffness tensor $C_{\mathbf{G}}^{ijmn}$. n is the total number of RLV used in the Fourier expansion, and $\mathbf{U} = [A_{\mathbf{G}'}^1 \quad A_{\mathbf{G}'}^2 \quad A_{\mathbf{G}'}^3]^T$ is the eigenvector. The expressions of the matrices A, B, and C were listed in [12].

By applying the surface wave conditions and the traction free boundary conditions on the surface, the dispersion relation for the surface waves propagating in the two-dimensional phononic crystals, with both of the filling and background materials belonging to the triclinic system, can be obtained. When k_z in Eq. (5) is equal to zero, the equation degenerates into the eigenvalue problem of the bulk waves as

$$\mathbf{CU} = 0. \quad (6)$$

The dispersion relations of the bulk waves propagating in the two-dimensional phononic crystals can be obtained by setting the determinant of matrix C equal to zero.

III. Band Structures with Hollow Cylinders

As mentioned in section II, the Fourier coefficients, $\rho_{\mathbf{G}}$ and $C_{\mathbf{G}}^{ijmn}$ in Eqs. (2) and (3), can be expressed in the form with the so-called structural function F_G [12]. The structural functions with circular cylinders in phononic crystals are $F_{\mathbf{G}} = \dfrac{2\,ff J_1(Gr_0)}{Gr_0}$. Here, ff is filling fraction, $G = \sqrt{G_1^2 + G_2^2}$, and J_1 is a Bessel function. It should be modified when we construct the phononic band structure with hollow cylinders. The diagram of the band structure with hollow cylinders is shown in Fig. 1. Here, r_0 and r_i are the outer and inner radii of the hollow cylinders in the band structure. Therefore, the structural functions with hollow cylinders can be obtained as

$$F_{\mathbf{G}} = \frac{2\pi r_0^2 J_1(Gr_0)}{a^2 Gr_0} - \frac{2\pi r_i^2 J_1(Gr_i)}{a^2 Gr_i} \quad (7)$$

for the hollow circular cylinders. Here, a is the lattice constant. The left and right diagrams in Fig. 2 show the 3-D and 2-D density maps constructed using the structural function with the unit cell in real space. The resulting density maps in real space are reasonable achievements on the dispersion relations in k space.

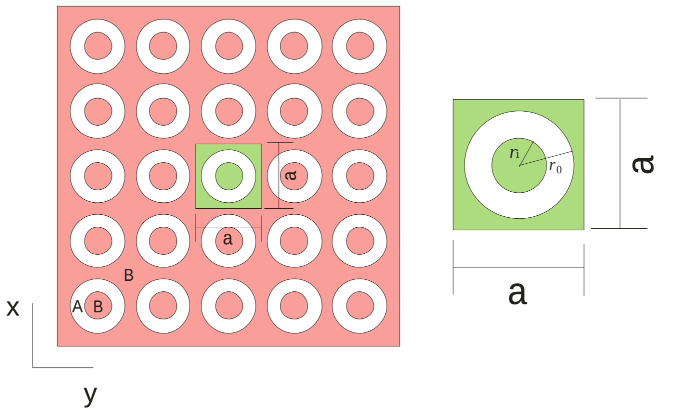

Fig. 1 Phononic band structures with hollow cylinders in a square lattice. r_0 and r_i are the outer and inner radii of hollow cylinders

A. PMMA/Ni Phononic Band Structures

Consider the PMMA/Ni phononic band structures forming the two-dimensional square lattices with lattice spacing a. Figs. 3 to 6 show the dispersion relations of the bulk modes along the boundaries of the irreducible part of the Brillouin zone with the ratios of r_i/r_0 = 0.1, 0.3, 0.5, and 0.85. It is worth noting that the filling fraction is 0.636 for r_i =0 and two narrow full band gaps exist in this band structure. In these examples, we used a total of 625 RLVs to construct the results and they resulted in a good convergence. In the dispersion relations, the diamond symbols represent the mixed polarization modes while the solid circles represent the transverse polarization modes. When we take the ratio r_i/r_0= 0.1 as shown in Fig. 3, there is only a narrow full frequency band gap located at about 3.6 normalized frequency in the band structure. The right diagram of Fig. 3 is a sketch map for the band structure. However, larger full frequency band gaps are investigated when the inner radii r_i of the cylinders increase to match the ratio r_i/r_0= 0.3 and 0.5 as shown in Figs. 4 and 5, respectively. It is interesting to note that the full frequency band gap is smaller when the ratio r_i/r_0= 0.85 as shown in Fig. 6, and begins to disappear as the ratio r_i/r_0 approaches 1. This is logical for the two-dimensional phononic band structures in which two materials with different acoustic impedances are necessary. Fig. 7 shows the frequency band-gap variations of the BAW modes in PMMA/Ni phononic crystals with hollow cylinders from r_i/r_0 = 0 to 1. The vertical axis is the relative frequency band-gap width $(\omega_2-\omega_1)/\omega_0$ to allow a better judgment of the effect of band structures with hollow cylinders. From the calculated results, we clearly observe that the full frequency band gap can be enlarged and reduced by changing the r_0/r_i ratio.

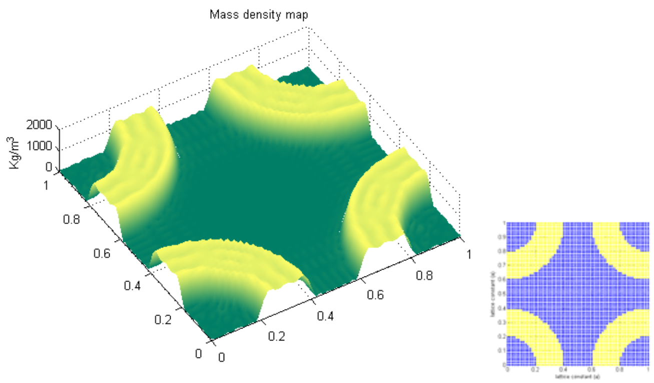

Fig. 2 Left and right diagrams show the 3-D and 2-D density maps constructed by structural function with unit cell in real space

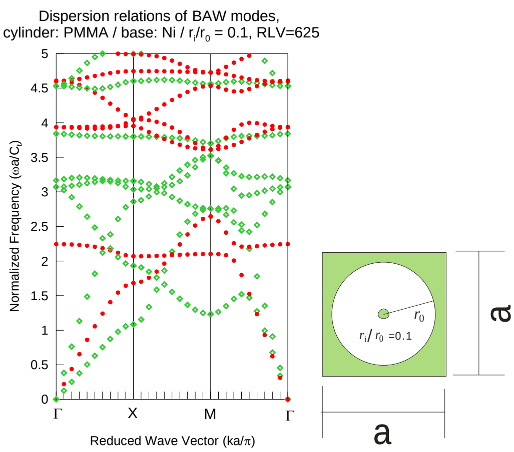

Fig. 3 Dispersion relations of all bulk modes in PMMA/Ni phononic band structures with hollow cylinders and r_i/r_0 = 0.1.

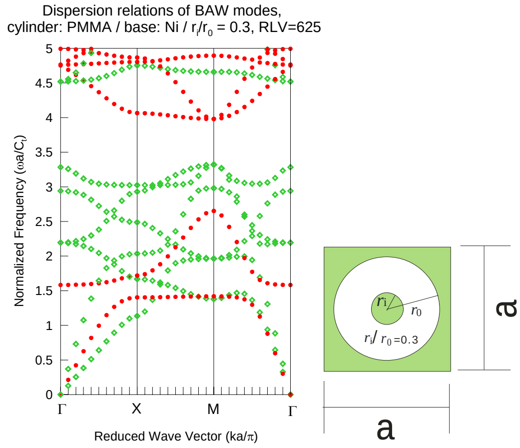

Fig. 4 Dispersion relations of all bulk modes in PMMA/Ni phononic band structures with hollow cylinders and r_i/r_0 = 0.3.

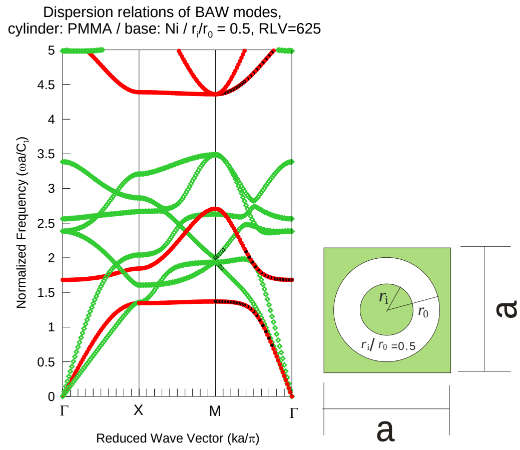

Fig. 5 Dispersion relations of all bulk modes in PMMA/Ni phononic band structures with hollow cylinders and r_i/r_0 = 0.5.

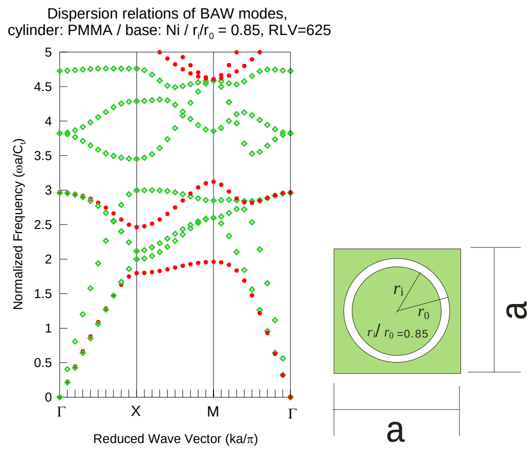

Fig. 6 Dispersion relations of all bulk modes in PMMA/Ni phononic band structures with hollow cylinders and r_i/r_0 = 0.85.

The other important point in discussing the band structures with hollow cylinders is to obtain large full frequency band gaps for all kinds of propagating modes. The existence of the large full frequency band gap for the BAW modes in phononic crystals is the necessary condition for obtaining the full frequency band gap of the SAW modes. Fig. 8 shows the dispersion relations of the SAW modes in PMMA/Ni phononic crystals with hollow cylinders and $r_i/r_0 = 0.5$. We can obtain a large full frequency band gap of the SAW modes in this band structure in which the frequency band gap is obtained with difficulty in the band structure with perfect arrays of circular cylinders. This type of band structure and technique are useful for tuning the frequency band gaps and the design of acoustic waveguides.

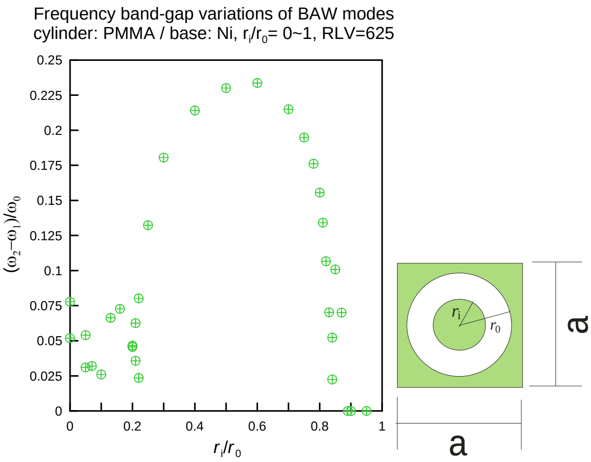

Fig. 7 Frequency band-gap variations of BAW modes in PMMA/Ni phononic crystals with hollow cylinders and $r_i/r_0 = 0 \sim 1$.

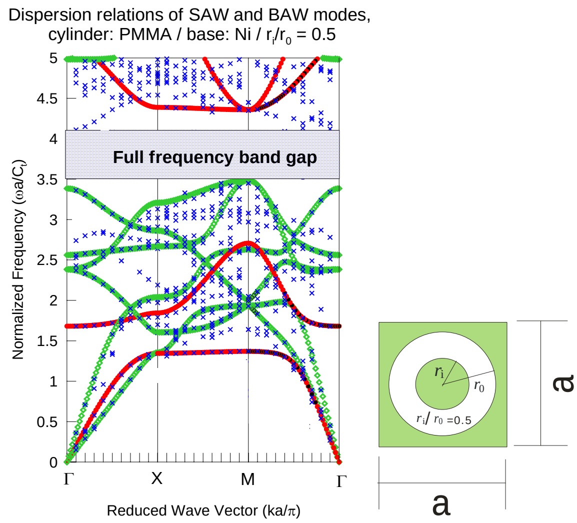

Fig. 8 Dispersion relations of SAW and BAW modes in PMMA/Ni phononic band structures with hollow cylinders and $r_i/r_0 = 0.5$.

B. *Vacuum/Si Phononic Band Structures*

Consider the vacuum/Si phononic band structures forming the two-dimensional square lattices. The vacuum/Si phononic band structures are adopted to get rid of the unexpected dispersion curves in the air/Si phononic band structures.[16] Fig. 9 shows the dispersion relations of the bulk modes along the boundaries of the irreducible part of the Brillouin zone with the ratios of $r_i/r_0 = 0.75$. In the dispersion relations, the diamond symbols represent the mixed polarization modes while the open circles represent the transverse polarization modes. Fig. 10 shows the enlarge plot of Fig. 9. Different from the results in solid/solid phononic band structures with hollow cylinders shown in section III-A, a narrow full frequency band gap in very low frequency range (about 0.58 normalized frequency) exists in the vacuum/solid phononic band structures with hollow cylinders.

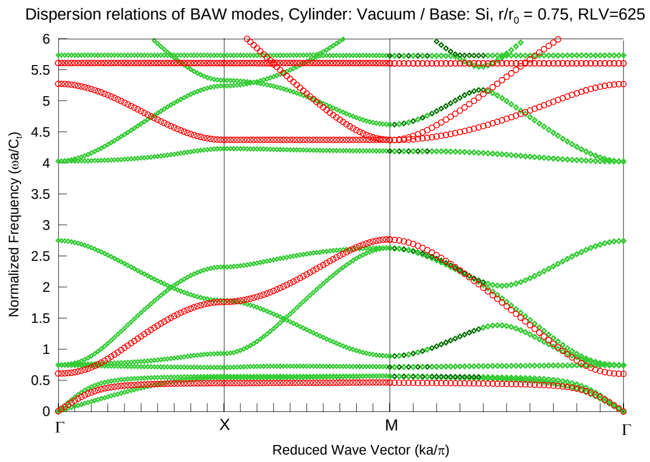

Fig. 9 Dispersion relations of all bulk modes in vacuum/Si phononic band structures with hollow cylinders and ri/r0 = 0.75

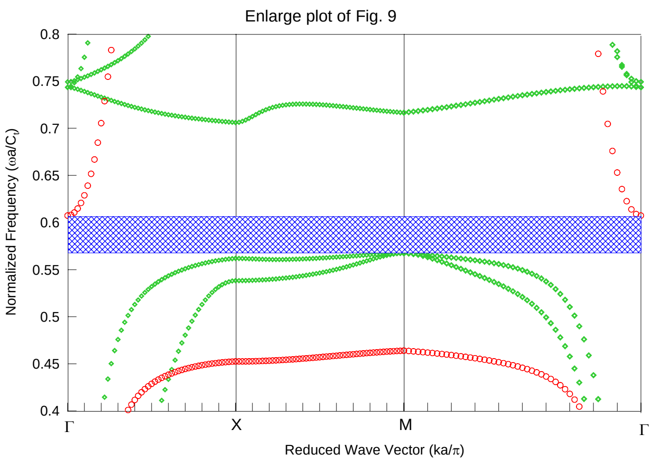

Fig. 10 Enlarge plot of Fig. 9

IV. CONCLUSION

In this paper, we studied the tunable frequency band gaps of the surface and bulk acoustic waves in two-dimensional phononic band structures with hollow cylinders. The results showed that the elastic/acoustic band gaps can be enlarged or reduced by changing the inner radius of hollow cylinders in the phononic band structures. The full and large frequency band gaps of SAW modes are the necessary condition for the design of acoustic channels. The technique is suitable for obtaining large frequency band gaps in the band structures consisted of low-acoustic-impedance cylinders in high-acoustic-impedance host materials. The effect discussed in this paper can be potentially utilized for enlarging the phononic band-gap frequency and may serve as a basis for studying the frequency band gaps of the SAW and BAW modes in phononic band structures.

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