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Emission Constrained Hydrothermal Scheduling Algorithm

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***Abstract*—**This paper presents an efficient emission constrained hydrothermal scheduling algorithm that deals with nonlinear functions such as the water discharge characteristics, thermal cost, and transmission loss. It is then incorporated into the hydrothermal coordination program. The program has been tested on a practical utility system having 32 thermal and 12 hydro generating units. Test results show that a slight increase in production cost causes a substantial reduction in emission.

***Keywords*—**Emission constraint, Hydrothermal coordination, and Hydrothermal scheduling algorithm.

I. Introduction

OPTIMAL scheduling of power plant generation is the determination of the generation for every generating unit such that the total system generation cost is minimum while satisfying the system constraints. However with insignificant marginal cost of hydro electric power, the problem of minimizing the operational cost of a hydrothermal system essentially reduces to minimizing the fuel cost for thermal units constrained by the generating limits, available water, and the energy balance condition in a given period of time [1].

In recent years, many approaches have been suggested to solve the hydrothermal scheduling problem. The proposed approaches include dynamic programming, functional analysis technique, method of local variations, principle of progressive optimality, general mathematical programming techniques and evolutionary algorithm. Dynamic programming [2] has the ability to handle all the constraints enforced by the hydro subsystem. The computational requirements are, however, considerable with this technique for a realistic system size. The incremental dynamic programming (IDP) technique [3] keeps the computational requirements in a reasonable range. Bonaert et al. [4] have employed the same technique under the framework of a more sophisticated model. A functional analysis minimum-norm formulation is proposed [5]. Bijwe and Nanda [6] reported the superiority of the method of local variations algorithm over the IDP based algorithm as used by Bonaert et al. Again the application of progressive optimality algorithm [6] to optimal hydrothermal scheduling problem performs better than the method of local variations.

Generally all those method have slow convergence characteristics. Investigations on the use of Newton-Raphson method have been carried out. Formulation of the scheduling problem in Newton-Raphson method for solving a set of nonlinear equations produces a large matrix expression. The drawbacks of Newton's method are the computation of the inverse of a large matrix, the ill-conditioning of the Jacobian matrix and the divergence caused by starting values. Powell's hybrid method has also been proposed to avoid the divergence problem encountered by Newton-Raphson method [7]. A method using LU factorization of the matrix in Newton's method formulation has been proposed [8] which reportedly shows superiority over the Powell's method, however, the size of the matrix still remains very large requiring substantial computations.

Abdul Halim and Khalid [1] linearizes the coordination equations so that the Lagrangian of the water availability constraint is determined separately from the unit generations. This water availability constraint Lagrange multiplier determines the Lagrange multiplier for the power balance constraint and hence leads to the computation of the generation of thermal and hydro units. The algorithm requires small computation resources. It has global-like convergence property so that even if the starting values are far from the solution, convergence is still achieved rapidly. Recently evolutionary algorithm has been proposed for solving scheduling problems [9]-[11].

In this paper, the formulation [1] has been modified to cope with emission constraint. It is then incorporated into the hydrothermal coordination program [12]. The program has been tested on a practical utility system. Test results show that a slight increase in production cost reduces the emission substantially.

II. Problem Formulation

A. Notations

The list of symbols used in this paper is as follows:
M, H - number of thermal and hydro units, respectively
T - number of periods for dividing the scheduling time horizon
i - Index of the thermal unit
h - Index of the hydro unit
t - time index
$\underline{P}$ - MW power output of a generating unit
$\overline{P}$ - maximum MW power of a generating unit

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$\underline{P}$ - minimum MW power of a generating unit

D_t - demand in period t

$Ploss_t$ - transmission loss in period t

$qtot_h$ - prespecified volume of water available for the h-th hydro unit

$C_i(.)$ - production cost function of the i-th thermal unit

$E_i(.)$ - emission function of the i-th thermal unit

$q_h(.)$ - water flow rate function of the h-th hydro unit

b0, b1, b2 – coefficients for production cost function

e0, e1, e2 – coefficients for emission function

a0, a1, a2 – coefficients for water flow rate function

B0, B1, B – transmission loss coefficients

λ_t - Lagrangian multiplier for the energy balance equation in period t

γ_h - Lagrangian multiplier for the h-th hydro unit

B. *Objective Function and Constraints*

Mathematically, the hydrothermal scheduling problem can be expressed as follows:

$$\sum_{t=1}^{T}\sum_{i=1}^{M} C_i(P_{i,t}) \tag{1}$$

subject to the energy balance equation

$$EB_t = \sum_{i=1}^{M} P_{i,t} + \sum_{h=1}^{H} P_{h,t} - D_t - Ploss_t = 0 \tag{2}$$

and to the water availability constraint

$$W_h = \sum_{t=1}^{T} q_h(P_{h,t}) = qtot_h \tag{3}$$

with

$$\underline{P}_i \le P_{i,t} \le \overline{P_i}$$

$$0 \le P_{h,t} \le \overline{P_h}$$

The new constraint function is the emission function. The total emission from the system is

$$\sum_{t=1}^{T}\sum_{i=1}^{M} E_i(P_{i,t}) \tag{4}$$

III. Proposed Approach

The cost objective function in (1) is then augmented by constraint equation (4) using Lagrange multiplier ω, called the emission weighting factor, as follows

$$\min_{P_{i,t}} z = \sum_{t=1}^{T}\sum_{i=1}^{M} \{C_i(P_{i,t}) + \omega_i E_i(P_{i,t})\} \tag{5}$$

Representing cost and sulphur oxide emission of thermal unit as quadratic functions of thermal generation and water discharge rate of hydro unit as quadratic function of hydro generation, we get:

$$C_i(P_{i,t}) = b0_i + b1_i P_{i,t} + b2_i P_{i,t}^2$$

$$E_i(P_{i,t}) = e0_i + e1_i P_{i,t} + e2_i P_{i,t}^2$$

$$q_h(P_{h,t}) = a0_h + a1_h P_{h,t} + a2_h P_{h,t}^2$$

The transmission loss is represented by the following expression:

$$Ploss_t = \sum_{s=1}^{M+H}\sum_{j=1}^{M+H} B_{s,j} P_{s,t} P_{j,t} + \sum_{s=1}^{M+H} B1 P_{s,t} + B0 \tag{6}$$

The augmented Lagrangian function L is

$$L(P_{i,t}, P_{h,t}, \lambda, \gamma) = z - \sum_{t=1}^{T} \lambda_t EB_t - \sum_{h=1}^{H} \gamma_h (W_h - qtot_h) \tag{7}$$

The set of coordination equations for a minimum cost operating condition is then given by:

$$\frac{\partial L}{\partial P_{i,t}} = TA_k \frac{d}{dP_{i,t}}[C_i(P_{i,t}) + \omega_i E_i(P_{i,t})] - \lambda_t \frac{\partial EB_t}{\partial P_{i,t}} = 0$$

$$\frac{\partial L}{\partial P_{h,t}} = -\lambda_t \frac{\partial EB_t}{\partial P_{h,t}} - \gamma_h \frac{dW_h}{dP_{h,t}} = 0$$

$$\frac{\partial L}{\partial \lambda_t} = -EB_t = 0$$

$$\frac{\partial L}{\partial \gamma_h} = -W_h + qtot_h = 0 \tag{8}$$

Substituting the above system parameters into the minimum condition yields the following coordination equations:

$$b1_i + 2b2_i P_{i,t} + \omega_i e1_i + 2\omega_i e2_i P_{i,t} = \lambda_t (1 - K_{i,t})$$

$$\gamma_h (a1_h + 2a2_h P_{h,t}) = \lambda_t (1 - K_{h,t})$$

$$\sum_{i=1}^{M} P_{i,t} + \sum_{h=1}^{H} P_{h,t} = D_t + Ploss_t$$

$$\sum_{t=1}^{T} (a0_h + a1_h P_{h,t} + a2_h P_{h,t}^2) = qtot_h$$

where

$$K_{i,t} = \frac{\partial Ploss_t}{\partial P_{i,t}} = 2\sum_{j=1}^{M+H} B_{i,j} P_{j,t} + B1_i$$

$$K_{h,t} = \frac{\partial Ploss_t}{\partial P_{h,t}} = 2\sum_{j=1}^{M+H} B_{h,j} P_{j,t} + B1_h$$

These are a set of nonlinear equations of unknown variables $P_{i,t}$ (steam), $P_{h,t}$ (hydro), λ_t, γ_h and they can only be solved iteratively.

Let $P_{i,t}^{old}, P_{h,t}^{old}, \gamma_h^{old}, \lambda_t^{old}$ be the approximate solutions at the previous iterative stage. The next iterates, i.e.,

$$P_{i,t}^{new} = P_{i,t}^{old} + \delta P_{i,t}, \quad P_{h,t}^{new} = P_{h,t}^{old} + \delta P_{h,t}$$

$$\gamma_h^{new} = \gamma_h^{old} + \delta\gamma_h, \quad \lambda_t^{new} = \lambda_t^{old} + \delta\lambda_t$$

generated by Newton's method are given as follows:

$$2(b2_i + \omega_i e2_i)\delta P_{i,t} + 2\lambda_t^{old}\sum_{j=1}^{M+H} B_{i,j}\delta P_{j,t} - \lambda_t^{new}(1 - K_{i,t}^{old}) = -2(b2_i + \omega_i e2_i)P_{i,t}^{old} - b1_i - \omega_i e1_i \quad (9)$$

$$2a2_h\gamma_h^{old}\delta P_{h,t} + 2\lambda_t^{old}\sum_{j=1}^{M+H} B_{h,j}\delta P_{j,t} - \lambda_t^{new}(1 - K_{h,t}^{old}) + (2a2_h P_{h,t}^{old} + a1_h)\lambda_t^{new} = 0 \quad (10)$$

$$\sum_{i=1}^{M}(1 - K_{i,t}^{old})\delta P_{i,t} + \sum_{h=1}^{H}(1 - K_{h,t}^{old})\delta P_{h,t} = Ploss_t^{old} + D_t - \sum_{i=1}^{M} P_{i,t}^{old} - \sum_{h=1}^{H} P_{h,t}^{old} \quad (11)$$

$$\sum_{t=1}^{T}(2a2_h P_{h,t}^{old} + a1_h)\delta P_{h,t} = qtot_h - \sum_{t=1}^{T}[a2_h(P_{h,t}^{old})^2 + a1_h P_{h,t}^{old} + a0_h] \quad (12)$$

Consider (9) and (10). To simplify calculations, the equations are diagonalized by neglecting all the terms with $B_{s,j}, s \neq j$ coefficients in the terms $\sum_{j=1}^{M+H} B_{i,j}\delta P_{j,t}$, $\sum_{j=1}^{M+H} B_{h,j}\delta P_{j,t}$. This yields the equations as follows

$$[2(b2_i + \omega_i e2_i) + 2\lambda_t^{old}B_{i,i}]\delta P_{i,t} - \lambda_t^{new}(1 - K_{i,t}) = -2(b2_i + \omega_i e2_i)P_{i,t}^{old} - b1_i - \omega_i e1_i$$

$$\text{or, } \delta P_{i,t} = \frac{\lambda_t^{new}(1 - K_{i,t}^{old}) - [2(b2_i + \omega_i e2_i)P_{i,t}^{old} + b1_i + \omega_i e1_i]}{2(b2_i + \omega_i e2_i) + 2\lambda_t^{old}B_{i,i}} \quad (13)$$

and $(2\gamma_h^{old}a2_h + 2\lambda_t^{old}B_{h,h})\delta P_{h,t} - \lambda_t^{new}(1 - K_{h,t}^{old})$ $(2a2_h P_{h,t}^{old} + a1_h)\gamma_h^{new} = 0$

$$\text{or, } \delta P_{h,t} = \frac{\lambda_t^{new}(1 - K_{h,t}^{old}) - (2a2_h P_{h,t}^{old} + a1_h)\gamma_h^{new}}{2\gamma_h^{old}a2_h + 2\lambda_t^{old}B_{h,h}} \quad (14)$$

Next inserting these expressions for $\delta P_{i,t}, \delta P_{h,t}$ into (11) and (12), we get

$$A_t\lambda_t^{new} - \sum_{h=1}^{H}\alpha_{h,t}\gamma_h^{new} = C_t \quad (15)$$

$$\sum_{t=1}^{T}\alpha_{h,t}\lambda_t^{new} - \beta_h\gamma_h^{new} = \delta_h \quad (16)$$

where

$$A_t = \sum_{i=1}^{M}\frac{(1 - K_{i,t}^{old})^2}{2(b2_i + \omega_i e2_i) + 2\lambda_t^{old}B_{i,i}} + \sum_{h=1}^{H}\frac{(1 - K_{h,t}^{old})^2}{2\gamma_h^{old}a2_h + 2\lambda_t^{old}B_{h,h}}$$

$$\alpha_{h,t} = \frac{(1 - K_{h,t}^{old})(2a2_h P_{h,t}^{old} + a1_h)}{2\gamma_h^{old}a2_h + 2\lambda_t^{old}B_{h,h}}$$

$$\beta_h = \sum_{t=1}^{T}\frac{(2a2_h P_{h,t}^{old} + a1_h)^2}{2\gamma_h^{old}a2_h + 2\lambda_t^{old}B_{h,h}}$$

$$C_t = Ploss_t^{old} + D_t - \sum_{i=1}^{M} P_{i,t}^{old} - \sum_{h=1}^{H} P_{h,t}^{old} + \sum_{i=1}^{M}\frac{(1 - K_{i,t}^{old})[2(b2_i + \omega_i e2_i)P_{i,t}^{old} + b1_i + \omega_i e1_i]}{2(b2_i + \omega_i e2_i) + 2\lambda_t^{old}B_{i,i}}$$

$$\delta_h = qtot_h - \sum_{t=1}^{T}[a2_h(P_{h,t}^{old})^2 + a1_h P_{h,t}^{old} + a0_h]$$

Finally, eliminating λ_t^{new} from (15) and (16), we get

$$\sum_{t=1}^{T}\sum_{j=1}^{H}\frac{\alpha_{h,t}\alpha_{j,t}\gamma_j^{new}}{A_t} - \beta_h\gamma_h^{new} = \delta_h - \sum_{t=1}^{T}\frac{\alpha_{h,t}C_t}{A_t} \quad (17)$$

This is a set of H equations for γ_h^{new}. Having obtained γ_h, we solve for λ_t from (15). Then we can get the equations for $\delta P_{i,t}, \delta P_{h,t}$, from (13) and (14).

IV. Test Results

This emission constrained hydrothermal scheduling algorithm has been coded in 'C' language for use in the hydrothermal coordination program [12] developed in 'C' language too. The program has been tested on a practical utility system using the data of the generating units and system demands. The test system consists of 32 thermal and

12 hydro generating units, of which seven are gas turbine units. The total thermal capacity of the system is 3,640 Megawatt (MW) and the total hydro capacity is 848 MW. Numerical results presented here are based on three data sets: Case 1, Wednesday; Case 2, Saturday; Case 3, Sunday. The scheduling horizon is 24 hours in all cases. A summary of the system characteristics and parameters for these data sets is shown in Table I.

TABLE I
SUMMARY OF POWER SYSTEM

System characteristics	Number of units	Total capacity or requirements (MW)
Steam units	25	3360
Gas turbine units	7	280
Hydro units	12	848
All units	44	4488
Peak demand		2198
Minimum demand		1563
Maximum reserve		166

To avoid the voluminous amount of results, the complete solution process will be presented for Case 1 only. However summary of results of all cases will be presented at the end. For Case 1, using the hydrothermal coordination program without the emission constraint (ω=0 for all units), a schedule shown in Table II was suggested whose cost of operation is $2,140,405. The total emission from this schedule was 812.11 tons. The emissions from each unit are shown in Table III. Emissions from units 1, 2, 9 and 10 were found to be quite high.

TABLE II
SCHEDULE
(* 1-unit is on; 0-unit is off. ** 1-nonzero generation; 0-zero generation)

Hour	Thermal Units *	Hydro Units **
	000000000 1111111111 2222222222 333	000000001111
	123456789 0123456789 0123456789 012	123456789012
1	111100001 0110000110 0000110000 000	111111111111
2	111100001 0110000110 0000110000 000	111111111111
3	111100001 1110000110 0000110000 000	000000000000
4	111100001 1110000110 0000110000 000	000000000000
5	111100001 1110000110 0000110000 000	000000000000
6	111100001 1110000110 0000110000 000	000011110000
7	111100001 1110000110 0000110000 000	111111111111
8	111100001 1110000110 0000110000 000	111111111111
9	111111111 1110000110 0000110000 000	111111111111
10	111111111 1110000110 0000110000 000	111111111111
11	111111111 1110000110 0000110000 000	111111111111
12	111111111 1110000110 0000110000 000	111111111111
13	111111111 1110000110 0000110000 000	111111111111
14	111111111 1110000110 0000110000 000	111111111111
15	111111111 1110000110 0000110000 000	111111111111
16	111111111 1110000110 0000110000 000	111111111111
17	111111111 1110000110 0000110000 000	111111111111
18	111111011 1110000110 0000110000 000	111111111111
19	111111011 1110000110 0000110000 000	111111111111
20	111111011 1110000110 0000110000 000	111111111111
21	111111011 1110000110 0000110000 000	111111111111
22	111111011 1110000110 0000110000 000	111111111111
23	111111011 1110000110 0000110000 000	111111111111
24	111111011 1110000110 0000110000 000	000011110000

TABLE III
EMISSIONS FROM THERMAL UNITS

Unit	Emission (tons)	Unit	Emission (tons)	Unit	Emission (tons)
1	125.17	12	22.03	23	0.00
2	128.95	13	0.00	24	20.43
3	37.42	14	0.00	25	20.45
4	37.25	15	0.00	26	0.00
5	28.05	16	0.00	27	0.00
6	26.91	17	21.96	28	0.00
7	16.36	18	22.00	29	0.00
8	27.96	19	0.00	30	0.00
9	133.35	20	0.00	31	0.00
10	122.42	21	0.00	32	0.00
11	21.41	22	0.00		

In order to reduce emissions from the above mentioned four units, hydrothermal coordination program was rerun using ω=200 for these particular units. The schedule obtained corresponds to an operating cost of $2,146,002. The deviation of this schedule from the earlier one was as follows:
-Thermal unit 4 was not committed at hour 24
-Thermal unit 5 was committed during hours 3 to 8
-Thermal unit 7 was committed during hours 18 to 22
-Thermal unit 10 was not committed during hours 3 to 5
-All hydro units had nonzero generation during hours 3 to 8
-Hydro units 1 to 4 had nonzero generation at hour 24
-Hydro units 9 to 12 had nonzero generation at hour 24

The total emission from this schedule was 793.54 tons. The increase of $5597 i.e. 0.26% in total cost in this run over the previous one was due to inclusion of emission constraint. But this caused a reduction of 18.57 tons i.e. 2.29% in total emission. Cost and emission summary of all three cases are shown in Table IV.

TABLE IV
COST AND EMISSION SUMMARY

Data Set	ω=0 for all units		ω ≠ 0 for all units	
	Cost ($)	Emission (tons)	Cost ($)	Emission (tons)
Case 1	2,140,405	812.11	2,146,002	793.54
Case 2	2,024,189	777.23	2,033,040	751.94
Case 3	1,641,302	648.95	1,654,434	608.71

Emission constraint was enforced through a set of weighting factors. Higher emitters were given higher weighting factors to limit the emission to a greater extent. In the study system, only four units were found to be larger emitters of sulphur oxide. The emissions from them contribute to more than 60% of the total emissions. Hence, it is quite logical to use higher weighting factors for those units only while keeping the weighting factor equal to zero for other units as the emissions from them are quite low. The reduction of emission is done by shifting some loadings of these units to more expensive units thus resulting in higher operating cost. However, the percentage reduction in emission obtained is much higher than the percentage increase in operating cost.

V. Conclusion

An efficient hydrothermal scheduling algorithm for dealing with nonlinear functions such as the water discharge characteristics, thermal cost, transmission loss and emission constraint is developed. It is then incorporated into the hydrothermal coordination program. The program has been tested on a practical utility system using the data of the generating units and system demands. Emission weighting factor is varied to mitigate the impact of the emission of the corresponding unit. Higher emitters of sulphur oxide are given a higher weighting factor in order to limit the emissions to a greater extent. The reduction of emissions is accomplished by shifting some loading of these higher emitter units to more expensive units. This results in higher operating cost. However, it has been observed that a slight increase in production cost causes a substantial reduction in emission.

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