

# Expansion of A Finit Size Partially Ionized Laser-Plasma

Mohamed Fawzi Mahboub, and Mourad Djebli

*Abstract*—The expansion mechanism of a partially ionized plasma produced by laser interaction with solid target (copper) is studied. For this purpose we use a hydrodynamical model which includes a source term combined with Saha's equation. The obtained self-similar solution in the limit of quasi-neutrality shows that the expansion, at the earlier stage, is driven by the combination of thermal pressure and electrostatic potential. They are of the same magnitude. The initial ionized fraction and the temperature are the leading parameters of the expanding profiles.

*Keywords*—expansion, quasi-neutral, laser-ablated plasma, self-similar.

## I. INTRODUCTION

**B**ASICALLY, plasma expansion results from the combination of two effects. The first one is due to thermal pressure produced by a localized deposition of high energy in a small space domain containing a target. This energy introduces high thermal pressure gradient, which leads to the expansion. Such a mechanism is common to neutral or ionized gas, i.e. plasma. Once the plasma starts to expand, lightweight more mobile particles, i.e. electrons are heated and firstly expand towards the vacuum. Thereby, the electron motion creates a charge separation and generates a self-consistent ambipolar electric field that would accelerate the ions and retard electrons. The plasma expansion takes place. This is the second mechanism of the expansion. In fact, acceleration of ions to supersonic velocities during the expansion is a major application of plasma expansion. The mechanism is well-known since the pioneer work of Plyutto and Gurevich *et al.* [1]. Nevertheless, plasma expansion into a vacuum is a topic of interest to space plasmas and laser produced plasmas. The fundamental point of interest is the continuous acceleration of ions at the expansion front where a space charge electric field is set up by the faster electrons.

When the plasma expansion takes place with a characteristic size of radial inhomogeneity larger than the Debye length, the quasi-neutrality is preserved and the self-similar solution exists. There are different self-similar classes of transformations that reduce the partial differential equations (PDE), governing the plasma expansion, to ordinary ones (ODE). The process of reduction to self-similar form fully preserves the nonlinearity. The self-similar solution provides important clues to a wider class of solutions of the original PDE[2]. To achieve the reduction, the two variables of space  $x$  and time  $t$  can be combined into a one variable. Such a transformation

is not permitted at very small  $t$ , small compared to the time scale characterizing the phenomenon. In the laser-plasma interaction, the time corresponds to the laser pulse duration  $\tau_L$ . Therefore, the self-similar solution is relevant only for  $t \gg \tau_L$ . The similarity solutions were firstly found for explosion and implosion problems. Simple scaling arguments illustrate the invariant nature of the scaled solutions.

The finite plasma expansion is determined by initial condition and a possible existence of external source of energy. In the quasi-neutral limit, an analytical self-similar solution exists for a Gaussian plasma of size much larger than the Debye length[3]. For a laser-produced plasma with a limited mass, the expansion has been investigated with two different classes of the self-similar transformation. The obtained self-similar solutions correspond to a quasi-isothermally expansion during the laser irradiation and an adiabatic expansion after turning of the laser[4]. Kumaret *al.* used a new self-similar solution which requires a suitable time-dependent temperature of laser-generated hot electrons. They obtained a different solution which, allowed mono-energetic spikes in the ion spectrum. However, their model assumes that the expansion is driven by the ion thermal pressure in the presence of an appropriate tailoring of the plasma electron temperature profile[5].

In laser plasma the expansion is driven by a fraction of the energy absorbed in the ablated materials. The gas dynamical model of plume expansion can be applied to study ionized plumes[6]. The ratio of electrons, ions and atoms at a given temperature and density is governed by Saha's equation. The ionization is found to extend the limit of plasma expansion with two different behaviors, depending on whether the plasma is close to or far away from the ablated target[7]. In the present model, we consider the effect of charge separation, but limited to the approximation of quasi-neutral plasma. The physical picture for which the expansion is driven by the electrostatic potential is the explosion of a spherical plasma having a uniformly distributed electrons and ions. Popov *et al.* have studied the spherical explosion of a plasma produced by the interaction of ultrashort laser pulse with submicron targets. It was shown that the quasi-neutral ion energy spectrum can be used in the approximate evolution, when the electron temperature is very small ( $T \ll 1$ )[8]. By using the multi-fluids model, we showed that the plasma expansion driven by thermal pressure gradient or by electrostatic potential are of the same magnitude for a finite partially ionized plasma.

## II. MODELING

In a vacuum, for a laser-induced plasma, during the time when the distance of the plasma front from the target is com-

parable to the laser spot dimension, the plasma propagation is one-dimensional[9]. The fluid one-dimensional approximation can be used to investigate the expansion of a plasma of ions, electrons and neutral atoms. At the beginning of the expansion, the plasma contains ions, electrons and neutral atoms of densities  $n_i$ ,  $n_e$  and  $n_n$ , respectively. During the expansion, the plasma is subject to ionization recombination processes. There are different processes that lead to ionization of neutral atoms. However, one of the processes is found dominant. When energies are of the order of ionization potential of the atom, which is several electrons volts, the dominant ionization process in a partially ionized plasma is by electron impact. For a plasma with electron (ion) concentration much smaller than its equilibrium value, the recombination is unimportant[10]. Therefore, ion fluid equations must contain a source term which considers the density variation due to ionization by electron impact.

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_i)}{\partial x} = \alpha_e n_e n_n \quad (1)$$

where  $m_i$  and  $v_i$ , stand for the ion mass and velocity, respectively. Ionization by electron impact is proportional to the ionization rate[10]:

$$\alpha_e = \sigma_e \bar{v}_e \left( \frac{U_i}{k_B T_e} + 2 \right) e^{-U_i/k_B T_e} \quad (2)$$

Where  $\bar{\sigma}_e$  is the average cross section,  $\bar{v}_e = (8k_B T_e / \pi m_e)^{1/2}$  is the mean thermal speed of the electron and  $U_i$ ,  $T_e$ ,  $k_B$  being the ionization potential, the electron temperature and the Boltzmann constant, respectively.

For an expansion driven by the thermal pressure gradient and the electrostatic ambipolar potential, ion momentum equation is given by,

$$m_i n_i \left[ \frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} \right] = -e n_i \frac{\partial \phi}{\partial x} - \frac{\partial p_i}{\partial x} \quad (3)$$

The pressure for an isothermal situation is  $p_i = n_i T_i$ . The ion temperature is assumed to be equal to the electron temperature  $T$  ( $T_i = T_e = T$ ). Such approximation is used whatever the ions are cold or warm because the ion temperature is usually not known. In the time scale corresponding to the ion expansion, the inertialess electrons follow Boltzmann distribution,

$$n_e(x, t) = n_{e0} \exp\left(\frac{e\phi(x, t)}{T}\right). \quad (4)$$

The densities of neutral atoms, electrons and ions are related by Saha's equation

$$\frac{n_e n_i}{n_n} = \frac{2U_+}{U_o} \left( \frac{2\pi m_e k_B T}{h^2} \right)^{\frac{3}{2}} \exp\left(-\frac{U_i}{k_B T}\right) = K_1(T) \quad (5)$$

where  $h$  is the Planck's constant,  $U_+$  and  $U_o$  are the electronic partition functions. Saha's equation is used when only one ionization level is important.

To investigate the expansion of a finite size plasma we follow the same transformation as Ref.[5], i.e.,

$$\Phi(\xi) = e\phi/T, v_{i(e)} = \dot{R}f(\xi), \xi = x/R \quad (6)$$

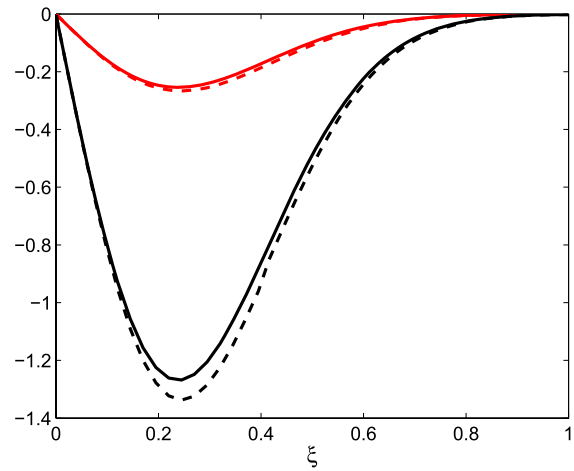


Fig. 1. (Color online) Plots of  $dN_i/d\xi$  (solid lines) and  $N_i d\phi/d\xi$  (dashed lines) versus the self-similar variable  $\xi$ . With  $n_{i0} = 0.1n_o$ , Red lines ( $T = 0.15$ ) and black lines ( $T = 0.2$  eV).

where  $R(t)$  stands for a time-dependent characteristic system size which have an initial value  $R_0 = R(0)$  and expands preserving quasi-neutral plasma ( $n_i = n_e$ ).

$$n_i = n_0 \frac{R_0}{R} N_i(\xi) \quad (7)$$

The density is normalize by the initial total density of neutral atoms and ions  $n_0 = n_{i0} + n_{n0}$ . The function  $f(\xi)$  is obtained from continuity equation,

$$f(\xi) = \xi + \frac{C}{N_i(\xi)} + \frac{F}{d} N_i^3(\xi) \quad (8)$$

where  $C$  is a constant obtained using the boundary condition at origin. Constants in the last right-hand side of Eq.(8) are  $F = (\alpha_e n_0^2 R_0^2) / 4K_1(T)$  and  $d = R\dot{R}$ . In the limit of a totally ionized plasma, i.e.,  $\alpha_e \rightarrow 0$ , The function  $f$  has the same expression as the one corresponding to Ref.[5].

From Eqs.(1) to (8), one obtains

$$\frac{dN_i}{d\xi} = N_i^2 \frac{[C(a+1) + N_i \xi + b N_i \frac{d\Phi}{d\xi}]}{[aC^2 - bN_i^2 - 2(\frac{aCF}{d})N_i^4 - (\frac{3aF^2}{d^2})N_i^8]} \quad (9)$$

$$\frac{d\Phi}{d\xi} = N_i \left[ \frac{C(a+1) + N_i \xi}{aC^2 - 2bN_i^2 - 2(\frac{aCF}{d})N_i^4 - (\frac{3aF^2}{d^2})N_i^8} \right] \quad (10)$$

where  $a = \dot{R}^2$ ,  $b = T_e/m_i = c_s^2$ ,  $\ddot{R}R = \dot{R}^2/a = T_e/(m_i b) = 1$

### III. DISCUSSION

Numerical investigation is conducted for a plasma of charged ions produced during pulsed laser ablation of solid targets (copper). Ions formation occurred, during the early phase of plasma expansion, through ionization by the impact of fast electrons or by multi-photon interactions. For energies

$$\frac{dv_i}{d\xi} = \left( \frac{v_i}{\xi + \frac{C}{N_i} + \frac{F}{d} N_i^3} \right) \left[ 1 + \left( \frac{1}{aC^2 - bN_i^2 - 2\left(\frac{aCF}{d}\right)N_i^4 - \left(\frac{3aF^2}{d^2}\right)N_i^8} \left( \frac{3F}{d} N_i^4 - C \right) \right) \left( C(a+1) + N_i\xi + bN_i \frac{d\Phi}{d\xi} \right) \right] \quad (11)$$

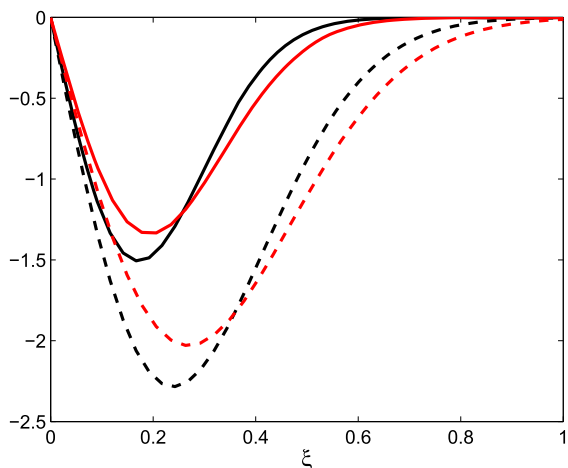


Fig. 2. (Color online) The same as Fig.1 but with  $T = 0.15 \text{ eV}$ , Red lines ( $n_{i0} = 0.1n_o$ ) and black lines ( $n_{i0} = 0.5n_o$ ).

in the range  $7.72 < E < 20(\text{eV})$ , which correspond to the first ionization, the cross section by electron-impact is  $\sim 2 \times 10^{-16} \text{ cm}^2$ [11]. The main aim of the study is to understand which mechanism that conducts the expansion of a partially ionized gas. The set of differential equations (9-11) is solved for three different situations. Firstly, we suppose that the expansion is conducted by the pressure term, as it is the case for neutral gases or partially ionized plasma[12]. The second situation concerns the expansion driven by the electrostatic potential. Such a scenario is more common in Coulomb explosion. The latter occurs when a cluster irradiated by a high-intensity laser, becomes very rapidly and highly ionized, free electrons are instantly swept and leave the ionic core which is then torn apart by repulsive Coulomb forces[13]. From Eq.(3), the thermal pressure and electrostatic terms are proportional to  $dN_i/d\xi$  and  $N_i d\Phi/d\xi$ , respectively. Solving the set of Eqs. (9- 11), allowed us to plot this two terms as a function of the self-similar variable (Fig.1). For an initial total density  $n_0 \sim 10^{15} \text{ cm}^{-3}$ [14], the thermal pressure and the electrostatic potential contribution to the momentum equation are of the same magnitude, particularly close to the plasma source region. For a partially ionized plasma, it cannot be correct to assume that the expansion is governed only by one of these effects. Among all initial parameters, the temperature and the initial ionized fraction are the most important ones leading to change the profile during the expansion. It is important to note that the temperature should be of the order of  $\sim 0.2 \text{ eV}$ , otherwise the plasma is totally ionized. Lower temperature reduces the domain of the self-similar variable for which a self-similar solution exists ( red lines of Fig.1). When the temperature increases, the discrepancy between the

effect of the thermal pressure and the electrostatic potential can be visible for a wider domain (black lines of Fig.1). Increasing the initial ionized fraction turns out to enhance both effects (Fig.2). The plasma expansion is the results of the combination of the thermal pressure which increases with the density and the electrostatic potential. The latter results from the local separation between ions and electrons, limited to the Debye sphere to preserve quasi-neutrality of the plasma. In Figure (3), the density profile during the expansion is plotted when the expansion is driven by the thermal pressure or the electrostatic potential (red lines) and the combination of the two effects (black lines). The profiles are slightly different from the known plasma expansion profile. Commonly, there is a rarefaction wave propagating in the opposite direction of the expanding front for which the density decreases with a convex profile. For a partially ionized plasma, close to the source the density increases due to ionization process, the profile start with concave shape[15]. The combination of the two effects enlarge the expansion limit which is more important for higher temperature. One of the problems related to the self-similar solution is boundary conditions. As the self-similar transformation preserves the shape, the choice of the initial density is not important, all curves are plotted with normalized density. The choice of the initial velocity is different because it is inherent to the fluid energy. Usually we choose  $v(\xi_0) = 0$ , the expansion is supposed to start from an unperturbed plasma. As the velocity is not known, this is just an approximation. For a fully ionized plasma of ions and electrons in thermal equilibrium, the velocity is given by  $v = \xi \pm C_s$ ,  $C_s = \sqrt{\gamma T e / m_i}$ . The positive sign is chosen when a rarefactive wave propagating into the unperturbed plasma. For  $\xi = 0$  the front initially moves with  $C_s$ , and the velocity increases linearly until the end of the expansion. For the isothermal case, the electric field is space independent and is different from zero when  $x \rightarrow 0$ . When the expansion starts with an unperturbed plasma ( $n(\xi_0) = n_o$ ), ions at the discontinuity are accelerated to infinity by the infinite electric field as soon as  $t > 0$  the ion velocity increases linearly in  $x$  for a given  $t$  (c.f. Ref.[16]). Clearly when  $N \sim 0$  (end of the expansion), the velocity loses its physical meaning. In the present work, the velocity profile is almost linear, the deviation can be seen only far away from the plasma source. Changing the boundary conditions didn't have significant effect on the velocity profile but affect the final self-similar front velocity.

The variation of the ionization fraction ( $\eta = n_i / (n_i + n_n)$ ) is very sensitive to the temperature (Fig.4). Depending on terms present in momentum equation that drive the expansion, The limit of  $\eta \rightarrow 1$  has been reached differently, it is attained faster when only one effect is present. Once the laser is

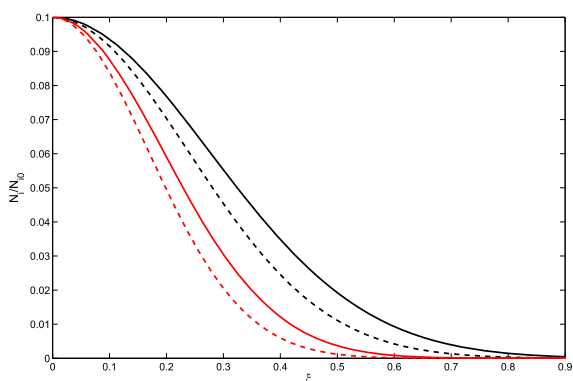


Fig. 3. (Color online) Normalized ion density versus  $\xi$ . Red lines corresponds to the expansion governed by the thermal pressure or the electrostatic potential and black lines both two effects taking into account. dashed lines with  $T = 0.15$  eV and solid lines with  $T = 0.2$  eV. The ion initial density is  $n_{i0} = 0.1n_0$ .

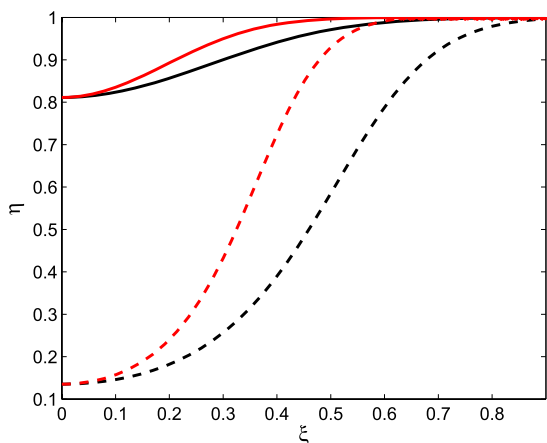


Fig. 4. (Color online) The variation of the ionization fraction during the self-similar expansion driven by the electrostatic force or thermal pressure (red lines) and the combination of the two effects (black lines) for  $T = 0.15$  eV (---) and  $T = 0.16$  eV (—).

turned off, the main ionization process is attributed to impact between energetic electrons and neutral atoms. With higher temperature the saturation is faster. But the combination of the two effect leads to make the variation of the ionization fraction slower. An ideal gas with higher pressure means that density or temperature is higher. Clearly, increasing the density turns out to increase the collision between electrons and neutral atoms to reach the ionization saturation. On the other hand higher temperature corresponds to more important kinetic energy and therefore, increases collisional probability.

#### IV. CONCLUSION

We have shown that the self-similar expansion of a finite size plasma is driven by the electrostatic potential and the ther-

mal pressure. One of this effect cannot be neglected in favor of the other, particularly close to the plasma source region. The ionization effect, considered through Saha's equation, is found to affect the density profile which behaves differently from a fully ionized plasma. The ionization of the residual neutral atoms occurs at lower temperatures and reaches the saturation limit faster as the initial ionized fraction increases. Higher energy deposited by the laser pulse during the plasma formation enhances the self-similar domain for which the quasi-neutral expansion occurs.

#### REFERENCES

- [1] G. Hairapetian and R. L. Stenzel, "Particle dynamics and currentfree double layers in an expanding, collisionless, twoelectronpopulation plasma," *Phys. Fluids B*, vol. B3, no4, pp.899-914, Apr. 1991.
- [2] P. L. Sachdev, self-similarity and beyond: "exact solution of nonlinear problems," Chapman and Hall/CRC, Boca Raton, 2000.
- [3] P. Mora, "Collisionless expansion of a Gaussian plasma into a vacuum," *Phys. Plasmas*, vol. 12, no. 11, pp. 112102-1-112102-8, Nov. 2005.
- [4] M. Murakami, Y. G. Kang, K. Nishihara, S. Fujioka and H. Nishimura, "Ion energy spectrum of expanding laser-plasma with limited mass," *Phys. Plasmas*, vol. 12, no. 6, pp. 062706-1-062706-8, Jun. 2005.
- [5] N. Kumar and A. Pukhov, "Self-similar quasi-neutral expansion of a collisionless plasma with tailored electron temperature profile," *Phys. Plasmas*, vol. 15, no. 5, 053103-1-053103-1-5, May 2008.
- [6] B. Doggett and J. G. Lunney, "Expansion dynamics of laser produced plasma," *J. Appl. Phys.*, vol. 109, pp.093304-1-093304-6, May (2011).
- [7] D. Bennaceur-Doumaz and M. Djebli, "Effect of ionization on laser-induced plume self-similar expansion," *Appl. Surf. Sci.*, vol. 255, pp. 4595-4599, Dec. 2009.
- [8] K. I. Popov, V. Yu. Bychenkov, W. Rozmus and L. Ramunno, "A detailed study of collisionless explosion of single- and two-ion-species spherical nanoplasmas," *Phys. Plasmas*, vol. 17, no. 8, 083110-1-083110-13, Aug. 2010.
- [9] J. Hermann, C. Boulmer-Leborgne and D. Hong, "Diagnostics of the early phase of an ultraviolet laser induced plasma by spectral line analysis considering self-absorption", *J. Appl. Phys.* vol. 83, no. 2, pp. 691-697, 1998.
- [10] Ya. B. Zel'dovich and Yu. p. Raizer, "Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena," Academic Press, New York 1966.
- [11] W. Lotz, "Electron-impact ionization cross-sections for atoms up to  $Z=108$ ," *Z. Physik*, vol. 232, no.2, pp 101-107, 1970.
- [12] V. Akkerman, C. K. Law and V. Bychkov, "Self-similar accelerative propagation of expanding wrinkled flames and explosion triggering," *Phys. Rev. E*, vol. 83, no.2, pp. 026305-1-026305-6, Feb. 2011.
- [13] A. E. Kaplan, B. Y. Dubetsky and P. L. Shkolnikov, "Shock Shells in Coulomb Explosions of Nanoclusters," *Phys. Rev. Lett.*, vol. 91, no. 14, pp. 143401-1-143401-4, Oct. 2003.
- [14] V. K. Unnikrishnan, K. Aiti, V. B. Kartha, C. Santhosh, G. P. Gupta and B. M. Suri, "Measurements of plasma temperature and electron density in laser-induced copper plasma by time-resolved spectroscopy of neutral atom and ion emissions," *PRAMANA journal of Physics*, vol. 74, no. 6, pp.983-993, Jun. 2010.
- [15] R. Fermous, D. Bennaceur-Doumaz and M. Djebli, "A one-dimensional plume plasma expansion: Self-similar approach," *Phys. Lett. A*, vol. 376, pp. 500-504, Nov. 2012.
- [16] Sack H. and Schamel, "Plasma expansion into vacuum: a hydrodynamical approach," *Phys. Rep.*, vol. 156, no.6, pp. 311-395, Jun. 1987.

**Moahamed Fawzi Mahboub** received the master degree in theoretical physics from Algiers university of Sciences and technology (USTHB), Algeria and is currently preparing his Ph.D. in plasma physics. His research interests include plasma expansion and numerical simulation.

**Mourad Djebli** received the master degree in theoretical physics from Algiers university of Sciences and technology (USTHB), Algeria, and the Ph.D. in plasma physics. He is currently with USTHB, where he is a head of a research group and a Professor of physics. His research interests include laser plasma interaction and strongly coupled systems.