

# A Semi-Classical Signal Analysis Method for the Analysis of Turbomachinery Flow Unsteadiness

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**Abstract**—This paper presents the use of a semi-classical signal analysis method that has been developed recently for the analysis of turbomachinery flow unsteadiness. We will focus on the correlation between the Semi-Classical Signal Analysis parameters and some physical parameters in relation with turbomachinery features. To demonstrate the potential of the proposed approach, a static pressure signal issued from a rotor/stator interaction of a centrifugal pump is studied. Several configurations of the pump are compared.

**Keywords**—Semi-classical signal analysis, turbomachines, new indices, physical parameters

## I. INTRODUCTION

A widespread industrial application of turbomachines has been found on literature. This kind of machines has been widely adopted by public facilities and heavy industries. A state-of-the-art review has shown the potential development in this field. It was pointed out that many machines having moderate efficiency and subject to operating anomalies are still in operation and could certainly be improved by making use of today's technology.

There are many sources of anomalies that can degrade performances of a turbomachine and sometimes destroy it. The need of powerful and reliable tools to analyze and control the behavior of these machines is crucial. The targeted objectives are threefold: 1) predicting the performance collapse due to cavitation, degassing or pumping, 2) advanced analysis of pressure waves downstream turbomachines, 3) diagnostic of turbomachines conduct.

To achieve the anticipated performance from turbomachines in addition to accurate calibration as well, many signal measurements and post-processing methods are needed. Standard methods are used to analyze turbomachinery signals such pressure measurements [1], [6], [7]. Among these methods, Fourier transform is one of the most used and established methods. Unfortunately methods based on the Fourier transform are not suitable for non-stationary signals, and as it is known, most of the studied signals are non-stationary. So other methods such as Short time Fourier transform, and Wavelets have been used but both have some limitations [7]. Recently, a new method based on a semi-classical approach has been developed by Laleg et al in [2],

[5], [4]. This method provides new spectral parameters that give relevant information on the signal that is being analyzed based on interpreting that signal as a potential of Schrödinger operator. We will denote this method SCSA for semi-classical signal analysis. The SCSA has been used first for the analysis of arterial blood pressure signals and has given interesting results [3], [4]. More than a good reconstruction of the signal it has been shown that some of the SCSA parameters are correlated to physiological parameters of great interest. Moreover, the SCSA parameters allow the discrimination between different physiological and pathological situations unlike classical parameters that are currently used in clinical practice. In this paper, we propose to apply the SCSA to the analysis of turbomachines signals. We will focus on the correlation of the SCSA parameters to some physical parameters. In the next section, we will introduce briefly the basis of the SCSA method. In section 3, we will present the first results obtained in the application of the SCSA to turbomachines. A discussion will follow introducing some advantages of SCSA and hence possible applications of the SCSA to turbomachines and describing current work.

## II. A SEMI-CLASSICAL SIGNAL ANALYSIS METHOD

### A. Definition

The main idea behind the SCSA method is first to interpret a measured signal (Pressure, Velocity, Flow rate...etc) as a potential of a Schrödinger operator. The signal is then represented using the discrete spectrum of this operator, which introduces new spectral parameters that contain relevant information on the signal.

Let us consider the signal  $y$  as a potential of the Schrödinger operator such that:

$$H_h(y) = -h^2 \frac{d^2}{dt^2} - y. \quad (1)$$

Under some assumptions on the signal [2], [4], [5], the SCSA technique consists in reconstructing the signal  $y$  with the discrete spectrum of  $H(h; y)$  using the following formula:

$$y_h(t) = 4h \sum_{n=1}^{N_h} \kappa_{nh} \psi_{nh}^2(t), \quad t \in \quad (2)$$

where  $-\kappa_{nh}^2$  with  $\kappa_{nh} > 0$  and  $\kappa_{1h} > \kappa_{2h} > \dots > \kappa_{N_h h}$ ,  $n = 1, \dots, N_h$  are the negative Eigenvalues of  $H(h, y)$  and  $\psi_{nh}$ ,  $n = 1, \dots, N_h$  are the associated  $L^2$  - normalized eigenfunctions which satisfies:

$$H_h(y)\psi = -\kappa_{nh}^2 \psi_{nh}. \quad (3)$$

As described in [2], [5], the parameter  $h$  plays an important role in the SCSA. Indeed as  $h$  decreases the approximation of the signal by  $y_h$  improves.

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### B. SCSA parameters and properties

The SCSA method introduces new spectral quantities that provide relevant information on the signal. These quantities could be the negative Eigenvalues and some Riesz means of these Eigenvalues [2], [3]. We will call these Riesz means invariants (since they are related to the invariants in time of a Korteweg-de Vries equation as explained in [2]). We will be interested in this paper in the first two invariants denoted by :

$INV_{jh}, j = 1, 2$  and which are defined by:

$$INV_{1h} = 4h \sum_{n=1}^{N_h} k_{nh}, \quad (4)$$

$$INV_{2h} = \frac{16}{3} h \sum_{n=1}^{N_h} k_{nh}^2. \quad (5)$$

The properties of the SCSA are widely discussed in [2]. Interesting results regarding the asymptotic behavior of the SCSA parameters have been considered, when  $h \geq 0$ . It has been shown for example that

$$\lim_{h \rightarrow 0} INV_{1h} = \int_{-\infty}^{+\infty} y(t) dt, \quad (6)$$

$$\lim_{h \rightarrow 0} INV_{2h} = \int_{-\infty}^{+\infty} y(t)^2 dt. \quad (7)$$

Some semi-classical questions related to signal analysis have been considered in [5] which give an idea on the convergence of the SCSA.

Applying the definition of the limit in (6) and (7) we can write:

For  $\forall \epsilon > 0 \exists h_1$  and  $h_2 / \forall h < \min(h_1, h_2)$

$$|INV_{1h} - \int_{-\infty}^{+\infty} y(t) dt| < \epsilon, \quad (8)$$

$$|INV_{2h} - \int_{-\infty}^{+\infty} y(t)^2 dt| < \epsilon. \quad (9)$$

For fixed very small  $\epsilon$ :

$$INV_{1h} = INV_1 = \int_{-\infty}^{+\infty} y(t) dt, \quad (10)$$

$$INV_{2h} = INV_2 = \int_{-\infty}^{+\infty} y(t)^2 dt. \quad (11)$$

Let  $y$  be a positive continuous periodic function of period  $T$ . We would like to study  $y$  on only one period. For this purpose, we introduce the function  $y_T$  such that:

$$y_T = \begin{cases} y(t), & \text{for } t = [0, T], \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

The invariants of  $y_T$  are given by:

$$INV_{1T} = \int_0^T y(x) dx, \quad (13)$$

$$INV_{2T} = \int_0^T y(x)^2 dx. \quad (14)$$

After applying Fourier transform on (13) and (14), a simplified form of both 1<sup>st</sup> and 2<sup>nd</sup> invariants can be easily obtained for sinusoidal signals of amplitude  $A$ .

$$INV_{1T} = \frac{A}{f} \quad (15)$$

$$INV_{2T} = \frac{3A^2}{2f} \quad (16)$$

For simplicity, we put  $INV_{1T} = INV_1, INV_{2T} = INV_2$ .

### III. APPLICATION OF THE SCSA TO TURBO MACHINES

Since turbomachinery design process depends mainly on strong signal analysis and post processing procedure as well, the need of relating the new introduced spectral parameters to turbomachinery parameters became very important and highly insistent need.

The new parameters that are generated by applying SCSA are first and second Invariants of the measured signals, in addition to the Eigenvalues of the same signals. These parameters will lead to unveil much information that is hidden in the analyzed signals.

#### A. Unveiling the physics behind the invariants

The main procedure in this analysis is relating the outcomes of applying SCSA on turbomachinery to the physics behind them. This can be achieved by retrieve each newly introduced spectral parameter to its basic units.

In the practical work, three turbomachines generated signals were used and tested which are velocity (V), pressure (P) and volumetric flow rate (Q). Based on equations (15) and (16), each kind of previous signals was retrieved back to its 1<sup>st</sup> and 2<sup>nd</sup> invariants as the following:

TABLE I  
PHYSICAL MEANINGS BEHIND SIGNAL INVARIANTS

	Pressure, P[Pa]	Velocity, V[m/s]	Flow rate, Q[m <sup>3</sup> /s]
$INV_1$	$\frac{[Kg]}{[m][s]}$ : dynamic viscosity	[m]: length	[m <sup>3</sup> ]: Volume
$INV_2$	$\frac{1}{[s]} \left( \frac{[Kg]}{[m][s]} \right)^2$ : time rate of the squared dynamic viscosity	$\frac{[m^2]}{[s]}$ : kinematic viscosity	$\frac{[m^6]}{[s]}$ : not used

Besides analyzing the invariants alone, it is possible to make combinations of these invariants by simple arithmetic operations like multiplication and division.

TABLE II  
BASIC ARITHMETIC OPERATION OVER INVARIANTS OF SELECTED SIGNALS

	P [Pa]	V [m/s]	Q [m <sup>3</sup> /s]
$INV_1 \times INV_2$	time rate of the cube of dynamic viscosity	Volumetric flow rate	Not used
$\frac{INV_2}{INV_1}$	Pressure	Velocity	Volumetric flow rate

The previous results show that it is possible to define new indices to describe the turbomachinery unsteadiness just by combining invariants.

Now we will be interested on how we can redefine some standards fluid “numbers”, such as the Reynolds number, Froude number, and Cavitations number, using previous SCSA parameters.

In the following, we will detail the example of the Reynolds number and the same procedure is applied for the other quantities. The Reynolds number is defined by

$$Re = \frac{\rho VL}{\mu} \quad (17)$$

Where,  
 $\rho$  : density of the fluid.  
 $V$  : velocity of the fluid.  
 $L$  : characteristic length.  
 $\mu$  : dynamic viscosity.

To define an invariant  $Re$ , the density being a material property, equivalent invariants length ( $INV_{1V}$ ), velocity ( $\frac{INV_{2V}}{INV_{1V}}$ ) and dynamic viscosity ( $INV_{1P}$ ) are used.

$$Re_{INV} = \frac{\rho INV_{2V}}{INV_{1V}} \times \frac{INV_{1V}}{INV_{1P}} = \frac{\rho INV_{2V}}{INV_{1P}} \quad (18)$$

Where  $INV_{jX}$ ,  $j = 1, 2$  denotes the invariants computed using the signal  $X$ .

Same procedure is used for the rest of dimensionless parameters; an illustration of some of them is presented in table III.

TABLE III  
DIMENSIONLESS PARAMETERS WRITTEN USING INVARIANTS

Invariant Reynolds $Re_{INV}$	$\frac{\rho VL}{\mu} = \frac{\rho INV_{2V}}{INV_{1P}}$
Invariant Froude $Fr_{INV}$	$\frac{V}{\sqrt{gL}} = \frac{INV_{2V}}{INV_{1V} \sqrt{g}}$
Invariant Euler $Eu_{INV}$	$\frac{P}{\rho V^2} = \frac{1}{\rho} \frac{INV_{2P} INV_{1V}^2}{INV_{1P} INV_{2V}^2}$

### B. Analysis of multi-period signals

For the analysis of multi-period signals we compute the SCSA parameters in each period. It is worth noting that if the time interval is equal or multiple of the measured signal both Invariants and Eigen values will have a constant trend, where they will have a periodic trend if the used period is a fraction of the measured signal as it is illustrated in figure 1, where the first plot shows the original signal  $y(t)$  coinciding over the reconstructed one  $y_h(t)$ .

In the case of constant Invariants and Eigen values, all dimensionless parameters were previously mentioned will be also constant. But in case of periodic Invariants, all those parameters will be periodic as well.

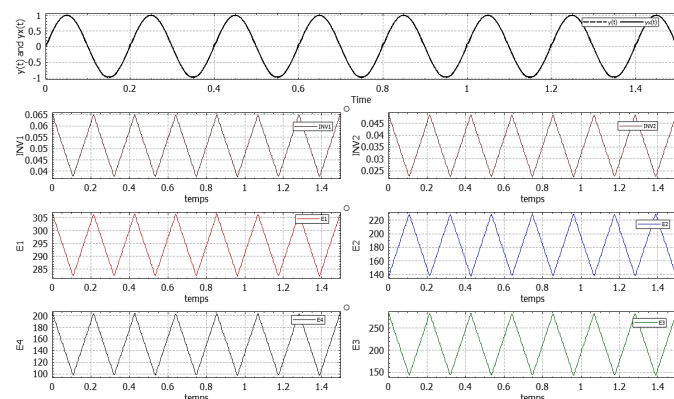


Fig. 1 Pressure signal analyzed with half of its period

### C. Effect of blades number in turbomachines pressure signal

A test was performed on three turbo machines. Each one has different number of blades 4, 5 and 6. The analysis target was to find any relation between number of blades, signal's amplitude, Invariants and phase shift.

The analysis showed that number of blades has no relation with the amplitude of the signal (figure 2). Beside that it was proved that signals have a periodic trend with no phase shift and some distortion from unwanted frequency content.

By applying those signals to the source code, the results were very close and somehow coincide with too low error percentage.

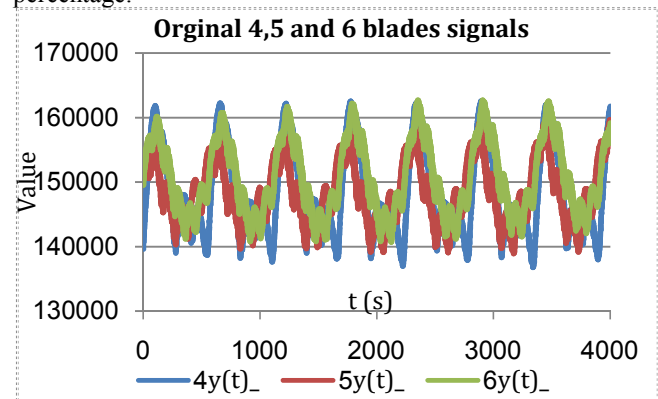


Fig. 2 Original signals from 4, 5 and 6 blades for turbo machines

Regarding the Invariants, it was shown that the higher number of blades will give higher values for 1<sup>st</sup> and 2<sup>nd</sup> Invariants with Gaussian distribution (figures 3 and 4).

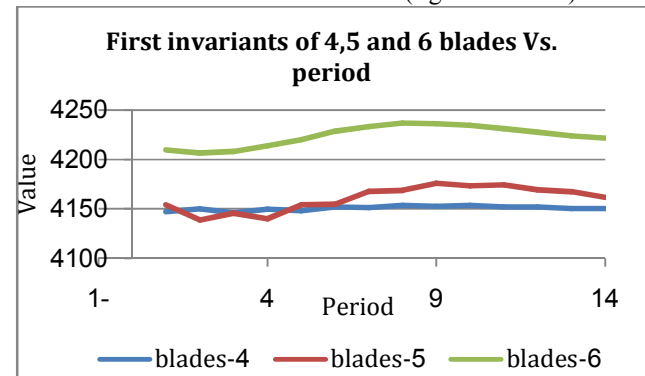


Fig. 3 1st invariants for 4, 5 and 6 blades turbo machine

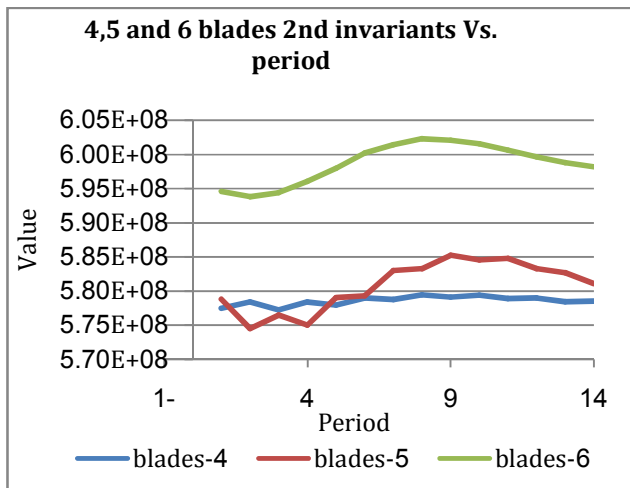


Fig. 4 2nd invariants for 4, 5 and 6 blades turbo machines

#### IV. DISCUSSION AND CONCLUSION

We have introduced in this paper preliminary results on the application of a new signal analysis method to turbomachines signals. Advanced studies on the subject are still carried. As shown above the SCSA provides new spectral parameters that contain relevant information on flow unsteadiness. The analysis of signals issued from turbomachines is a new and promising approach. It responds to many issues. For example, the SCSA seems to be suitable for the analysis of non-stationary signals since the SCSA parameters are very sensitive. Most of the signals that are studied are non-stationary and standard signal analysis methods are either non suitable for such signals (Fourier based approaches) or suffer some limitations when used in the analysis of turbomachines signals (Tim-frequency and time scales methods). So this can be a great advantage of the SCSA. It is worth to mention also that SCSA parameters are very sensitive to very small and limited amplitude or frequency disturbance. In addition, we believe that the SCSA parameters can be used to predict the performance collapse due to cavitations, degassing and pumping.

#### REFERENCES

- [1] M. Cudina, "Detection of cavitation phenomenon in a centrifugal pump using audible sound". *Mechanical Systems and Signal Processing*, 17(6), 1335-1347, 2003.
- [2] T.M.Laleg-Kirati, E. Crépeau and M. Sorine "Semi-classical signal analysis," submitted for publication.
- [3] T.M.Laleg-Kirati, C. Médigue, Y. Papelier, F. Cottin and A. Van de Louw, "Validation of a semi-classical signal analysis method for stroke volume variation assessment: A comparison with the PiCCO technique" *Annals of biomedical engineering*, vol. 38, no. 12, pp. 3618-3629, 2010.
- [4] T. M. Laleg, C. Médigue, F. Cottin, and M. Sorine. "Arterial blood pressure analysis based on scattering transform II", in *Proc. EMBC, Lyon, France, August 2007*
- [5] B. Helffer and T.M.Laleg-Kirati, "On semi-classical questions related to signal analysis," *Asymptotic Analysis Journal*, to be published.
- [6] I. Howard, "A review of rolling element bearing vibration "detection, 290 diagnosis and prognosis. published by DSTO Aeronautical and Maritime Research Laboratory, 1994.
- [7] Z.K. Peng and F.L. Chu, "Application of the wavelet transform in machine condition monitoring and fault diagnostics: a review with bibliography", *Mechanical systems and signal processing*, vol. 18, pp. 199-221, 2004.