World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:1, No:6, 2007

Variable Rough Set Model and Its Knowledge Reduction for Incomplete and Fuzzy Decision Information Systems

Da-kuan Wei, Xian-zhong Zhou, Dong-jun Xin, and Zhi-wei Chen

Abstract**—**The information systems with incomplete attribute values and fuzzy decisions commonly exist in practical problems. On the base of the notion of variable precision rough set model for incomplete information system and the rough set model for incomplete and fuzzy decision information system, the variable rough set model for incomplete and fuzzy decision information system is constructed, which is the generalization of the variable precision rough set model for incomplete information system and that of rough set model for incomplete and fuzzy decision information system. The knowledge reduction and heuristic algorithm, built on the method and theory of precision reduction, are proposed.

Keywords**—**Rough set, Incomplete and fuzzy decision information system, Limited valued tolerance relation, Knowledge reduction, Variable rough set model

I. INTRODUCTION

ROUGH set theory, proposed by Polish mathematician Professor Z. Pawlak in 1982, is an excellent method to acquire knowledge, and the traditional rough set theory can only process complete information system, but there are rather more incomplete or fuzzy information system in applications, the classical rough set can not handle them because of weak power of data acquisition, therefore the Pawlak's rough set theory must be extended in order to overcome the problem. At the present, there are mainly two approaches to extend the classical rough set theory from complete information system to incomplete information system. The first is indirect method that transforms an incomplete decision information system to complete decision information system by adding attribute values to missing values. The second is direct method that extends the relative concepts in classical rough set theory to those in incomplete decision information system, which is studied with great concentrations by many researchers all over the world. M.Kryszkiewicz proposed tolerance relation in incomplete information system with missing values to extend the rough set model [1]--[2]. R.Slowinski and D.Vanderpooten generalized definition of rough approximations based on non-symmetric similarity relation [3]. G.Y. Wang et al. extended rough set model based on valued tolerance relation and limited tolerance relation respectively in incomplete information system [4]--[5].

In many virtual realities, there exists a type of incomplete information system with decisions. Furthermore, if the decisions are unambiguous [1]--[9], then the system is incomplete decision information system which has been researching by a lot of scholars for the last years; if the decisions are fuzzy, then the system is incomplete and fuzzy decision information system that has not been deeply discussed until now. This paper, based on the previous researching on the incomplete and fuzzy decision information system [10]--[12], introduces a new approach by virtue of variable precision rough set model for incomplete information system [13] to construct a new kind of rough set model which will be called variable rough set model for incomplete and fuzzy decision information system, which is a generalization of rough set model for incomplete decision information system and is also that for complete fuzzy decision information system. Theory and method of attribute reduction under variable rough set model are suggested in this paper.

This paper is structured as follows: Section 2 introduces the variable precision rough set model and its properties for incomplete information system. Section 3 presents the concept of incomplete and fuzzy decision information system. Section 4 proposes the variable rough set model for incomplete and fuzzy decision information system. Section 5 provides the method of knowledge reduction for the incomplete and fuzzy decision information system based on the variable rough set model. Section 6 shows a sample. Finally, section 7 concludes this paper.

II. THE VARIABLE PRECISION ROUGH SET MODEL FOR INCOMPLETE INFORMATION SYSTEM [13]

We ever studied deeply the rough set models and their knowledge reduction for the incomplete and fuzzy decision information system by use of tolerance relation and nonsymmetrical similarity relation [10]--[12], but the description of the relation between objects is not sufficiently precise because of adopting tolerance relation or similarity relation instead of equivalence relation. Naturally we associate that it is necessary to establish a new model with fuzzy measurement for every object. For this reason, we introduce a series of concepts below.

Definition 1 Let $S=(U,A,V,f)$ be an incomplete

Manuscript received March 10, 2006. The work was supported in part by the National Science Foundation of China (No.70571032)

Da-kuan Wei is with School of Automation, Nanjing University of Science and Technology, Nanjing 210094, and School of Information Engineering, Hunan University of Science and Engineering, Yongzhou 425006, China (Tel: 86-013601457238 or 013187101979; E-mail: weidakuan@126.com).

Xian-zhong Zhou is with School of Management and Engineering, Nanjing University, Nanjing 210093, China.

Dong-jun Xin and Zhi-wei Chen are with School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China.

information system, where U is a nonempty finite set of objects called universe of discourse, A is a nonempty finite set of condition attributes; and for every $a \in A$, such that $f: U \to V_a$, $V = \bigcup_{a \in A} V_a$, where V_a, which possibly contains null values (a null value is usually denoted by $*$), is called the value set of attribute a [1]--[6].

For $x_i, x_j \in U$, the joint probability of two objects with the same value on all condition attributes is said to be tolerance degree, and denoted by $\mu(x, y) = \prod_{a \in A} P(a(x_i) = a(x_j))$.

Let T be a tolerance relation and k $(0 \le k \le 1)$ be a constant. Then $R_k = \{(x, y) \mid (x, y) \in T \wedge \mu(x, y) > k\}$ is called limited valued tolerance relation with tolerance degree k.

$R_k(x) = \{y \in U \mid \mu(x, y) > k\} = \{y \in U \mid (x, y) \in R_k\}$ is called limited valued tolerance class.

Let $X, Y \subset U, 0.5 < \delta \le 1$. Then X is said to be majority inclusion in Y if and only if $D(Y/X) \ge \delta$, where

$$D(Y/X) = \begin{cases} |X \cap Y| / |X| & |X| > 0 \\ 1 & |X| = 0 \end{cases}.$$

Where $|\bullet|$ is the cardinal number of a set.

Definition 2 Let $S = (U, A, V, f)$ be an incomplete information system, R_k is limited valued tolerance relation on U, $X \subseteq U$. Then the $\delta-$upper and the $\delta-$lower approximation of X based on tolerance degree k, denoted by $\overline{R}_{k\delta}(X)$, $\underline{R}_{k\delta}(X)$ respectively, are defined as follows:

$$\overline{R}_{k\delta}(X) = \{x \mid x \in U \wedge D(X / R_k(x)) > 1 - \delta\} \quad \text{and}$$

$$\underline{R}_{k\delta}(X) = \{x \mid x \in U \wedge D(X / R_k(x)) \ge \delta\}.$$

$\underline{R}_{k\delta}(X)$ is also called the $\delta-$positive region of X based on tolerance degree k, denoted by $POS_{k\delta}(X)$.

The $\delta-$boundary region and $\delta-$negative region of X based on tolerance degree k, represented by $BNR_{k\delta}(X)$ and $NEG_{k\delta}(X)$ separately, are defined as follows:

$BNR_{k\delta}(X) = \{x \mid x \in U \wedge 1 - \delta < D(X / R_k(x)) < \delta\}$ and

$NEG_{k\delta}(X) = \{x \mid x \in U \wedge D(X / R_k(x)) \le 1 - \delta\}$.

As $\delta = 1$, the variable precision rough set model is the common rough set model for incomplete information system.

From definition 1 and definition 2, we can obtain the following conclusions.

Lemma Let $X, Y \subset U, 0.5 < \delta \le 1, 0 \le k \le 1$, then the following properties hold:

(1) $\underline{T}(X) \subseteq \underline{R}_{k\delta}(X) \subseteq \overline{R}_{k\delta}(X) \subseteq \overline{T}(X)$;

(2) $\underline{R}_{k\delta}(\Phi) = \overline{R}_{k\delta}(\Phi) = \Phi$, $\underline{R}_{k\delta}(U) = \overline{R}_{k\delta}(U) = U$;

(3) $\overline{R}_{k\delta}(X) \cup \overline{R}_{k\delta}(Y) \subseteq \overline{R}_{k\delta}(X \cup Y)$,

$\overline{R}_{k\delta}(X) \cap \overline{R}_{k\delta}(Y) \supseteq \overline{R}_{k\delta}(X \cap Y)$;

(4) $\underline{R}_{k\delta}(X) \cup \underline{R}_{k\delta}(Y) \subseteq \underline{R}_{k\delta}(X \cup Y)$,

$\underline{R}_{k\delta}(X \cap Y) \subseteq \underline{R}_{k\delta}(X) \cap \underline{R}_{k\delta}(Y)$.

III. The Concept of Incomplete and Fuzzy Decision Information System

Based-on the concept of incomplete information system and that of fuzzy decision information system, we shall introduce the concept of incomplete and fuzzy decision information system as follows.

Definition 3 [10]--[12] Incomplete and fuzzy decision information system, denoted still by S, is defined as:

$$S = (U, A, V, F; D, W, G).$$

Where: (1) U is a nonempty finite set of objects, i.e., $U = \{x_1, x_2, ..., x_n\}$;

(2) A is a nonempty finite set of condition attributes, i.e., $A = \{a_1, a_2, ..., a_m\}$;

(3) $F = \{f_1, f_2, ..., f_m\}$ is a set of condition attribute mappings, $f_j: U \to V_j$, $j \le m$, and there is at least a V_j containing null values $*, V = \bigcup_{1 \le j \le m} V_j$;

(4) D is a nonempty finite set of decision attributes, i.e., $D = \{d_1, d_2, ..., d_p\}$;

(5) $G = \{g_1, g_2, ..., g_p\}$ is a set of fuzzy decision mappings, $g_k: U \to W_k$, $W_k \in F(U)$, $W = \bigcup_{1 \le k \le p} W_k$ and $W_k(x_i) \in [0,1]$, $i \le n$, $k \le p$, $F(U)$ is the whole of all fuzzy sets of U.

If $p = 1$, i.e., let $D = \{d\}$, $G = \{g\}$, $W \in F(U)$. Then $S = (U, A, V, F; \{d\}, W, \{g\})$ is called incomplete and fuzzy (single) decision information system, this type is researched in this paper. While $p > 1$, S is called incomplete and fuzzy multi-decision information system.

From definition 3, S is incomplete information system as D, W and G are empty sets; and S is complete fuzzy decision information system as (U, A, V, F) is complete information system.

IV. The Variable Rough Set Model for Incomplete and Fuzzy Decision Information System

Definition 4 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 < \delta \le 1$, $R_k^B(x)$ be a limited valued tolerance class of x with respect to B in incomplete space (U, A, V, F). For every $W \in F(U)$, let

$$\overline{R}^B_{k\delta}(W)(x) = \max\{\delta \cdot W(y) : y \in R_k^B(x)\},$$

$$\underline{R}^B_{k\delta}(W)(x) = \min\{\delta \cdot W(y) : y \in R_k^B(x)\}$$

Where $W(y)$ represents membership function value of y. $\overline{R}^B_{k\delta}(W)$, $\underline{R}^B_{k\delta}(W)$ are called $\delta-$upper approximation set and $\delta-$lower approximation set of fuzzy set W based on tolerance degree k in incomplete space (U, A, V, F) separately. Such a rough set model, based on $\delta-$upper approximation set and $\delta-$lower approximation set, is called the variable rough set model in incomplete and fuzzy decision information system. Obviously, $\overline{R}_{k\delta}^{B}(W)$, $\underline{R}_{k\delta}^{B}(W)$ are a pair of fuzzy sets.

From definition 4, as $\delta = 1$, $\overline{R}_{k\delta}^{B}(W)$ and $\underline{R}_{k\delta}^{B}(W)$ are the usual upper approximation set and lower approximation set for incomplete and fuzzy decision information system respectively [11].

Theorem 1 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$. Then the following properties hold:

(1) $\underline{R}_{k\delta}^{B}(W) \subseteq \overline{R}_{k\delta}^{B}(W)$; (2) If $W_1 \subseteq W_2$, Then: $\overline{R}_{k\delta}^{B}(W_1) \subseteq \overline{R}_{k\delta}^{B}(W_2)$, $\underline{R}_{k\delta}^{B}(W_1) \subseteq \underline{R}_{k\delta}^{B}(W_2)$.

Theorem 2 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$. Then:

$$\overline{R}_{k\delta}^{B}(\overline{R}_{k\delta}^{B}(W)) \supseteq \underline{R}_{k\delta}^{B}(\overline{R}_{k\delta}^{B}(W)) \supseteq \overline{R}_{k\delta}^{B}(\underline{R}_{k\delta}^{B}(W)) \supseteq \underline{R}_{k\delta}^{B}(\underline{R}_{k\delta}^{B}(W))$$

Proof According to theorem 1, we can easily prove that $\overline{R}_{k\delta}^{B}(\overline{R}_{k\delta}^{B}(W)) \supseteq \underline{R}_{k\delta}^{B}(\overline{R}_{k\delta}^{B}(W))$, $\overline{R}_{k\delta}^{B}(\underline{R}_{k\delta}^{B}(W)) \supseteq \underline{R}_{k\delta}^{B}(\underline{R}_{k\delta}^{B}(W))$ are valid.

Now we only prove $\underline{R}_{k\delta}^{B}(\overline{R}_{k\delta}^{B}(W)) \supseteq \overline{R}_{k\delta}^{B}(\underline{R}_{k\delta}^{B}(W))$. In fact, for every $x \in U$, then

$$\overline{R}_{k\delta}^{B}(\underline{R}_{k\delta}^{B}(W))(x) = \max_{y \in R_k^B(x)} \{\delta \cdot \underline{R}_{k\delta}^{B}(W)(y)\}$$

$$= \max_{y \in R_k^B(x)} \{\delta \cdot \min_{z \in R_k^B(y)} \{\delta \cdot W(z)\}\} \le \min_{y \in R_k^B(x)} \{\delta \cdot \max_{z \in R_k^B(y)} \{\delta \cdot W(z)\}\}$$

$$= \min_{y \in R_k^B(x)} \{\delta \cdot \overline{R}_{k\delta}^{B}(W)(y)\} = \underline{R}_{k\delta}^{B}(\overline{R}_{k\delta}^{B}(W))(x).$$

Hence, $\underline{R}_{k\delta}^{B}(\overline{R}_{k\delta}^{B}(W)) \supseteq \overline{R}_{k\delta}^{B}(\underline{R}_{k\delta}^{B}(W))$.

Theorem 3 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$. For every $x \in U$, then:

$$\overline{R}_{k\delta}^{B}(W^C)(x) + \underline{R}_{k\delta}^{B}(W)(x) = \delta,$$

$$\underline{R}_{k\delta}^{B}(W^C)(x) + \overline{R}_{k\delta}^{B}(W)(x) = \delta.$$

Where W^C is the complementary set of the fuzzy set W.

Proof $\overline{R}_{k\delta}^{B}(W^C)(x) = \overline{R}_{k\delta}^{B}(W^C(x))$

$$= \max\{\delta \cdot (1 - W(y)) \mid y \in R_k(x)\}$$

$$= \delta \cdot \max\{1 - W(y) \mid y \in R_k(x)\}$$

$$= \delta \cdot (1 - \min\{W(y) \mid y \in R_k(x)\})$$

$$= \delta - \underline{R}_{k\delta}^{B}(W)(x),$$

$$\Rightarrow \overline{R}_{k\delta}^{B}(W^C)(x) + \underline{R}_{k\delta}^{B}(W)(x) = \delta.$$

Similarly, the other formula can be proved.

According to definition 4, we immediately obtain:

Theorem 4 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta_1 \le \delta_2 \le 1$. Then:

$$\underline{R}_{k\delta_1}^{B}(W) \subseteq \underline{R}_{k\delta_2}^{B}(W) \subseteq \underline{R}_{k1}^{B}(W),$$

$$\overline{R}_{k\delta_1}^{B}(W) \subseteq \overline{R}_{k\delta_2}^{B}(W) \subseteq \overline{R}_{k1}^{B}(W).$$

V. Knowledge Reduction

In this section, we mainly introduce elementary theory and algorithm for knowledge reduction under variable rough set model in an incomplete and fuzzy decision information system.

A. Basic Theory for Precision Reduction

Theorem 5 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$, $0 \le \alpha, \beta \le 1$, and assume that:

$$\overline{R}_{k\delta}^{B}(W)_\beta = \{x \in U : \overline{R}_{k\delta}^{B}(W)(x) \ge \beta\},$$

$$\underline{R}_{k\delta}^{B}(W)_\alpha = \{x \in U : \underline{R}_{k\delta}^{B}(W)(x) \ge \alpha\}.$$

Then: (1) $\overline{R}_{k\delta}^{B}(W)_\beta$ and $\underline{R}_{k\delta}^{B}(W)_\alpha$ monotonously decrease with regard to β and α respectively under the containing relation of sets; (2) $\underline{R}_{k\delta}^{B}(W)_\alpha \subseteq \overline{R}_{k\delta}^{B}(W)_\alpha$; (3) $\alpha \ge \beta \Rightarrow \underline{R}_{k\delta}^{B}(W)_\alpha \subseteq \overline{R}_{k\delta}^{B}(W)_\beta$.

Proof From the definition of $\overline{R}_{k\delta}^{B}(W)_\beta$ and $\underline{R}_{k\delta}^{B}(W)_\alpha$, (1), (2) and (3) can be easily proved.

Especially, as $\alpha \ge \beta$, (1) $\overline{R}_{k\delta}^{B}(W)_\beta = \Phi \Rightarrow \underline{R}_{k\delta}^{B}(W)_\alpha = \Phi$; (2) $\underline{R}_{k\delta}^{B}(W)_\alpha = U \Rightarrow \overline{R}_{k\delta}^{B}(W)_\beta = U$.

Theorem 6 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $C \subseteq B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$, $0 \le \alpha, \beta \le 1$. Then:

(1) $\overline{R}_{k\delta}^{B}(W)_\beta \subseteq \overline{R}_{k\delta}^{C}(W)_\beta$; (2) $\underline{R}_{k\delta}^{C}(W)_\alpha \subseteq \underline{R}_{k\delta}^{B}(W)_\alpha$.

Proof (1) $C \subseteq B \subseteq A \Rightarrow R_k^B(x) \subseteq R_k^C(x)$ for $\forall x \in U \Rightarrow$

$$\max\{\delta \cdot W(y) : y \in R_k^B(x)\} \le \max\{\delta \cdot W(y) : y \in R_k^C(x)\}$$

$$\Rightarrow \overline{R}_{k\delta}^{B}(W)(x) \le \overline{R}_{k\delta}^{C}(W)(x)$$

$$\Rightarrow \overline{R}_{k\delta}^{B}(W) \subseteq \overline{R}_{k\delta}^{C}(W) \Rightarrow \overline{R}_{k\delta}^{B}(W)_\beta \subseteq \overline{R}_{k\delta}^{C}(W)_\beta.$$

(2) The formula can be analogously proved.

Definition 5 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$, $0 \le \beta \le \alpha \le 1$. Then (α, β) precision degree of W is defined as: $\alpha_{k\delta}^{B}(\alpha, \beta) = \left|\underline{R}_{k\delta}^{B}(W)_\alpha\right| / \left|\overline{R}_{k\delta}^{B}(W)_\beta\right|$.

And ruled: as $\overline{R}_{k\delta}^{B}(W)_\beta = \Phi$, then $\alpha_{k\delta}^{B}(\alpha, \beta) = 1$.

Obviously, $0 \le \alpha_{k\delta}^{B}(\alpha, \beta) \le 1$, and $\alpha_{k\delta}^{B}(\alpha, \beta)$ is monotonously non-increasing w. r. t. α and is monotonously non-decreasing w. r. t. β.

From theorem 6 and definition 5 we instantly have:

Theorem 7 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$, $0 \le \beta \le \alpha \le 1$. Then $\alpha_{k\delta}^{A}(\alpha, \beta) \ge \alpha_{k\delta}^{B}(\alpha, \beta)$.

Definition 6 Let $S = \{U, A, V, F; \{d\}, W, \{g\}\}$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1$, $0 \le \beta \le \alpha \le 1$. If $\alpha^A_{k\delta}(\alpha,\beta) = \alpha^B_{k\delta}(\alpha,\beta)$ and $\alpha^B_{k\delta}(\alpha,\beta) \ne \alpha^C_{k\delta}(\alpha,\beta)$ for every $C \subset B$. Then B is said to be (α,β) precision reduction under variable rough set model in an incomplete and fuzzy decision information system S.

From definition 6 known, precision reduction ensures the ratio of the number of the certain objects that variable membership values are not less than α to that of the possible objects that variable membership values are not less than β is invariable.

From theorem 7, definition 5 and 6, we can easily associate that the following results hold.

Theorem 8 Let $S = (U, A, V, F; \{d\}, W, \{g\})$ be an incomplete and fuzzy decision information system, $B \subseteq A$, $0 \le k \le 1$, $0 < \delta \le 1, 0 \le \beta \le \alpha \le 1$. Then B is (α,β) precision reduction under variable rough set model in an incomplete and fuzzy decision information system S if and only if B is the minimal proper subset of condition attributes which simultaneously satisfies the two equations $\overline{R}^B_{k\delta}(W)_\beta = \overline{R}^A_{k\delta}(W)_\beta$ and $\underline{R}^B_{k\delta}(W)_\alpha = \underline{R}^A_{k\delta}(W)_\alpha$.

B. Knowledge Reduction Algorithm

The following provides a heuristic algorithm for precision reduction.

REDUCTION ALGORITHM

Input: Incomplete and fuzzy decision information system $S = (U, A, V, F; \{d\}, W, \{g\})$, the valid values of k, δ α and β ($0 \le k \le 1$, $0 < \delta \le 1$, $0 \le \beta \le \alpha \le 1$).

Output: Reduction B ($B \subseteq A$) of the system S.

Step 1. Calculate the subsets of condition attributes A: $B_1, ..., B_q$, where $|B_r| = |A| - 1$ ($r = 1, ..., q$);

Step 2. Calculate all limited valued tolerance classes $R^A_k(x_i)$ and $R^{B_r}_k(x_i)$ ($1 \le i \le n$, $1 \le r \le q$);

Step 3. Calculate $\overline{R}^A_{k\delta}(W)$, $\underline{R}^A_{k\delta}(W)$, $\overline{R}^{B_r}_{k\delta}(W)$ and $\underline{R}^{B_r}_{k\delta}(W)$ ($r = 1, ..., q$).

Step 4. Compute $\overline{R}^A_{k\delta}(W)_\beta, \underline{R}^A_{k\delta}(W)_\alpha$, $\overline{R}^{B_r}_{k\delta}(W)_\beta$ and $\underline{R}^{B_r}_{k\delta}(W)_\alpha$ ($r = 1, ..., q$).

Step 5. If $\overline{R}^A_{k\delta}(W)_\beta \subset \overline{R}^{B_r}_{k\delta}(W)_\beta$ or $\underline{R}^A_{k\delta}(W)_\alpha \supset \underline{R}^{B_r}_{k\delta}(W)_\alpha$ ($r = 1, ..., q$), then stop (A itself is reduction of the system S); Otherwise, $\overline{R}^A_{k\delta}(W)_\beta = \overline{R}^{B_r}_{k\delta}(W)_\beta$ and $\underline{R}^A_{k\delta}(W)_\alpha = \underline{R}^{B_r}_{k\delta}(W)_\alpha$, go to step 1 (consider continuously the corresponding the subsets of B_r until finding a $B \subseteq B_r \subseteq A$ satisfies that $\overline{R}^B_{k\delta}(W)_\beta = \overline{R}^{B_r}_{k\delta}(W)_\beta = \overline{R}^A_{k\delta}(W)_\beta$ and $\underline{R}^B_{k\delta}(W)_\alpha = \underline{R}^{B_r}_{k\delta}(W)_\alpha = \underline{R}^a_{k\delta}(W)_\alpha$, but $\overline{R}^B_{k\delta}(W)_\beta \subset \overline{R}^C_{k\delta}(W)_\beta$ or $\underline{R}^B_{k\delta}(W)_\alpha \subset \underline{R}^C_{k\delta}(W)_\alpha$ for every $C \subset B_r \subseteq A$).

VI. Example

The following table gives an example of incomplete and fuzzy decision information system S [12].

TABLE I
INCOMPLETE AND FUZZY DECISION INFORMAAION SYSTEM

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
a_1	1	1	3	1	*	3	3	3	2
a_2	2	*	2	2	2	1	2	1	3
a_3	1	3	3	*	1	*	*	2	*
d	0.8	0.6	0.9	0.7	0.8	0.5	0.7	0.6	0.5

Where $U = \{x_1, x_2, ..., x_9\}$, $A = \{a_1, a_2, a_3\}$,

$W = 0.8/x_1 + 0.6/x_2 + 0.9/x_3 + 0.7/x_4 + 0.8/x_5 + 0.5/x_6$
$+ 0.7/x_7 + 0.6/x_8 + 0.5/x_9$.

If $k = 0.2$, $\delta = 0.9$, $\alpha = \beta = 0.5$. then $R^A_k(x_1) = \{x_1, x_4, x_5\}$, $R^A_k(x_2) = \{x_2, x_4\}$, $R^A_k(x_3) = \{x_3, x_7\}$, $R^A_k(x_4) = \{x_1, x_2, x_4\}$, $R^A_k(x_5) = \{x_1, x_5, x_7\}$, $R^A_k(x_6) = \{x_6, x_8\}$, $R^A_k(x_7) = \{x_3, x_7\}$, $R^A_k(x_8) = \{x_6, x_8\}$, $R^A_k(x_9) = \{x_9\}$;

$$\overline{R}^A_{k\delta}(W) = 0.72/x_1 + 0.63/x_2 + 0.81/x_3 + 0.72/x_4 + 0.72/x_5 + 0.54/x_6 + 0.81/x_7 + 0.54/x_8 + 0.45/x_9,$$

$$\underline{R}^A_{k\delta}(W) = 0.63/x_1 + 0.54/x_2 + 0.63/x_3 + 0.54/x_4 + 0.63/x_5 + 0.45/x_6 + 0.63/x_7 + 0.45/x_8 + 0.45/x_9;$$

$\overline{R}^A_{k\delta}(W)_\beta = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$,

$\underline{R}^A_{k\delta}(W)_\alpha = \{x_1, x_2, x_3, x_4, x_5, x_7\}$.

(1) Let $B_1 = \{a_1, a_2\}$, we can calculate

$\overline{R}^{B_1}_{k\delta}(W)_\beta = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$,

$\underline{R}^{B_1}_{k\delta}(W)_\alpha = \{x_1, x_2, x_3, x_4, x_5, x_7\}$,

Then $\overline{R}^A_{k\delta}(W)_\beta = \overline{R}^{B_1}_{k\delta}(W)_\beta$, $\underline{R}^A_{k\delta}(W)_\alpha = \underline{R}^{B_1}_{k\delta}(W)_\alpha$.

Furthermore we can still compute

$\overline{R}^{B_1}_{k\delta}(W)_\beta \subset \overline{R}^{\{a_1\}}_{k\delta}(W)_\beta$, $\overline{R}^{B_1}_{k\delta}(W)_\beta \subset \overline{R}^{\{a_2\}}_{k\delta}(W)_\beta$;

$\underline{R}^{B_1}_{k\delta}(W)_\alpha \supset \underline{R}^{\{a_1\}}_{k\delta}(W)_\alpha$ and $\underline{R}^{B_1}_{k\delta}(W)_\alpha \supset \underline{R}^{\{a_2\}}_{k\delta}(W)_\alpha$.

Therefore $B_1 = \{a_1, a_2\}$ is a $(0.5, 0.5)$ reduction of the incomplete and fuzzy decision information system S.

(2) Let $B_2 = \{a_2, a_3\}$, $B_3 = \{a_1, a_3\}$. By similarly calculating, we can obtain: B_2 and B_3 are not the reductions of S.

VII. Conclusions

The information system with both incomplete information and fuzzy decisions is general and important practice problem to be studied. In this paper, on the base of notion of the variable precision rough set model and the concept of incomplete and fuzzy decision information system, the variable rough set model of incomplete and fuzzy decision information system, based on limited valued tolerance relation, is proposed, which is generalization for rough set model of incomplete and fuzzy decision information system, which is of course generalization for rough set model of incomplete information system and that of complete fuzzy decision information system. Knowledge reduction and its algorithm ascertaining precision degree unchanged are developed.

REFERENCES

[1] M. Kryszkiewicz. “Rough set approach to incomplete information systems,” Information Sciences, vol. 112, pp. 39-49, 1998.

[2] M. Kryszkiewicz. “Rules in incomplete information systems,” Information Sciences, vol. 113, pp. 271-292, 1999.

[3] R. Slowinski and D. Vanderpooten. “A Generalized Definition of Rough Approximation Based on Similarity,” IEEE TRANSACTIONS ON KNOWLEDGE AND DATA ENGIERING, vol. 12. pp. 331-336, March/April 2000.

[4] G. Y. Wang. Rough Set Theory and Knowledge Acquisition. Xi'an: Xi'an JiaoTong University Press, 2001.

[5] G. Y. Wang. “Extension of Rough Set under Incomplete Information systems,” Journal of Computer Research and Development, vol.39, no. 10, pp. 1238-1243, Oct. 2002.

[6] W. Z. Wu, J. S. Mi and W. X. Zhang. “A New Rough Set Approach to Knowledge Discovery in Incomplete Information System,” Proceedings of the Second International Conference on Machine Learning and Cybernetics, Xi’an, pp. 1713-1718, 2-5 November 2003.

[7] B. Huang, D. K. Wei and X. Z. Zhou. “An algorithm for Maximum Distribution Reductions and Maximum Distribution Rules in Information system,” Computer science, vol. 31, no. 10A, pp. 80-83, 2004.

[8] B. Huang and X. Z. Zhou. “Extension of Rough Set Model Based on Connection Degree under Incomplete Information System,” Systems Engineering --Theory and Practice, no. 1, pp. 88-92, 2004.

[9] D. K. Wei, X. Z. Zhou and Y. G. Zhu. “Knowledge Reduction in Incomplete Information System Based on Improved-Tolerance Relation,” Computer Science, vol. 32, no. 8.A, pp. 53-56, 2005.

[10] D. K. We and X. Z. Zhou. “Rough Set Model in Incomplete and Fuzzy Decision Information System Based on Improved-Tolerance Relation,” 2005 IEEE INTERNATIONAL CONFERENCE ON GRANULAR COMPUTERING, Tsinghua University, China, pp. 278~283, July 2005.

[11] D. K. Wei, B. Huang and X. Z Zhou. “Rough Set Model and Knowledge Reduction in Incomplete and Fuzzy Objective Information system,” Computer Engineering, no.7, 2006. (to be published)

[12] D. K. Wei, and X. Z Zhou. “A Rough Set Approach to Incomplete and fuzzy decision information system,” 2006 IEEE the 6th World Congress on Intelligent Control Automation, Dalian, China, July 21-23, 2005. (to be published)

[13] H.Y. Zhang, and J.Y. Liang. “Variable Precision Rough Set Model and a Knowledge Reduction Algorithm for Incomplete Information System,” Computer Science, vol.30, no. 4, pp: 153-155, 2003.

Da-kuan Wei is a Ph.D. student in the School of Automation, Nanjing University of Science and Technology, and also an associate professor with the School of Information Engineering, Hunan University of Science and Engineering, and the senior member of CCF (China Computer Federation). His research interests mainly include rough set theory, intelligent information processing etc. He has published around 10 peer-refereed articles in these areas.

Xian-zhong Zhou is a professor, Ph.D. supervisor with the School of Engineering and Management, Nanjing University, and also the senior member of CIE (Chinese Institute of Electronics). His current research interests mainly include intelligent information processing, information system theory and technology, rough set theory etc.

Dong-jun Xin is a Ph.D. student in the School of Automation, Nanjing University of Science and Technology. His current research interests include rough set theory and applications.

Zhi-wei Chen is a Ph.D. student in the School of Automation, Nanjing University of Science and Technology. His current research interests include rough set theory and applications.